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A praxeological analysis of the concept of slope in mathematics textbooks within the framework of the anthropological theory of the didactic

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ABSTRACT

This study aims to examine how the concept of slope is structured in middle school and secondary school mathematics textbooks on the basis of the praxeological approach within the framework of the Anthropological Theory of the Didactic (ATD). Designed as a qualitative study, ecological and praxeological analyses were conducted on nationally used mathematics textbooks through the method of document analysis. As a result of the analyses, three fundamental mathematical organizations related to the concept of slope and a total of 20 task types within these organizations were identified. These organizations were classified as determining and interpreting slope, constructing the equation of a line using slope, and interpreting slope depending on the position of the line. The findings indicate that as grade levels increase, there is a dominant praxeological progression in the teaching of slope from concrete and visually based tasks toward more abstract and analytical approaches. The results provide didactic implications regarding how the praxeological structures presented in textbooks shape students' access to the concept of slope and reveal important consequences for textbook design and instructional decision-making in mathematics education.

Keywords: anthropological theory of the didactic, praxeological analysis, slope, mathematics textbook

INTRODUCTION

In an era in which social change and development are increasingly accelerating, the rapid changes experienced require us to re-examine and reshape our perspective on mathematics, our expectations from mathematics education, the ways in which we use mathematics, and, more importantly, the processes of learning and teaching mathematics (Ministry of National Education [MoNE], 2018a, 2018b). Continuous and inevitable change, together with innovations in science and technology, affects our lives and reveals the necessity of making changes in existing situations (Ersoy, 2003). Along with technological developments and the emergence of new knowledge, opportunities, and tools, the profile of students needed has also changed into individuals who value mathematics, possess developed mathematical thinking skills, have abilities of discussion and reasoning, and are able to use mathematics in modeling and problem solving (Altun, 2006; MoNE, 2018b). For the development of such individuals, instructional environments need to be organized and their efficiency needs to be increased. Educational materials are an important component of instructional environments and are expected to be used effectively by teachers (İler et al., 2025). One of the most important educational materials is textbooks. Textbooks, which are both the means and the target of change in education, occupy an important place in shaping the learning process (Rezat et al., 2021).

Textbooks play a crucial role in shaping the learning process and in ensuring students' active participation. They also play a role in enabling students' active involvement in the learning process (Kuncoro et al., 2024). Mathematics textbooks are an important educational resource for both teachers and students in learning processes and serve as a tool that guides students toward curricular goals (İler et al., 2025; Yunianta et al., 2023). In order to take an effective place in learning processes, textbook contents need to be organized so as to include problems and exercises that support higher-order learning skills such as problem solving, reasoning, and application. The contents of textbooks should be enriched with subject explanations, sample questions, exercises, and applications in a way that ensures conceptual learning (İler et al., 2025; Yunianta et al., 2023). Analyzing textbook content has an important role in determining the contribution of textbooks to levels of conceptual learning (İler et al., 2025).

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Didactics is a field of study developed to examine, analyze, and understand how learning functions; that is, it aims to make learning optimal in a teaching or educational situation (Sağlam Arslan, 2008; Therer, 1992). Since the late 1970s, various theories have been developed in France along with developments in the didactics of mathematics. One of these theories is the Anthropological Theory of the Didactic (ATD) developed by Yves Chevallard (Sağlam Arslan, 2008). ATD provides a theoretical framework for textbook analysis.

ATD posits that mathematical knowledge should be examined not only in the context of an individual's cognitive processes but also within the framework of how it is produced, transformed, and transmitted within specific institutional structures (Chevallard, 1991). In this regard, the foundations of the theory are based on the concept of didactic transposition, which explains the transformation of scientific knowledge into knowledge to be taught, and it particularly analyzes the processes of restructuring knowledge in school mathematics.

Praxeology, which constitutes the fundamental unit of analysis in ATD, enables the examination of mathematical activities through four components: tasks (T), techniques (τ), the explanations that justify these techniques (technology, θ), and the theoretical structure upon which these explanations are based (theory, Θ) (Chevallard, 1991). This framework reveals not only how mathematical knowledge is applied but also the underlying justifications of these applications, thereby making the epistemological dimension of the teaching process visible. In this context, praxeological analysis is considered an effective tool for examining the structure, depth, and conceptual coherence of mathematical content presented in textbooks (Winsløw, 2011).

ATD also focuses on the institution–knowledge relationship in order to explain how mathematical knowledge is positioned within an institutional context. For a piece of knowledge to be considered institutionally defined, it must meet the conditions for its existence within an institution. In this context, the purposes for which a mathematical concept is taught, the types of tasks through which it is presented, and the techniques that are emphasized reflect the epistemological preferences of the relevant institution (Sağlam Arslan, 2008). Textbooks, in turn, can be considered concrete representations of this institutional knowledge; therefore, examining the presentation of mathematical concepts in textbooks provides significant insights into the teaching process.

In recent years, studies conducted in the field of mathematics education have demonstrated that ATD provides an effective theoretical framework, particularly for the analysis of textbooks and the examination of the teaching processes of mathematical concepts (Agustito et al., 2025; Akar, 2018; Aydoğan, 2025; Hendriyanto et al., 2023; Putra, 2017; Tesfamicael & Lundeby, 2019; Winsløw, 2011; Yunianta et al., 2023). These studies make it possible to analyze how mathematical concepts are structured across different grade levels and how this structuring exhibits coherence in terms of didactic progression. The study conducted by Hendriyanto et al. (2023) demonstrates that analyzing mathematical concepts presented in textbooks through praxeological components (T, τ, θ, Θ) is an important tool for revealing the nature of learning opportunities. The study conducted by Agustito et al. (2025) demonstrates that the mathematical tasks presented in textbooks are largely based on procedural techniques and that conceptual explanations are limited. This situation emphasizes that students tend to learn at a procedural level and that this may lead to epistemological and didactic learning obstacles by restricting the development of deep conceptual understanding. These findings indicate that the mathematical organizations presented in textbooks should not be limited solely to the technical dimension, but should also be addressed together with the components of technology and theory that give meaning to these techniques.

When the existing literature is examined, it is observed that specific concepts presented in mathematics textbooks are mostly addressed through particular grade levels, and that holistic studies revealing how these concepts are structured across grade levels are limited (Agustito et al., 2025; Akar, 2018; Hendriyanto et al., 2023; Putra, 2017; Tesfamicael & Lundeby, 2019). When studies on the concept of slope are examined, it is seen that they predominantly focus on students' conceptual understandings, representations, and teacher knowledge (Cho & Nagle, 2017; Moore-Russo et al., 2011; Nagle et al., 2013; Stump, 1999). Although these studies reveal the difficulties encountered in learning the concept of slope and the associated conceptual limitations, they do not sufficiently demonstrate how the concept is structured across different grade levels within the context of textbooks and through which praxeological components this structuring is realized. In this regard, the present study aims to fill this gap in the literature and examines holistically, within the framework of ATD, how the concept of slope is structured from middle school to upper secondary education. By systematically analyzing the components of technique, technology, and theory accompanying the task types related to the concept of slope, it aims to reveal how the concept is structured within a mathematical organization across different grade levels. Additionally, the systematic construction of the praxeological structure related to the concept of slope provides a theoretical framework for understanding the instructional processes of mathematical concepts.

Purpose of the Study

The purpose of this study is to reveal how the concept of slope presented in middle school and secondary school mathematics textbooks is structured praxeologically within the ATD framework and to examine what kind of didactic progression this structuring exhibits across grade levels. Accordingly, the study aims to analyze, from a holistic perspective, the task types related to the concept of slope and the accompanying components of technique, technology, and theory. Within this scope, the study seeks to answer the following question: "How is the concept of slope structured in middle school and secondary school mathematics textbooks in terms of its praxeological components (task, technique, technology, and theory), and what kind of didactic progression does this structuring display across grade levels?"

Significance of the Study

Textbooks directly influence teachers' instructional decisions and students' access to mathematical concepts as one of the fundamental components of mathematics teaching in different education systems. For this reason, examining through which mathematical meanings, which forms of representation, and which didactic choices mathematical concepts are presented in textbooks is important for evaluating and improving the quality of the instructional process.

This study aims to reveal how the concept of slope is addressed in mathematics textbooks used at the middle school and secondary school levels in Türkiye within the ATD framework and to identify with which praxeological components and which didactic structures this concept is presented in textbooks. ATD provides an epistemological framework of analysis that enables the examination of mathematical knowledge in instructional contexts; by considering knowledge together with its task–technique–technology–theory components, it allows the analysis of the instructional organization of mathematical content. One of the fundamental assumptions of the theory is that any mathematical knowledge can be investigated praxeologically (Putra, 2017).

