Pedagogical Research
2023, 8(2), em0152
e-ISSN: 2468-4929
https://www.pedagogicalresearch.com

MODESTUM

Research Article **OPEN ACCESS**

An alternative path of embodying geometrical concepts: Student gestures

Tuba Akcakoca [1], Gonul Yazgan-Sag [2*], Ziya Argun [2]

[1] Ministry of National Education, Ankara, TURKEY
[2] Department of Mathematics and Science Education, Gazi Faculty of Education, Gazi University, Ankara, TURKEY
***Corresponding Author:** gonulyazgan@gazi.edu.tr

Citation: Akcakoca, T., Yazgan-Sag, G., & Argun, Z. (2023). An alternative path of embodying geometrical concepts: Student gestures. *Pedagogical Research, 8*(2), em0152. https://doi.org/10.29333/pr/12786

ARTICLE INFO

Received: 08 Aug. 2022

Accepted: 17 Nov. 2022

ABSTRACT

This study aims to reveal how the embodied cognition of certain geometrical concepts of secondary-school students arises via gestures and what kinds of gestures they produce while engaging with different concepts. The study participants comprised four eleventh-grade students studying at a state high school in Turkey. The study focused on the gestures of students related to angle, a measure of an angle, congruence-similarity, and translation. Data were gathered via video-recorded focus group discussions and individual interviews, and the cognition of the students for each concept was coded using content analysis. According to the research findings, it was found that the deictic gestures of the participants reflect the grounding of cognition in the physical environment; representational gestures manifest mental simulations of action and perception, and some metaphoric gestures reflect body-based conceptual metaphors.

Keywords: cognition, embodied cognition, gestures, geometrical concepts, secondary students

INTRODUCTION

Recently, various studies (Alibali et al., 2014a; Alibali & Nathan, 2012; Edwards, 2019) have focused on gestures, which are a ubiquitously important tool for conveying mathematical ideas in learning environments (Williams-Pierce et al., 2017). Gestures are spontaneous movements of the arms and hands that accompany speech (Kendon, 2004; McNeill, 1992). McNeill (1992) defined spontaneous gestures, as follows: "these are usually movements of the arms and hands and are closely synchronized with the flow of speech" (p. 11). Gestures are a special form of action that involves moving part of the body (usually the hands or arms) to express an idea or meaning, and they are not produced to manipulate or move objects, such as actions, but instead are produced as part of the cognitive processes that form the basis of thought and speech (Alibali et al., 2014a). In other words, "a gesture is a manifestly expressive action that enacts imagery (not necessarily by the hands or hands alone) and is generated as part of the process of speaking" (McNeill, 2012, p. 4). Recently, while studying the construction of meaning and communication, many researchers have pointed out those gestures are routinely produced during speech (Arzarello & Edwards, 2005; Goldin-Meadow, 2003; Hostetter & Alibali, 2008; McNeill, 2005; Núñez, 2006; Radford, 2003). People can use gestures to portray actions or objects through their forms, represent abstract ideas, provide emphasis to discourse structure, and refer to locations, items, or people in the world (Novack & Goldin-Meadow, 2017). Gestures might reveal cognitive processes, which could be affected by gestures (Akcakoca, 2018; Alibali & Nathan, 2012; Hostetter & Alibali, 2008). In this respect, increasing attention has been paid to the investigation of gestures in physical environments of learning and teaching activities.

Studies on gestures in mathematics education provide evidence for the many roles of gestures in conveying mathematical ideas and mathematical thinking. In this respect, gestures are an integral component in the communication of mathematical ideas (Akinci & Arikan, 2017; Alibali & Nathan, 2012; Church & Goldin-Meadow, 1986). Gestures play a key role in instructional communication, teaching concepts, and revealing embodied mathematical knowledge (Alibali & Nathan 2012, Yu & Uttal, 2021; Williams-Pierce et al., 2017). Some gestures are even representational actions for mathematical concepts (Alibali & Nathan, 2012; Edwards, 2009; Kita et al., 2017; Ping & Goldin-Meadow, 2008). These gestures convey conceptual information and bring out implicit knowledge, which is not expressed in speech (Boyatzis & Watson, 1993; Broaders et al., 2007). Thus, they enable the comprehension of mathematical meaning (Alibali et al., 2014b) and play an active role in the construction of mathematical thoughts (Alibali & Nathan, 2012; Alibali et al., 2019; Arzarello et al., 2009; Goldin-Meadow, 2003; Nemirovsky & Ferrara, 2009; Núñez, 2006; Radford, 2003). Gestures are an alternative form and means for embodying and organizing information that cannot be expressed formally or verbally (Arzarello et al., 2009), and therefore they can reduce the cognitive burden (Goldin-Meadow, 2000; Yoon et al., 2011). They shape the mathematical discourse in learning environments and provide novel and accurate solutions (Broaders et al., 2007; Singer & Goldin-Meadow, 2005).