In this context, determining how the concept of slope is structured praxeologically in textbooks allows not only the evaluation of textbook content in the national context but also the comparison of different instructional traditions and didactic approaches. The findings obtained from the study are expected to provide didactic implications for the teaching of the concept of slope in the contexts of textbook writing, instructional design, and teacher education, and to contribute to the development of more coherent and meaningful conceptual structuring in mathematics education. The study group consists of textbooks used in the context of Türkiye. The praxeological analysis approach based on the ATD adopted in the study offers a theoretical framework that can be applied in different countries and education systems in order to examine the ways in which mathematical concepts are structured in textbooks. In this respect, the findings obtained from the study group have the potential to produce didactic implications related to the teaching of the concept of slope that are not limited only to the local context but are also comparable at the international level.

One of the significant contributions of this study is that the analyses conducted within the ATD framework are not limited to the context of Türkiye but are also applicable across different educational systems. ATD provides a universal theoretical framework that enables the examination of how mathematical knowledge is structured within institutional contexts. In this respect, the praxeological structures and mathematical organizations presented in the study establish an analytical basis that can be comparatively examined with textbooks and curricula used in different countries. This, in turn, allows the study to go beyond being merely a national-level textbook analysis and to produce generalizable didactic implications that can contribute to international comparisons in the field of mathematics education.

METHOD

Research Design

This study was designed within the scope of the qualitative research approach and is based on the document analysis method in order to conduct an ecological and praxeological analysis of the concept of slope in middle school and secondary school mathematics textbooks within the framework of the ATD. Document analysis is a method that allows the systematic analysis of written and visual materials containing information about the phenomenon under investigation (Yıldırım & Şimşek, 2013).

During the document analysis process, the contents related to the concept of slope in the textbooks were first examined contextually through ecological analysis; subsequently, within the scope of praxeological analysis, the task types related to the concept of slope were identified and the accompanying components of technique, technology, and theory were systematically analyzed. As a result of these analyses, mathematical organizations related to the concept of slope were constructed, and the structural features of these organizations across grade levels were comparatively examined.

Study Group

In this qualitative study, a study group was formed in order to examine in depth the praxeological structure of the concept of slope in mathematics textbooks. The study group consists of the official mathematics textbooks used in mathematics education at the middle school and secondary school levels in Türkiye. In determining the study group, the criterion sampling approach, one of the purposive sampling methods, was used. Within this scope, the criteria for selecting the textbooks were determined as follows: (i) being approved by the Ministry of National Education and officially used, (ii) explicitly and directly including the concept of slope, (iii) covering textbooks belonging to different school types that reflect instructional diversity, and (iv) including content representing different curriculum periods.

Within this scope, the data sources of the study consist of the mathematics curricula and the nationally used Grade 8, 9, 10, 11, and 12 mathematics textbooks in Türkiye, in which the concept of slope is explicitly addressed in the curricula and reconstructed across different mathematical contexts. The textbooks were selected in order to comparatively examine how the concept of slope is presented at different grade levels. While the concept of slope is addressed within the context of linear relationships at the middle school level, it is reconstructed within different mathematical contexts such as analytic geometry and calculus at the secondary education level. Therefore, these grade levels provide a coherent framework for examining the didactic progression of the concept. In addition, Grade 9 and 10 mathematics textbooks prepared in line with the Türkiye Yüzyılı Maarif Modeli (TYMM), which has been implemented since 2024, were also included in the analysis as they reflect current instructional approaches.

In accordance with these criteria, a total of 13 mathematics textbooks were included in the study group. The study group consists of four Grade 8, four Grade 11, and three Grade 12 mathematics textbooks, as well as the Grade 9 and Grade 10 mathematics textbooks prepared in line with the Türkiye Yüzyılı Maarif Model (TYMM), which has been implemented since 2024. Detailed information on the grade levels and codes of the textbooks examined is presented in **Table 1**.

Table 1. The mathematics textbooks examined in the study

Code	Title of the Textbook
8M1	Middle School and Imam Hatip Middle School Mathematics Grade 8 Textbook (Erenkuş & Eren Savaşkan, 2019)
8M2	Middle School and Imam Hatip Middle School Mathematics Grade 8 Textbook (Böge & Akıllı, 2021)
8M3	Middle School and Imam Hatip Middle School Mathematics Grade 8 (Ersoy, 2025)
8M4	Middle School and Imam Hatip Middle School Mathematics Grade 8 Textbook (Altunkaynak et al., 2024)
11M1	Secondary School Mathematics Grade 11 Textbook (Altun, 2019)
11M2	Mathematics Grade 11 Skills-Based Activity Book (MoNE, 2022a)
11M3	Secondary School Science High School Mathematics Grade 11 Textbook (Öz et al., 2020)
11M4	Secondary School Mathematics Grade 11 Textbook (Seymen et al., 2021)
12M1	Mathematics Grade 12 Skills-Based Activity Book (MoNE, 2022b)
12M2	Secondary School Science High School Mathematics Grade 12 Textbook (Kemancı et al., 2021)
12M3	Secondary School Mathematics Grade 12 Textbook (Emin et al., 2021)
M9	Mathematics Grade 9 Textbook (Demirci et al., 2024)
M10	Secondary School Mathematics Grade 10 Textbook (Sökmez et al., 2025)

The textbooks included in the study group were selected in a way that would allow a comparative examination of the praxeological structures and the didactic progression that emerge in the instructional presentation of the concept of slope from middle school to the upper grades of secondary education. This diversity is functional in terms of revealing through which mathematical organizations the concept of slope is structured at different grade levels and what kind of instructional progression this structuring displays.

Data Collection Tools and Process

In this study, data were collected in line with the praxeological analysis approach developed within the ATD framework. As data sources, middle school and secondary school mathematics textbooks approved by the Ministry of National Education and used in formal education, as well as the relevant mathematics curricula, were used. In the data collection process, the textbooks were not treated as direct measurement instruments, but as documents that allow the examination of the instructional presentation of the concept of slope. Within this scope, the contents related to the concept of slope in the textbooks were systematically examined in accordance with the praxeological analysis framework based on the task–technique–technology–theory components of ATD. This framework was used in the study as a tool for data collection and data structuring. Within the scope of the study, a total of 13 mathematics textbooks that include the concept of slope and were determined through criterion sampling were examined. The textbooks examined cover Grades 8, 9, 10, 11, and 12 and were selected to represent different curriculum periods and school types. Detailed information about these textbooks is presented in **Table 1**.

Data Analysis

In this study, the data obtained through document analysis were analyzed within the framework of the ATD, drawing on both ecological and praxeological perspectives. The analysis was conducted in two stages in order to examine the instructional presentation of the concept of slope in textbooks in a holistic manner. First, the contents related to the concept of slope were examined contextually through ecological analysis. Then, a praxeological analysis was carried out to identify the task types and to systematically examine the associated components of technique, technology, and theory. As a result of these analyses, mathematical organizations related to the concept of slope were constructed, and their structural features across grade levels were examined comparatively.

Ecological analysis is defined by Chevallard (1991) as the questioning of the conditions and contexts necessary for a piece of knowledge to exist. In this approach, the instructional environment in which knowledge is located is explained by the concept of habitat, and its function within this environment is explained by the concept of niche (Akar, 2018). In this study, within the scope of ecological analysis, the positions of the concept of slope in textbooks and mathematics curricula were examined in terms of under which learning domains and topic headings it is presented, with which mathematical contexts it is associated, and which instructional purposes it serves. Accordingly, the topic headings, information boxes, examples, activities, and applications in which the concept of slope appears were identified and the habitats of slope were defined; the instructional functions attributed to the concept of slope within these habitats were analyzed within the framework of the concept of niche.

Praxeological analysis is a model developed by Chevallard (1992, 2006) that allows the epistemological examination of human activities. Within the Anthropological Theory of the Didactic, a praxeology is considered as a structure consisting of the components task type (T), technique (τ), technology (θ), and theory (Θ) (Akar, 2018). In this study, the praxeological analysis process was carried out through the systematic examination of the contents related to the concept of slope in Grade 8 middle school and Grade 9, 10, 11, and 12 secondary school mathematics textbooks and mathematics curricula.

Within this scope, first, all questions, examples, activities, and applications related to the concept of slope in the textbooks were identified and treated as units of analysis. By examining the identified units of analysis, task types (T) were defined within the scope of praxeology; the methods used in solving these tasks were identified within the framework of the technique (τ) component. Then, the mathematical justifications used to explain these techniques were evaluated within the scope of the technology (θ) component; the mathematical explanations and generalizations on which these technologies are based were analyzed within the framework of the theory (Θ) component. The tasks included in the textbooks were classified as information boxes, examples/questions/exercises, activities, information technology–supported applications, and application questions. Some tasks, although not presented directly as in-class activities, were included in the scope of analysis because they reflect the

Table 2. Learning outcomes related to the topics in which the concept of slope

Program	Grade	Learning Area/ Theme	Subdomain/ Context	Outcome Code	Treatment of the Concept of Slope
2018	8	Algebra	Linear Equations	M.8.2.2.6	Meaning of slope (sign, magnitude), relating with graphs and equations, modeling
2018	11	Analytic Geometry	Analytic Examination of the Line	11.2.1.3	Definition of slope, angle of inclination, line equations, positions of lines
2018	12	Derivative	Instantaneous Rate of Change	12.5.2.1	Relationship between derivative and tangent slope
2024 (TYMM)	Preparatory	Numbers, Quantities and Change	Linear Relationships	MAT.H.1.1	Use of mathematical tools and technologies in linear relationships
2024 (TYMM)	9	Mathematics of Change	Linear Functions	MAT.9.2.1	Relating through reference linear function and transformations
2024 (TYMM)	10	Analytic Examination	Lines and Coordinate System	MAT.10.5.2	Examination of slope and lines through the coordinate system
2024 (TYMM)	12	Change / Derivative	Instantaneous Change and Derivative	MAT.12.2.5	Instantaneous rate of change and the concept of derivative
2024 (TYMM)	12	Derivative	Function–Derivative Relationship	MAT.12.2.8	Relationships between function and derivative representations
2024 (TYMM)	12	Derivative	Real-life problems	MAT.12.2.9	Modeling and problem solving with derivative

curriculum and allow students to work individually. The praxeological structures obtained in this process were examined comparatively across grade levels.

In this study, in determining the instructional function (niche) of the concept of slope in textbooks, the perceptual, physical, operational, and algebraic classification proposed by Takeuchi and Shinno (2019) was adopted. This classification made it possible to systematically analyze which cognitive and mathematical processes are foregrounded by the tasks related to the concept of slope.

The examples presented in this study were selected not with the aim of covering all slope-related questions in the textbooks, but in a way that would representatively reflect each of the identified task types. In the selection of examples, the primary criterion was that they clearly reveal the mathematical organization and instructional function of the relevant task type. Accordingly, the examples were determined in a way that makes praxeological diversity visible rather than quantitative diversity.

In order to ensure the validity and reliability of the findings obtained in the study, the criteria of Lincoln and Guba (1985) were adopted. To increase the credibility of the study, the opinions of three field experts were obtained during the data analysis process. The experts were presented with the praxeological analysis framework used in the study, the identified task types, and the technique, technology, and theory components related to these tasks, and the theoretical appropriateness and consistency of the classifications were evaluated. In line with the feedback received from the experts, the necessary revisions were made in the analysis process and the classifications were clarified.

In order to ensure dependability, the ecological and praxeological analysis processes were structured systematically and as separate stages; the units of analysis, coding process, and classification criteria were defined in detail. In order to minimize the effect of researcher subjectivity, the findings obtained in the analysis process were supported by examples taken directly from the textbooks. In order to ensure transferability, the characteristics of the textbooks examined and the data analysis process were described in detail.

FINDINGS

Findings Obtained from Examining the Concept of Slope Through the Ecological Approach

In this section, the position (habitat) and function (niche) of the concept of slope in curricula and textbooks were examined using the ecological approach within the ATD framework. Within this scope, the learning domains, sub-learning domains, sections, and topics found in the documents in which the concept of slope appears were systematically analyzed.

Position of the concept of slope in the curricula (habitat)

When the Upper Secondary School Mathematics Curriculum adopted in 2018 (MoNE, 2018b) is examined, it is seen that the curriculum consists of three learning domains, namely “Numbers and Algebra”, “Geometry”, and “Data, Counting and Probability”. By considering the sub-learning domains of these learning domains according to grade levels, the position of the concept of slope within the curriculum was determined. Similarly, the Middle School Mathematics Curriculum (MoNE, 2018a) consists of five learning domains, namely “Numbers and Operations”, “Algebra”, “Geometry and Measurement”, “Data Processing”, and “Probability”, and the sub-learning domains in which the concept of slope is located were examined within this structure. When the 2018 curricula are examined, it was determined that the concept of slope is included in the learning outcomes of Grades 8, 11, and 12. Information related to these learning outcomes is presented in **Table 2**.

In the 2018 Mathematics Curriculum, the concept of slope is addressed at the middle school and secondary school levels within different learning domains and learning outcomes. The concept of slope first appears at the middle school level in Grade 8 under the sub-learning domain “Linear Equations” within the learning domain “Algebra”. In learning outcome M.8.2.2.6., students are expected to explain the slope of a line through models and to relate linear equations and graphs to slope. While it is stated that emphasis should be placed on the meaning of the sign and magnitude of slope, it is emphasized that in real-life related modeling, the sign of slope is not emphasized, considering slope as the ratio of vertical length to horizontal length. At the secondary school level, in Grade 11, within the sub-learning domain “Analytical Examination of the Line” under the learning domain “Analytic Geometry”, learning outcome 11.2.1.3. aims for students to examine lines on the analytic plane and perform operations. Defining the angle of inclination and the slope of a line, constructing the equation of a line on the analytic plane, finding the equations of lines parallel to the axes and passing through the origin, and interpreting the graphs of these equations are expected. In addition, examining the relative positions of two lines and finding the intersection point of two intersecting lines are also included within this learning outcome. In Grade 12, the concept of slope is addressed under the learning domain “Derivative” within the sub-learning domain “Instantaneous Rate of Change and Derivative”. In learning outcome 12.5.2.1., students are expected to explain the concept of derivative and perform operations. Emphasis is placed on the relationship between the derivative value of a given function at a point and the slope of the tangent at that point.

With the Upper Secondary School Mathematics Curriculum (MoNE, 2024a) and the Middle School Mathematics Curriculum (MoNE, 2024b) prepared within the scope of the Türkiye Yüzyılı Maarif Model (TYMM), significant changes were made in the structure of the curricula and they were reorganized with a thematic approach. The TYMM Upper Secondary School Mathematics Curriculum is structured under themes such as “Numbers”, “Quantities and Changes”, “Mathematics of Change”, and “Analytical Examination”. In the TYMM Middle School Mathematics Curriculum, themes such as “Numbers and Quantities”, “Operational Algebraic Thinking and Changes”, “Geometric Shapes”, and “Geometric Quantities” are included. When these curricula are examined, it is seen that the concept of slope is included in the learning outcomes of the preparatory class and Grades 9, 10, and 12 in the TYMM Upper Secondary program, and in the learning outcomes of Grade 8 in the TYMM Middle School program.

In the 2024 Mathematics Curriculum (MoNE, 2024a), the concept of slope is addressed within the contexts of linear relationships, functions, the Cartesian coordinate system, rate of change, and derivative. At the preparatory class level, in learning outcome MAT.H.1.1., students are expected to be able to use mathematical tools and technologies in solving problems involving linear relationships. Within the scope of this learning outcome, they are required to recognize mathematical tools and technologies that can be used in solving problems involving linear relationships, to select appropriate ones, and to use these tools and technologies effectively. At the Grade 9 level, in learning outcome MAT.9.2.1., students are expected to be able to make mathematical reasoning about the qualitative properties of the linear reference function defined as $f(x) = x$ on the real numbers and about the qualitative properties of the linear functions $g(x) = a \cdot f(x \mp r) \mp k$ ($a, r, k \in R$, $a \neq 0$) derived from this function. Within this scope, students are expected to determine the relationships between the qualitative properties of the linear reference function and its mathematical representations; to obtain new linear functions through operations performed on graphical or algebraic representations and to express the relationships between these representations. It is aimed that students make conjectures about the qualitative properties of other linear functions based on the qualitative properties of the linear reference function. At the Grade 10 level, in learning outcome MAT.10.5.2., students are expected to be able to use the Cartesian coordinate system as an appropriate representational tool to examine the properties of lines and to solve problems related to lines. Within this scope, they are required to recognize the Cartesian coordinate system as a tool for determining the angle of inclination and slope of a line and the relative positions of lines, to select it as an appropriate representation in problem situations, and to use this representation to express these properties. At the Grade 12 level, in learning outcome MAT.12.2.5., students are expected to be able to reason about the rate of change of a function around a given point. Within this scope, it is required to identify the components related to the rate of change of a function around a given point, namely the qualitative properties of the function, its algebraic and graphical representations, and the relationships between these components. The specified rate of change is expressed as the instantaneous change of the function at that point through tables, graphs, and limit representations. This instantaneous rate of change also represents the derivative of the function at that point. Students are expected to make conjectures about the derivative of a function at a given point by examining the instantaneous changes of functions such as $f(x) = x^n$, $f(x) = x$, and $f(x) = 1$ under appropriate conditions. In learning outcome MAT.12.2.8., students are expected to be able to make inferences about the mathematical representations of a function and its derivative function and the relationships between them. Within this scope, using reference functions, the limit definition of derivative, the rules of differentiation, and properties related to functions, students are expected to make conjectures about the mathematical representations of a function and its derivative function and the relationships between them, to generalize these conjectures, to compare generalizations with conjectures, and to present propositions based on the obtained generalizations. In addition, they are expected to use propositions about the mathematical representations of a function and its derivative function and the relationships between them in examining and obtaining the graphical representation of polynomial functions. In learning outcome MAT.12.2.9., it is aimed that real-life problems can be solved using derivatives. Within this scope, it is required to identify the mathematical components in problem situations where derivative knowledge can be applied, to examine the relationships between these components in the context of derivatives, to transform the context of change in the problem into function, equation, and derivative representations, and to express the meanings of these representations in the context of the problem.