Educational researchers have increasingly focused on gestures and their role in teaching mathematics and learning environments from different perspectives (Akçakoca & Yazgan-Sağ, 2021; Krause & Wille, 2021). Some researchers consider gestures from a "semiotic bundle" (Arzarello, 2006) perspective as semiotic resources (speech, writing, graphics, figures, etc.) that teachers and students can use to express, construct, develop, and relate their ideas (Akinci & Arikan, 2017; Arzarello & Paola, 2007; Arzarello et al., 2009; Gurefe, 2022; Thomas et al., 2009; Weinberg et al., 2015). This point of view considers semiotic resources as an integral part of a whole structure and analyzes these semiotic resources with a multi-model approach (Arzarello, 2006). However, some studies emphasize how mathematical cognition is embodied or formulated through perception and action (Akcakoca, 2018; Alibali & Nathan, 2012; Alibali et al., 2014b; Cook et al., 2008; Edwards, 2009, 2019; Healy & Fernandes, 2011; Kim et al., 2011; McNeill, 2005; Nemirovsky & Ferrara, 2009; Yu & Uttal, 2021; Williams-Pierce et al., 2017). They consider gestures as an important cognitive resource, especially for learning mathematics (Pier et al., 2014). These researchers analyzed gestures from the embodied cognition perspective (Barsalou, 2008; Glenberg, 2010; Wilson, 2002), which claims that cognitive processes are rooted in the interactions of the human body with the physical world. In this respect, gestures reflect the grounding of cognition in the physical environment (Alibali & Nathan, 2012; Alibali et al., 2014a; Krause & Salle, 2019), the embodied thinking about mathematical concepts and procedures, some mental images (Alibali & Nathan, 2012; Edwards, 2009; Hostetter & Alibali, 2008; Nemirovsky & Ferrara, 2009; Núñez, 2006), and body-based conceptual metaphors (Alibali & Nathan, 2012; Kim et al., 2011; Walkington et al., 2021). In other words, gestures provide evidence that the human body and the physical environment are involved in speaking and thinking (Akcakoca, 2018; Alibali & Nathan, 2012; Hostetter & Alibali, 2008).

Focusing on the embodied nature of mathematical cognition might enlighten someone regarding the examination of why some problems are more difficult than others, determining the appropriate assessment to evaluate mathematical knowledge, designing more effective learning environments, and selecting suitable methods to instruct mathematical content (Alibali & Nathan, 2012). Therefore, taking both the human body and the physical environment into consideration in revealing, conveying, and constructing mathematical ideas in a representational manner requires a different view of learning environments. In this regard, we could argue that paying attention to the gestures produced in the classroom environment can potentially enrich both learning and teaching activities. In addition, gestures help teachers gain insights into the students' learning of mathematical concepts. Reviewing different studies in the literature, we found that there are a limited number of studies (Alibali & Nathan, 2012; Alibali et al., 2014b, Edwards, 2009, Nemirovsky & Ferrara, 2009) categorizing gestures (McNeill, 1992) in detail in the construction of mathematical thinking and the analysis of mental processes. These studies emphasize how mathematical cognition is embodied or formulated through perception and action. The studies also consider gesture as an important cognitive resource, especially for mathematics learning, according to McNeill's (1992) classification of gestures. In this context, this study focuses on the embodied nature of mathematical cognition by categorizing gestures. According to McNeill (1992), iconic and metaphoric gestures are formal and semantic representations of ideas. Since gestures are specific to speakers, McNeill (1992) emphasized that they are idiosyncratic. This unique nature of representational gestures makes them especially useful for the assessment of individual thought processes. However, deictic gestures indicate the presence of embodied cognition in the physical environment (classroom, lecture session, and environments where an experimental study is conducted, etc.). Gestures were examined algebraic concepts such as counting (Alibali & DiRusso, 1999), equations, slope (Alibali & Nathan, 2012), fractions (Alibali & Nathan, 2012; Edwards, 2009; Zurina & Williams, 2011), ratio- proportion (Abrahamson, 2004), function graphs (Alibali & Paola, 2007; Arzarello et al., 2009). Gestures were also studied in geometrical concepts such as rectangles, pyramids, prisms (Healy & Fernandes, 2011; Kim et al., 2011), spheres, cylinders (Kim et al., 2011) symmetry (Healy & Fernandes, 2011; Valenzeno et al., 2003), polygons (Alibali & Nathan, 2012; Gurefe, 2022) angle, point, line, plane, vector, and projection (Akinci & Arikan, 2017). On the other hand, gestures and their roles were also investigated in students' problems related to algebraic concepts (Radford, 2003) and problem-solving processes (Alibali & Nathan, 2012; Cook et al., 2008). When all these studies were examined, it was seen that geometrical concepts were not studied with high school students. The geometrical concepts of this study were determined by considering the readiness of the participants of this study. Thus, the question that we want to answer in our study, in which we focused on the gestures of students about the angle, a measure of an angle, congruence-similarity, and translation concepts is, as follows: What are the gesture types that secondary-school students produce to reveal their embodied cognition about certain geometrical concepts and how do they use them?