Within the scope of the 2024 curricula, it is seen that the concept of slope is addressed at different grade levels within the contexts of linear relationships, functional representations, and rate of change. In the TYMM Upper Secondary School Mathematics Curriculum, the concept of slope is structured mostly through linear reference functions and the relationships between the graphical and algebraic representations of these functions at the preparatory class and Grades 9 and 10. In contrast, at the Grade 12 level, the concept of slope is addressed in the context of average and instantaneous rate of change and is used in a way that supports the transition to the concept of derivative. In the TYMM Middle School Mathematics Curriculum, the concept of slope is addressed in the Grade 8 learning outcomes in the context of modeling and interpreting linear relationships. These findings reveal

Table 3. Niche types of the concept of slope (Takeuchi & Shinno, 2020)

Niche Type	Definition
Perceptual	Intuitive and visual interpretations based on the appearance of graphs and models
Physical	Operations carried out using rulers, protractors, coordinate systems, and technological tools
Procedural	Operations based on numerical calculations and procedures
Algebraic	Operations carried out using derivatives, algebraic expressions, and analytical methods

that the concept of slope in the curricula exhibits increasing conceptual depth and contextual diversity as the grade level progresses.

After determining at which grade levels and under which themes the concept of slope is addressed in the curricula, the function (niche) with which this concept is structured in textbooks was examined. In this direction, the instructional function of the concept of slope was analyzed through the activities and questions included in the textbooks.

Findings regarding the function (niche) of the concept of slope

The function (niche) of the concept of slope in textbooks was analyzed on the basis of the questions and activities included in the textbooks. In this analysis, the perceptual, physical, operational, and algebraic classification proposed by Takeuchi and Shinno (2020) and used in the praxeological analysis of mathematics textbooks was adopted. This classification is presented in **Table 3**.

According to this classification:

- Perceptual slope refers to intuitive and visual judgments based on the appearance of a given model or graph,
- Physical slope refers to evaluations made using tools such as rulers, protractors, squared paper, coordinate systems, or dynamic geometry software,
- Operational slope refers to procedures based on the numerical calculation of slope, carried out using physical tools for drawing and measurement such as mirrors, rulers, and compasses,
- Algebraic slope refers to analyses conducted using derivatives, trigonometry, algebraic expressions, and vectors.

At the Grade 8 level, the concept of slope is addressed with an emphasis on the meaning of its sign and magnitude and is structured within a perceptual niche, as it aims at students' intuitive and visual sense-making. In real-life related modeling, slope is addressed around an algebraic and operational niche as the ratio of vertical length to horizontal length. The use of information and communication technologies is also classified as a physical niche.

At the secondary school level in Grade 11, defining the angle of inclination and the slope of a line, constructing the equation of a line, examining the relative positions of two lines, and finding the intersection point of two intersecting lines are structured within an algebraic and operational niche. In addition, writing the equations of lines parallel to the axes and passing through the origin and interpreting the graphs of the obtained equations fall within the perceptual and physical niche. Applications related to the distance of a point from a line and the distance between two parallel lines emphasize the operational niche. The use of information and communication technologies also emphasizes the physical niche. In the context of functions, the calculation of the average rate of change of a function falls within the algebraic and operational niche, whereas drawing and interpreting function graphs with the help of information and communication technologies falls within the physical niche. At the Grade 12 level, the concept of slope is structured within the context of derivative, and emphasizing the relationship between the derivative value of a given function at a point and the slope of the tangent at that point indicates an algebraic and operational niche. The use of information and communication technologies also indicates a physical niche.

It is seen that the concept of slope is addressed with different functions (niches) at different grade levels in textbooks. At lower grade levels, the concept of slope is presented mostly within perceptual and physical niches and is made sense of through intuitive interpretations based on the appearance of graphs and through concrete tools. As the grade level increases, operational and algebraic niches become dominant, and especially in Grades 11 and 12 the concept of slope is addressed in the contexts of analytic geometry and derivative. These findings reveal that the concept of slope in textbooks exhibits a gradual instructional structure progressing from concrete to abstract and from intuitive approaches toward analytical structures.

Findings Obtained from Examining the Concept of Slope Through the Praxeological Approach

In the study, the praxeological approach within the ATD framework was used in order to reveal the mathematical organizations related to the concept of slope in middle school and secondary school mathematics textbooks. Accordingly, the examples, activities, and exercises in the main texts of the textbooks were examined; tasks related to the concept of slope were identified and classified under task types.

Praxis block (practice)

Identification of task types

In the first stage of the praxeological analysis, all questions related to the concept of slope in the textbooks were analyzed. In this analysis process, the learning outcomes related to the concept of slope and their sub-components were taken into account; the learning outcome structures available on the EBA platform were also used as a reference in identifying the tasks. As a result of the examination, a total of 63 tasks were identified, and by considering the similarities and differences among these tasks, 20 different task types were defined.

The identified task types were grouped under three fundamental mathematical organizations according to the instructional functions of the concept of slope. Accordingly, task types T1–T12 were classified as related to determining and interpreting slope,

Table 4. Task types for the concept of slope

Task Type (T)	Definition
T1	Understanding and making sense of slope
T2	Making estimations about the magnitude of slope using real-life–related models
T3	Calculating the magnitude of slope using real-life–related models; defining and calculating the slope of a line on the coordinate plane
T4	Explaining the relationship between the slope of a line and the rate of change in a linear relationship
T5	Making inferences about slope by using the graphs of linear equations
T6	Calculating the slope of a line based on a linear equation
T7	Explaining what the magnitude of slope represents and its effect on the graph of a line
T8	Determining the sign of slope based on the measure of the slope angle
T9	Finding the slope of a line given the angle it makes with the x-axis
T10	Constructing and calculating the expression that gives the slope of a line given two points
T11	Identifying that the slopes of lines passing through any two of three points are equal for the points to be collinear
T12	Interpreting the relationship between the derivative value of a function at a point and the slope of the tangent at that point, and finding the slope of the tangent at a point on the graph using derivatives
T13	Constructing and writing the equation of a line given its slope and a point on the line
T14	Constructing and writing the equation of a line given two points on the line
T15	Constructing and writing the equations of lines parallel to the coordinate axes
T16	Constructing and writing the equation of a line given its intercepts with the coordinate axes
T17	Identifying and writing the equations of lines passing through the origin
T18	Constructing and writing the equation of the tangent line drawn at a point on a function
T19	Writing the equation of a line parallel or perpendicular to a given line
T20	Determining the relative positions of lines given by equations; establishing relationships among the slopes of parallel, perpendicular, and coincident lines

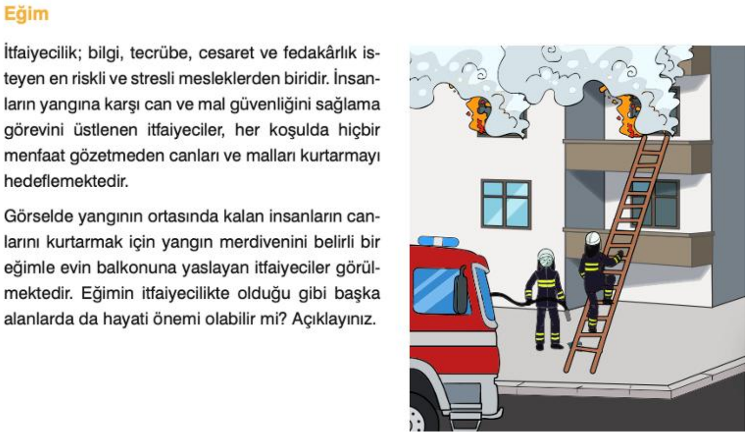

Figure 1. Example for Task Types T1 and T2 (Ersoy, 2025)

task types T13–T19 as related to constructing and writing the equation of a line using slope, and task type T20 as related to interpreting slope depending on the positions of lines. These task types and their definitions are presented in **Table 4**.

When **Table 4** is examined, it is seen that the concept of slope is addressed in textbooks within a multidimensional instructional structure through different task types. The distribution of task types indicates that the concept of slope is taught not only as a numerical ratio but also by being associated with graphical, algebraic, and geometric representations. This reveals that the teaching of slope in textbooks is structured in a way that activates different cognitive processes.

Examples related to task types

In order to show how the identified task types are concretized in textbooks, examples related to each task type are presented below. The examples presented were selected not to cover all slope-related questions in the textbooks, but to representatively reflect the identified task types and praxeological diversity. An example visual including task types T1 and T2 is presented in **Figure 1**.

In the example presented in **Figure 1**, the concept of slope is addressed through a real-life context, namely firefighting. The student is expected to make an intuitive inference about slope based on the appearance of the given visual. This example corresponds to task types T1 and T2, which are aimed at making sense of and interpreting the concept of slope. An example visual including task types T9 and T13 is presented in **Figure 2**.

In the example presented in **Figure 2**, the equation of a line is expected to be written based on the coordinates of a point on the line and the slope angle. The student is required to convert the slope angle into a slope value using trigonometric relationships and then to write the equation of the line by using this slope together with the given point information. In this context, the task is structured around calculating the slope from the slope angle and using this information in the process of constructing the equation of a line, and thus represents task types T9 and T13. An example visual including task type T12 is presented in **Figure 3**.

30. Örnek

A(−2, 1) noktasından geçen ve eğim açısı 135° olan doğrunun denklemini bulunuz.

Çözüm

Eğim açısı 135° olduğuna göre eğim $m = \tan 135° = -1$ olur.

$m = -1$ ve A(−2, 1) değerleri $y - y_1 = m \cdot (x - x_1)$ denkleminde yerine yazıldığında

doğrunun denklemi $y - 1 = -1 \cdot (x - (-2)) \Rightarrow y - 1 = -(x + 2)$

$\Rightarrow y = -x - 1$ olarak bulunur.

Figure 2. Example for Task Types T9 and T13 (Seymen et al., 2021)

ÖRNEK

$f(x) = \dfrac{x^3}{\sqrt{x}}$ fonksiyonunun grafiğine $x = 4$ apsisli noktasında çizilen teğetinin eğimini bulunuz.

ÇÖZÜM

f fonksiyonunun grafiğine $x = 4$ apsisli noktasında çizilen teğetinin eğimi $f'(4)$ olur. Buna göre

$f(x) = \dfrac{x^3}{\sqrt{x}} = \dfrac{x^3}{x^{\frac{1}{2}}} = x^{3-\frac{1}{2}} = x^{\frac{5}{2}}$

$f(x) = x^{\frac{5}{2}} \Rightarrow f'(x) = \dfrac{5}{2} \cdot x^{\frac{5}{2}-1} = \dfrac{5}{2} \cdot x^{\frac{3}{2}} = \dfrac{5x\sqrt{x}}{2}$ olur.

$f'(x) = \dfrac{5x\sqrt{x}}{2} \Rightarrow f'(4) = \dfrac{5 \cdot 4 \cdot \sqrt{4}}{2} = 20$ bulunur.

Figure 3. Example for Task Type T12 (Emin et al., 2021)

8. ÖRNEK

Analitik düzlemde $A(-3, 0)$ ve $B(0, -7)$ noktalarından geçen doğrunun denklemini bulunuz.

ÇÖZÜM

$A(-3, 0)$ noktası x ekseni, $B(0, -7)$ noktası y ekseni üzerindedir.

A ve B noktalarından geçen doğrunun eğimi $m_{AB} = \dfrac{-7-0}{0-(-3)} = -\dfrac{7}{3}$ olur.

Eğimi $m = -\dfrac{7}{3}$ olan ve $A(-3, 0)$ noktasından geçen doğrunun denklemi

$y - y_1 = m \cdot (x - x_1)$ eşitliği kullanılarak

$y - 0 = -\dfrac{7}{3} \cdot (x - (-3)) \Rightarrow y = -\dfrac{7}{3} \cdot (x + 3)$

$3y + 7x = -21$ bulunur.

$3y + 7x = -21$ denkleminde bütün terimler -21 ile bölünürse

$\dfrac{7x}{-21} + \dfrac{3y}{-21} = \dfrac{-21}{-21} \Rightarrow \dfrac{x}{-3} + \dfrac{y}{-7} = 1$ elde edilir.

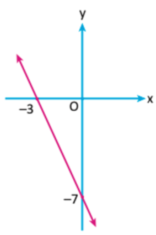

Figure 4. Example for Task Types T14 and T16 (Öz et al., 2020)

In the example presented in **Figure 3**, the slope of the tangent drawn to the graph of a function at a specific point is required to be determined. The student is expected to calculate the slope of the tangent by using the concept of derivative and to interpret this slope in a way that represents the local rate of change of the function. In this respect, the task corresponds to task type T12, which addresses slope within the context of instantaneous rate of change and requires the use of derivative knowledge. An example visual including task types T14 and T16 is presented in **Figure 4**.

In **Figure 4**, the construction of the equation of a line passing through two points with given coordinates in the analytic plane is required. The student is expected to first calculate the slope of the line using the coordinates of the two given points and then to use this slope value in the process of writing the equation of the line by means of the point–slope form. In this context, the task represents task type T14, which focuses on finding the slope using information from two points, and task type T16, which involves writing the equation of a line using this slope information. An example visual including task type T15 is presented in **Figure 5**.

In **Figure 5**, the construction of the equation of a line that has a given point in the analytic plane and does not intersect the x-axis is required. The student is expected to recognize, based on the given point information, that the line is parallel to the x-axis and accordingly to express the equation of the line in the form *y = constant*. In this context, the task focuses on determining the equations of lines parallel to the coordinate axes and represents task type T15. An example visual including task type T17 is presented in **Figure 6**.

Örnek

$A(-2, 4)$ noktasından geçen ve x ekseni ile ortak noktası olmayan doğrunun denklemini bulalım.

Çözüm

Doğrunun x ekseni ile ortak noktası olmadığından x eksenine paralel olur. Bu durumda doğrunun denklemi $y = 4$ tür.

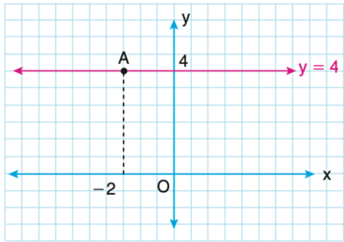

Figure 5. Example for Task Type T15 (Altun, 2019)

10. ÖRNEK

a) Eğimi $-\frac{4}{7}$ olan ve orijinden geçen doğrunun denklemini bulunuz.

b) Orijinden geçen ve x ekseniyle pozitif yönde 60° lik açı yapan doğrunun denklemini bulunuz.

ÇÖZÜM

a) Eğimi m olan ve $O(0,0)$ noktasından geçen doğrunun denklemi $y = mx$ olduğundan eğimi $-\frac{4}{7}$ olan ve orijinden geçen doğrunun denklemi $y = -\frac{4}{7}x$ olur.

b) $(0,0)$ noktasından geçen doğrunun eğim açısı 60° ise eğimi $m = \tan 60^\circ = \sqrt{3}$ olur. Buna göre doğrunun denklemi $y = \sqrt{3}x$ olur.

Figure 6. Example for Task Type T17 (Öz et al., 2020)

ÖRNEK

$f(x) = x^2 - ax - 2$ eğrisine $x = 2$ apsisli noktasında çizilen teğeti, x ekseni ile pozitif yönde 135° açı yaptığına göre teğet doğrusunun denklemini bulunuz.

ÇÖZÜM

f fonksiyonuna $x = 2$ apsisli noktasında çizilen teğetin eğimi m_t ise $m_t = \tan 135^\circ = f'(2)$ olur.

$f(x) = x^2 - ax - 2 \Rightarrow f'(x) = 2x - a$

$f'(2) = \tan 135^\circ \Rightarrow 4 - a = -1$

$\Rightarrow a = 5$ olur.