THEORETICAL FRAMEWORK

This study is grounded on the theoretical framework of embodied cognition (Barsalou, 2008; Dove, 2016; Shapiro, 2014; Wilson, 2002) and uses the gesture classification of McNeill (1992), which includes deictic, iconic, and metaphoric gestures. According to Nathan (2008), "embodied cognition is an emerging framework for understanding intellectual behavior concerning the physical and social environment and to the perception- and action-based systems of the body" (p. 375). Wilson (2002) evaluated six claims to better understand the embodied cognition perspective:

(i) cognition is situated,

(ii) cognition is time-pressured,

(iii) we off-load cognitive work onto the environment,

(iv) the environment is part of the cognitive system,

(v) cognition is for action, and

(vi) off-line cognition is body based.

According to Wilson's (2002) review, cognitive activities take place in the physical environment and naturally involve perception and action (cognition is situated). It is also necessary to understand how the interaction of cognition with the environment occurs over time (cognition is time-pressured). Another claim states that owing to the limitations in our information-processing capabilities (e.g., attention and working memory limitations), individuals use the physical environment to reduce their cognitive workload (we off-load cognitive work onto the environment). The flow of information between the mind and the physical environment is very intense and continuous. Therefore, according to scientists studying the nature of the cognitive activity, the mind is not a meaningful unit of analysis alone and the physical environment is also part of the cognitive system (the environment is part of the cognitive system). One of these claims explains the function of the mind to direct actions. In this context, it is necessary to understand cognitive mechanisms, such as perception and memory, in terms of their final contribution to actions (cognition is for action). Finally, even if the interaction between the mind and the environment ends, the activity of the mind stays grounded in sensory-motor mechanisms that emerged from the interaction with the environment (off-line cognition is body-based) (Wilson, 2002). Therefore, the embodied cognition perspective claims that cognitive processes are rooted in the interaction of the human body with the physical world (Krause & Salle, 2019; Wilson, 2002). With this theory, which asserts that the body plays a central role in shaping the mind, it is important to note that "[…] the boundaries between perception, action, and cognition become porous" (Seitz, 2000, p. 35). This emphasizes the importance of the theory in the investigation of successful interactions with the environment in addition to the sensory and motor functions (Seitz, 2000). From this point of view, "[…] human thought, including mathematical thinking, is embodied at multiple levels: through imagery, bodily motion and gesture, and through the experience of living, with specific biological capabilities, in a world with particular physical properties" (Edwards, 2009, p. 128). Recent research (Alibali & Nathan, 2012; Alibali et al., 2014b, Edwards, 2019) emphasized the means with which mathematical cognition is embodied and defined gestures as an important cognitive resource for learning mathematics (Krause & Salle, 2019; Pier et al., 2014). According to McNeill, gestures are spontaneous movements of the hands and arms that enact an image and are closely synchronized with the flow of speech and express the speech process explicitly (McNeill, 1992, 2012). Studying individuals telling stories, McNeill (1992) classified gestures into four main categories: deictic, iconic, metaphoric, and beats.

Deictic gestures are used to point out an object, a location, or an individual in the physical environment (McNeill, 1992). In addition, these gestures can be used to indicate an abstract or nonpresent referent (Goldin-Meadow, 1999, McNeill, 1992). McNeill (1992) stated that "pointing has the obvious function of indicating objects and events in the concrete world, but it also plays a part even where there is nothing objectively present to point at" (p. 18). Thus, deictic gestures are classified into concrete deictic gestures and abstract deictic gestures according to the presence or absence of the object pointed out. Deictic gestures provide an expression and convey knowledge via pointing out or showing (Butcher et al., 1991; Glenberg & Robertson, 2000; Krause & Salle, 2018; Salle & Krause, 2016). The function of deictic gestures to manifest speakers' indexing of speech content to objects, locations, or inscriptions in the physical environment via pointing out or showing is called "the indexing function of deictic gestures." Deictic gestures, with this indexing function, provide the transfer of meaning (Alibali et al., 2014b; Glenberg & Robertson, 2000; Gurefe, 2022; Krause & Salle, 2018; Salle & Krause, 2016). Speakers use abstract deictic gestures to index nonpresent objects, locations, or inscriptions. Abstract deictic gestures, which represent the metaphoric image of abstract ideas, show us that even abstract ideas have some physical space (McNeill, 1992, 2005). These gestures use the physical environment in different ways to reveal embodied cognition. Alibali and Nathan (2012) expressed these different ways, as follows:

(1) "[…] point to perceptually similar objects to index nonpresent objects,"

(2) "[…] point to locations to index nonpresent objects or people that are associated with those locations," and

(3) "[…] set up locations within their gesture space to serve as 'placeholders' for objects or people and then point to these locations to index those objects or people" (p. 253).