$a = 5 \Rightarrow f(x) = x^2 - 5x - 2$

$\Rightarrow f(2) = -8$ olur.

O hâlde teğet doğrusu $(2, -8)$ noktasından geçmektedir.

$A(x_0, y_0)$ noktasından geçen ve eğimi m olan doğrunun denklemi $\mathbf{y - y_0 = m(x - x_0)}$ dir.

Buna göre $A(2, -8)$ noktasından geçen ve eğimi -1 olan teğet doğrusunun denklemi

$y - (-8) = -1 \cdot (x - 2) \Rightarrow y = -x - 6$ bulunur.

Figure 7. Example for Task Type T18 (Emin et al., 2021)

In **Figure 6**, the construction of the equations of lines passing through the origin in two different situations is required. In the first situation, the student is expected to write the equation of a line passing through the origin with a given slope by using the appropriate form. In the second situation, the slope of the line with a given slope angle is required to be determined using the relevant relationship, and the equation of the line passing through the origin is then constructed using the obtained slope value. In this context, the example represents task type T17, which focuses on establishing the relationship between slope and equation for lines passing through the origin. An example visual including task type T18 is presented in **Figure 7**.

In the example presented in **Figure 7**, it is required to determine the slope of the tangent drawn at a specific point on a function and to construct the equation of the tangent line using this slope information. The student is expected to determine the slope of the tangent using the given slope angle through the relevant relationship, and then to write the equation of the tangent line by relating this slope to the derivative value of the function. In this context, the task represents task type T18, which relates the concept of slope to derivatives and tangent lines and addresses it within the context of instantaneous rate of change. An example visual including task type T19 is presented in **Figure 8**.

In the example presented in **Figure 8**, the construction of the equation of a line that is perpendicular to a given line with a known slope and passes through a given point is required. The student is expected to determine the slope of the new line by using the relationship that the product of the slopes of perpendicular lines is −1, based on the slope of the given line, and then to write the equation of the line using this slope together with the given point information. In this context, the task represents task type T19, which focuses on interpreting the relative positions of lines through the concept of slope and constructing the equation of a line accordingly. An example visual including task type T20 is presented in **Figure 9**.

Örnek

$2x+4y-3=0$ doğrusuna dik olan ve $A(3,-1)$ noktasından geçen doğrunun denklemini bulalım.

Çözüm

$2x+4y-3=0$ doğrusunun eğimi, $m_1=\frac{-2}{4}=\frac{-1}{2}$ dir. Bu doğruya dik olan doğrunun eğimi m_2 ise $m_1 \cdot m_2=-1$,

$\frac{-1}{2} \cdot m_2=-1$ ise $m_2=2$ olur. Buna göre $A(3,-1)$ noktasından geçen ve eğimi $m_2=2$ olan doğrunun denklemi,

$y-y_1=m_2 \cdot (x-x_1)$

$y-(-1)=2 \cdot (x-3)$ ise $y+1=2x-6$

$-2x+y+7=0$ olur.

Figure 8. Example for Task Type T19 (Altun, 2019)

49. Örnek

$ax+3y+1=0$ ve $6x+by+3=0$ doğruları çakışık olduğuna göre $a+b$ değerini bulunuz.

Çözüm

İki doğru çakışık olduğundan $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ olur. Buradan

$\frac{a}{6}=\frac{1}{3} \Rightarrow a=2$ ve $\frac{3}{b}=\frac{1}{3} \Rightarrow b=9$ olur.

O hâlde $a+b=2+9=11$ olur.

Figure 9. Example for Task Type T20 (Seymen et al., 2021)

In the example presented in **Figure 9**, the case of two lines given by algebraic equations being coincident is examined. The student is expected to determine the relative positions of the lines by using the information that the ratios of the coefficients must be equal for the lines to be coincident, and to solve the relationship among the given parameters accordingly. In this context, the task focuses on interpreting whether lines are parallel, perpendicular, or coincident through slope and coefficient relationships, and represents task type T20, which involves interpreting slope depending on the relative positions of lines.

In order to systematically analyze the task types related to the concept of slope, three fundamental mathematical organizations were established. These organizations were identified as determining and interpreting slope, constructing/writing the equation of a line using slope, and interpreting slope according to the position of a line. As a result of this classification, it was determined that task types T1–T12 focus on determining and interpreting slope, task types T13–T19 focus on using slope information in the process of constructing the equation of a line, and task type T20 focuses on interpreting the relative positions of lines through the concept of slope. This classification demonstrates that the concept of slope is structured in textbooks in ways that serve different cognitive and mathematical purposes.

Techniques

The techniques used in carrying out the task types reveal how the concept of slope is addressed in textbooks and which mathematical processes are foregrounded. In this context, the identified techniques include solution approaches with perceptual, physical, operational, and algebraic characteristics. The distribution of the techniques associated with the task types is presented in **Table 5**.

An examination of **Table 5** indicates that procedural and algebraic techniques are dominant in the majority of tasks related to the concept of slope. In particular, in the task types ranging from T3 to T12, procedural approaches such as calculating slope as the ratio of vertical to horizontal change, determining slope through coefficients in linear equations, finding slope using the tangent of the slope angle, and interpreting slope via the derivative of a function at a given point are foregrounded. This indicates that numerical computation and algebraic representation play a central role in the teaching of the concept of slope.

It is observed that perceptual and physical techniques are used primarily in task types T1 and T2, that is, in situations where the concept of slope is introduced for the first time or is associated with real-life contexts. In these tasks, the aim is to explain slope through intuitive notions such as steepness, inclination, and slope, and to arrive at visual judgments about slope through visuals, models, or dynamic software. However, it is noteworthy that these techniques are not systematically sustained at subsequent grade levels and are largely replaced by procedural techniques.

It is observed that in the process of constructing and writing the equation of a line using the concept of slope (T13–T19), the techniques are predominantly based on algebraic representations. In these tasks, operations such as writing the equation of a line passing through a given point with a known slope, constructing the equation of a line passing through two points, distinguishing lines parallel to the axes, and determining the relative positions of lines through implicit equations come to the forefront. This indicates that, at this stage, the concept of slope is used as a tool concept and gains its primary function within the context of analytic geometry.

Table 5. Praxis block for the concept of slope in textbooks

Task Type (T)	Technique (τ)
T1	τ1. Explaining and making sense of slope in terms of physical characteristics using everyday expressions such as steepness, inclination, and slope; making visual judgments about slope based on the appearance of the given model by making estimations (T1–T2).
T2	τ2. Making a physical judgment about the slope of a model by using tools such as a ruler, protractor, dotted or squared paper, a coordinate system, or dynamic geometry software, based on the given model (T1–T2).
T3	τ3. Calculating slope by finding the ratio of the change in the y-coordinates to the change in the x-coordinates for two points taken on a model/line in the coordinate plane (i.e., determining m) (T2–T3–T4–T5–T6–T10–T11).
T4	τ4. Determining slope and interpreting its sign by calculating the ratio of the unit change in the dependent variable to the unit change in the independent variable for two variables that have a linear relationship (T3–T4–T5–T6).
T5	τ5. Calculating and interpreting slope by using dynamic geometry software or various virtual manipulatives and dynamic tools (T2–T3–T4–T5–T6).
T6	τ6. Calculating slope by finding the ratio of height to horizontal distance of a model/line (T3–T4–T6–T7).
T7	τ7. Using the coefficient of *x* to determine the slope of a line given in the form $y = mx + b$ (T5–T6–T11).
T8	τ8. Finding slope by calculating the tangent of the slope angle (T3–T4–T5).
T9	τ9. Determining which angle's tangent represents the inclination angle of a model or a line (T9).
T10	τ10. Determining the sign of the tangent of the slope angle according to the region of the unit circle in which the angle lies (T8).
T11	τ11. Finding the slope of the tangent to a function at a given point by using the derivative of the function at that point (T12).
T12	τ12. Writing the equation of a line with slope *m* passing through the point $A(x_1, y_1)$ in the form $y - y_1 = m(x - x_1)$ (T13–T19–T20).
T13	τ13. Writing the equation of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the form $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$(T14).
T14	τ14. Writing the equations of lines parallel to the x-axis in the form $y = b$ ($b \in \mathbb{R}$) (T15).
T15	τ15. Writing the equations of lines parallel to the y-axis in the form $x = a$ ($a \in \mathbb{R}$) (T15).
T16	τ16. Writing the equation of the line d that intersects the x-axis at the point A(a, 0) and the y-axis at the point B(0, b) in the form $\frac{x}{a} + \frac{y}{b} = 1$ (T16).
T17	τ17. Writing the equation of a line passing through the origin with slope m in the form $y = mx$ (T17).
T18	τ18. Calculating the slope by finding f′(x); writing the equation of the tangent line by using the slope and the point of tangency (T18).
T19	τ19. Using the facts that the product of the slopes of perpendicular lines is −1, the slopes of parallel lines are equal, and lines are coincident if their slopes and intercepts are the same (T19–T20).
T20	τ20. Using the information that, for two lines given by implicit equations, the lines are parallel if the ratios of the coefficients of x and y are equal while the ratios of the constant terms are different; the lines intersect at a point if the ratios of the coefficients of x and y are different; and the lines are coincident if the ratios of the coefficients of x and y and the constant terms are equal (T20).

Logos block (Theoretical)

Praxeological analysis aims to reveal both the expected task types (praxis) in the teaching of the concept of slope and the mathematical justifications on which these tasks are grounded (logos) (Kuncoro et al., 2024). In this context, the logos block makes visible not only how the techniques used in textbooks are applied, but also the technological and theoretical structures that explain why these techniques are valid.

In the solutions to questions related to the concept of slope, the insufficient emphasis on the theoretical foundations that support procedural techniques may lead students to experience difficulties in relating the concept of slope to more advanced mathematical topics. The limited presentation of the technology and theory components in textbooks may cause students to apply the techniques without adequately conceptualizing them and to struggle with transferring these techniques to new situations.

Findings in the Context of Technology (θ) and Theory (Θ)

The justification of the techniques used in carrying out the task types constitutes the logos block of the praxeological structure. Accordingly, the technological discourses that explain these techniques and the theoretical frameworks on which these discourses are based are presented holistically in **Table 6**.

An examination of **Table 6** shows that two main technological discourses are predominantly foregrounded in task types T1–T12. The first is θ_1, which involves the intuitive recognition of slope and its conceptualization through physical models, while the second is θ_2, which is based on calculating slope using mathematical tools such as rate of change, coefficients, tangent, and derivative. The joint use of these two technologies indicates that the teaching of the concept of slope follows an instructional trajectory that progresses from an intuitive level toward a formal, calculation-based structure.

In task types T13–T19, the dominant technological discourse is identified as θ_3. This technology is based on constructing the equation of a line passing through a point with a known slope by considering a variable point on the line. This approach directly relates the concept of slope to analytic geometry theory and positions slope as a fundamental parameter that enables the algebraic representation of linear relationships.

In task type T20, the technology θ_4 involves interpreting the relative positions of lines (intersecting, parallel, coincident) through relationships between slopes and constant terms. This technology shows that the concept of slope is no longer treated merely as a computed quantity but becomes a tool used to interpret geometric relationships.

Table 6. Mathematical organizations related to the concept of slope and praxeological components

Mathematical Organizations	Task Type (T)	Technique (τ)	Technology (θ)	Theory (Θ)
Finding and interpreting slope	T1. Understanding and conceptualizing slope T2. Making estimations about the magnitude of slope using real-life–related models T3. Calculating the magnitude of slope using real-life–related models; defining and calculating the slope of a line on the coordinate plane T4. Explaining the relationship between the slope of a line and the rate of change in a linear relationship T5. Drawing inferences about slope using the graphs of linear equations T6. Calculating the slope of a line based on its linear equation T7. Explaining what the magnitude of slope means and its effect on the graph of a line T8. Determining the sign of the slope based on the measure of the slope angle T9. Finding the slope of a line whose angle with the *x*-axis is given T10. Constructing and calculating the relation that gives the slope of a line given two points T11. Identifying that, for three points to be collinear, the slopes of the lines passing through any two of these points must be equal T12. Interpreting the relationship between the derivative value of a function at a given point and the slope of the tangent at that point, and finding the slope of the tangent at a point on the graph using the derivative	τ1. Explaining and conceptualizing slope in terms of physical characteristics using everyday-life expressions such as *steepness*, *inclination*, and *slope*, and making visual judgments about slope by estimating based on the appearance of the given model (T1–T2). τ2. Making physical judgments about the slope of a model by using tools such as a ruler, protractor, dotted or squared paper, the coordinate system, or dynamic geometry software, based on the given model (T1–T2). τ3. Calculating the slope by finding, for two points taken on a model/line in the coordinate system, the ratio of the change in the y-coordinates of the points to the change in their x-coordinates, that is $\frac{y_2-y_1}{x_2-x_1}$ (T2 - T3 - T4 - T5 - T6 - T10 - T11) τ4. Finding the slope and interpreting its sign by calculating the ratio of the unit change in the dependent variable to the unit change in the independent variable for two variables that are in a linear relationship (T3–T4–T5–T6). τ5. Calculating and interpreting slope using dynamic geometry software or other virtual manipulatives and dynamic digital tools (T2–T3–T4–T5–T6). τ6. Calculating the slope by finding the ratio of the height (vertical change) to the horizontal distance of a model or a line (T3–T4–T6–T7). τ7. Determining the slope of a line given in the form $y = mx + b$ by using the coefficient of x(T5–T6–T11). τ 8. Finding the slope by calculating the tangent of the slope angle (T3–T4–T5). τ9. Identifying which angle corresponds to the tangent representing the slope angle of a model or a line (T9). τ10. Determining the sign of the tangent of the slope angle based on the region of the unit circle in which the angle lies (T8). τ11. Finding the slope of the tangent to a function at a given point by using the derivative of the function at that point (T12).	Θ1: Intuitively recognizing the slope of a line or a model and relating this recognition to a more precisely calculated measure of slope, and physically explaining the magnitude of the slope. Θ2: Using the ratio of vertical length to horizontal length, the rate of change between two variables, the coefficient of *x* derived from the equation of the model, the tangent of the angle of inclination, and the derivative of the function at the required point in order to numerically calculate the slope of a line or a model.	Θ1: Trigonometry Θ2: Analytic Geometry Θ3: Geometric Interpretation of the Derivative
Constructing / Writing the equation of a line using slope	T13. Constructing and writing the equation of a line given its slope and a point on the line T14. Constructing and writing the equation of a line given two points T15. Constructing and writing the equations of lines parallel to the coordinate axes T16. Constructing and writing the equation of a line given the points where it intersects the axes T17. Identifying and writing the equations of lines passing through the origin T18. Constructing and writing the equation of the tangent line drawn at a point on a function T19. Writing the equation of a line that is parallel or perpendicular to a given line	τ12. Writing the equation of the line passing through the point $A(x_1, y_1)$with slope $m$in the form $y - y_1 = m(x - x_1)$ (T13–T19–T20). τ13. Writing the equation of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the form $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$(T14). τ14. Writing the equations of lines parallel to the *x*-axis in the form $y = b$ ($b \in \mathbb{R}$) (T15). τ15. Writing the equations of lines parallel to the *y*-axis in the form $x = a$ ($a \in \mathbb{R}$) (T15). τ16. Writing the equation of the line $d$that intersects the *x*-axis at the point $A(a, 0)$ and the *y*-axis at the point $B(0, b)$ in the form $\frac{x}{a} + \frac{y}{b} = 1$(T16). τ17. Writing the equation of a line passing through the origin with slope $m$in the form $y = mx$(T17). τ18. Calculating the slope by finding $f'(x)$; writing the equation of the tangent line using the slope and the point of tangency on the function (T18).	Θ3: Determining the equation of a line with slope m that passes through the point $A(x_1, y_1)$ by considering a variable point $P(x, y)$on the line.	Θ1: Trigonometry Θ2: Analytic Geometry Θ3: Geometric Interpretation of the Derivative
Interpreting slope according to the position of a line	T20. Determining the relative positions of lines given by their equations, and establishing relationships among the slopes of parallel, perpendicular, and coincident lines	τ19. Using the facts that the product of the slopes of two perpendicular lines is −1, that the slopes of parallel lines are equal, and that lines with equal slopes and identical intercepts with the axes are coincident (T19–T20). τ20. Using the fact that, for two lines given in implicit form, if the ratios of the coefficients of $x$and $y$are equal while the ratios of the constant terms are different, the lines are parallel; if the ratios of the coefficients of $x$and $y$are different, the lines intersect at a single point; and if the ratios of the coefficients of x, y, and the constant term are equal, the lines are coincident (T20).	Θ4: For two given lines in the plane, there are three possible cases: intersecting at a point, being parallel, or being coincident.	Θ1: Trigonometry Θ2: Analytic Geometry

These technological discourses are justified, respectively, by the theories of Trigonometry, Analytic Geometry, and the geometric interpretation of the derivative. However, when **Table 6** is evaluated as a whole, it is observed that the connections between technology and theory in textbooks are often presented implicitly, and explanations regarding why the techniques are valid remain limited. This situation suggests that students may experience difficulties in relating the concept of slope across different mathematical contexts.

Findings Obtained from Middle School and Upper Secondary Mathematics Textbooks

By examining the curricula, the middle school and upper secondary mathematics textbooks used in schools and containing the concept of slope were identified. In the textbooks reviewed, information boxes, worked examples, exercises, activities, and applications related to the concept of slope were identified, and the task types to which these contents correspond were determined. Accordingly, a table was constructed to show the distribution of task types related to the concept of slope across grade levels.

The findings indicate that task types related to the concept of slope are concentrated within certain ranges across grade levels. An examination of Grade 8 middle school textbooks shows that task types are predominantly clustered in the range of T1–T6. At this grade level, the concept of slope is addressed through dimensions of understanding, conceptualization, definition, and basic calculation. In particular, task types T3 and T5 are observed to be dominant. This suggests that, at the level where the concept of slope is first introduced, more concrete, visual, and real-life–based approaches are preferred.

An examination of upper secondary Grade 11 mathematics textbooks shows that task types are predominantly concentrated in the ranges T4, T6, and T13–T16. At this grade level, it can be stated that the concept of slope transitions from a geometrically grounded understanding to one based on analytic representations. At this stage, slope ceases to be merely a ratio or a visual characteristic and becomes a concept addressed within the context of line equations and the analytic plane. In Grade 12 mathematics textbooks, task types are found to be particularly concentrated around T12 and T18 (the relationship between derivative and tangent) and T19 and T20 (the relative positions of lines). At this level, the concept of slope is addressed within the context of differential calculus and is associated with instantaneous rate of change and the slope of the tangent through the concept of derivative. This indicates that slope is integrated with higher-level mathematical concepts. The Grade 9 and Grade 10 textbooks prepared in accordance with the Türkiye Yüzyılı Maarif Modeli (TYMM) Upper Secondary Mathematics Curriculum (MoNE, 2024a) and the Türkiye Yüzyılı Maarif Modeli (TYMM) Middle School Mathematics Curriculum (MoNE, 2024b) were examined under a separate heading. Accordingly, in the Grade 9 textbook, task types T_3, T_4, and T_5 are dominant. At this grade level, the concept of slope is addressed in relation to graphs, ratios, and the concept of change. In the Grade 10 textbook, task types T3, T5, T10, and T13 are observed more frequently. This finding indicates that, at the Grade 10 level, the concept of slope is approached in a more analytic and calculation-oriented manner and is used as a tool serving the algebraic representation of linear relationships.

DISCUSSION AND CONCLUSION

In this study, middle school and secondary school mathematics textbooks containing the concept of slope were examined within the framework of the Anthropological Theory of the Didactic (ATD) using ecological and praxeological analysis approaches. Within the scope of the ecological analysis, the position (habitat) and instructional function (niche) of the concept of slope in curricula and textbooks were determined; through praxeological analysis, the mathematical organizations related to the concept of slope were revealed. It is observed that the concept of slope has a structure that progresses from middle school to the upper levels of secondary education, exhibiting continuity and contextual transformation through different mathematical contexts and instructional functions. This situation is consistent with the ATD, which argues that mathematical knowledge is reconstructed within institutional contexts (Chevallard, 1991)

The ecological findings indicate that the concept of slope is included within the learning outcomes of grades 8, 11, and 12 in the 2018 curricula. When the outcomes related to graphs are examined, it is observed that slope is associated with the algebra sub-learning area in grade 8, and with geometry and numbers–algebra domains in grades 11 and 12. Within the scope of the Türkiye Yüzyılı Maarif Modeli, it has been determined that the concept of slope is included in the learning outcomes of grade 8 at the middle school level and in preparatory, 9th, 10th, and 12th grades at the secondary level. This situation reveals that the concept of slope is a fundamental mathematical content that shows continuity across different curriculum periods.

In this study, the instructional function (niche) of the concept of slope was evaluated based on the perceptual, physical, procedural, and algebraic classification proposed by Takeuchi and Shinno (2020). Accordingly, the concept of slope is addressed at the perceptual level through intuitive and visual judgments, at the physical level through measurement and drawing tools, at the procedural level through numerical calculations, and at the algebraic level through trigonometry, derivatives, and analytical expressions. This classification made it possible to systematically analyze which cognitive and mathematical processes are foregrounded in tasks related to the concept of slope. Task types are generally structured around procedural and algebraic levels in textbooks, while perceptual and physical levels are given more limited emphasis. This situation is parallel to studies indicating that procedural knowledge is dominant in mathematics teaching rather than conceptual understanding (Doruk & Bayran Gün, 2026; Kuncoro et al., 2024; Takeuchi & Shinno, 2020). Indeed, Hendriyanto et al. (2023) and Agustito et al. (2025) reveal, through praxeological analysis, that the relationship between task types and techniques in textbooks directly affects learning opportunities and that procedural structures are predominant.It can be stated that the praxeological structures in textbooks show similar tendencies.

As a result of the praxeological analysis, three mathematical organizations covering 20 task types related to the concept of slope were identified. These organizations are: determining and interpreting slope, constructing/writing the equation of a line using slope, and interpreting slope according to the position of a line. Within these organizations, it was observed that the majority of tasks are structured through procedural and algebraic techniques, while perceptual and physical approaches remain more limited. This finding is consistent with previous studies indicating that a computation-based approach is dominant in the teaching of mathematical concepts in textbooks (Kuncoro et al., 2024; Takeuchi & Shinno, 2020).

When the distribution of task types by grade levels is examined, it is observed that tasks in grade 8 are predominantly concentrated in the range of T1–T6 and that slope is mainly introduced in visual, experiential, and real-life contexts. The concentration of tasks in grade 11 within the ranges of T4, T6, and T13–T16 indicates that the concept of slope evolves from a geometric understanding to an analytical structure. In grade 12, the concentration of tasks around T12, T18, and T19–T20 reveals that the concept of slope is associated with advanced mathematical concepts such as derivatives, tangents, and the positions of lines. In the 9th-grade textbook prepared within the scope of TYMM, task types T3, T4, and T5; and in the 10th-grade textbook, task types T3, T5, T10, and T13 were found to be more dominant.

Although it is observed that the concept of slope exhibits a progression from concrete to abstract, it is noteworthy that this progression is not structured in a sufficiently balanced way to ensure conceptual continuity. The noticeable decrease of examples at perceptual and physical levels in upper grades may limit students' opportunities to relate the concept to intuitive foundations. In the literature, establishing connections between different representations of the concept of slope is also identified as one of the areas in which students experience difficulties (Cho & Nagle, 2017; Nagle et al., 2013; Moore-Russo et al., 2011). Conceptual connections should be established more explicitly and systematically in textbooks. In particular, addressing slope in the contexts of derivatives and analytic geometry at upper grade levels indicates that the concept is associated with advanced mathematical structures through an instrumental function. However, the insufficient justification of this transition constitutes an instructional risk that may lead students to experience difficulties in relating the concept of slope across different mathematical contexts.

Accordingly, in the preparation of textbooks, it is important to ensure the continuity of the topic of slope across grade levels and to address the concept in a way that is not limited to procedural skills alone. Tasks and exercises included in textbooks should be designed in a way that supports students' conceptual understanding and contributes to the development of higher-order skills such as mathematical, critical, and analytical thinking.

In addition, some limitations of the study should be considered. Although the theoretical framework used in this research conducted with textbooks selected within the context of Türkiye offers a universal analytical opportunity, the diversity in the content and pedagogical approaches of textbooks in different countries may yield different results. Despite these limitations, the findings indicate that the concept of slope is reconstructed within different mathematical organizations at different grade levels and that these structures are based on specific techniques, technologies, and theoretical foundations. In this respect, the study demonstrates that the ATD approach provides a powerful analytical tool for examining the instructional organizations of mathematical concepts (Bosch & Gascón, 2006; Winsløw, 2011).

In conclusion, it is of great importance that the concept of slope is structured in textbooks in a way that supports conceptual continuity across grade levels. First of all, instructional arrangements should be implemented to ensure that the concept of slope is understood not only at the procedural level but also by establishing relationships across different mathematical contexts and representations. In future studies, examining textbooks from different countries comparatively and expanding the scope to include classroom practices will contribute to a deeper understanding of the findings.

It can also be stated that the ecological and praxeological analysis framework developed in this study provides an analytical tool that can be used not only in textbook analyses but also in teacher education studies aimed at developing preservice teachers' pedagogical content knowledge of the concept of slope. Recognizing how the concept of slope is structured across different grade levels may contribute to a more holistic understanding of the concept through its multiple representations and theoretical foundations.

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