Deictic gestures physically link speech and associated mental processes to the physical environment (Alibali & Nathan, 2012). It is difficult to interpret deictic gestures without the environmental background that provides meaning. Therefore, pointing gestures are defined by Goodwin (2007) as "environmentally coupled." According to Wilson (2002), for all these reasons, these gestures support the claim that cognition is present in the real world and that the environment is part of the cognitive system. Deictic gestures manifest the basis of cognition in the physical environment. In other words, deictic gestures reflect the grounding of cognition in the physical environment (Alibali & Nathan, 2012; Alibali et al., 2014b).

Iconic gestures bear a close formal relationship to the semantic content of speech and present images of concrete entities or actions (McNeill, 1992). These gestures have a form resembling their concrete referents visually (Edwards, 2009, 2019). Iconic gestures reflect the semantic content of speech in a transparent way (McNeill, 1992, 2005; Goldin-Meadow, 1999). For instance, an individual drawing a triangle in the air with his/her hand along with the word "triangle" is an example of this type of gesture.

Metaphoric gestures describe a mental image, or an abstract idea (McNeill, 1992) and they represent the concrete metaphor of a concept, a visual and kinetic image of that concept that we feel, and in a way, this description is similar to the concept. Metaphoric gestures should illustrate the abstract content in the mind by describing the meaning content via a metaphor (Alibali & Nathan, 2012; Gurefe, 2022; McNeill, 1992, 2005). For instance, an individual using his/her palms when talking about the set concept is an example of this kind of gesture. Even though the gesture here does not resemble the set concept, this is a metaphoric gesture illustrating the mental image of the individual. In addition, the gestural equivalents of some spatial expressions (i.e., up, down, left, and right) are considered "metaphoric gestures" (Alibali & Nathan, 2012; Akinci & Arikan, 2017). Iconic and some metaphoric gestures are also called "representational gestures" because of their formal similarity to the semantic content of the concept. Hostetter and Alibali (2008) argue that representational gestures derive from the same simulated actions and perceptions underlying language and mental imagery. This view is based on the idea that mental simulations of actions and perceptual situations stimulate the neural areas used in real actions and perception. Therefore, gestures derive from the action and perception simulations underlying language and mental images (Hostetter & Alibali, 2008; Kita et al., 2017). According to this view, gestures are seen as a special form of action used to express an idea or a meaning using part of the body, generally the hands or the arms. Put simply, representational gestures are produced because thinking is based on perception and action. From this perspective, representational gestures are acknowledged as evidence of embodied cognition as they are derived from simulated actions and perceptions (Barsalou, 2008; Hostetter & Alibali, 2008; Kita et al., 2017). Metaphoric gestures, which are not representational, provide evidence for the embodied nature of mathematical cognition as bodily movements (gestures) that reveal body-based conceptual metaphors (Lakoff & Johnson; 1980) and their spatial relations (Alibali & Nathan, 2012).

Beats gestures are stress or meter gestures that give rhythm (McNeill, 1992). Contrary to iconic and metaphoric gestures, the movements of the hand or fingers tend to have the same form as up and down or forwards and backward, independent of the semantic content of the speech (Goldin-Meadow, 1999; McNeill, 1992). The speaker adds an emphasis to the accompanying sentence or word with a gesture. Beats gestures, which are motoric and simple rhythmic gestures that occur simultaneously with speech (Alibali & Nathan, 2012), do not express the semantic content of the speech, and they are discarded in this study because they do not reflect cognition.

METHODOLOGY

Model of Research

This qualitative study, which focuses on students' embodied cognitions in terms of gestures, is designed as a phenomenological study. A phenomenological study focuses on how people describe what they experience and how they experience (Patton, 2002). A phenomenon is a situation or event that occurs spontaneously; it can be a physical object, a thought, or a concept (Ehrich, 1996). The phenomenon examined in this study is the gestures of the students related to geometric concept.

Participants

The participants of this study were determined by the purposeful sampling method. The purposeful sampling method aims to reach a rich data source, even if the number of participants is small (Patton, 2002). However, people's own words and sentences play an important role in defining phenomena (Jasper, 1994). All participants were observed before the interviews by the first researcher during one of the mathematics lessons, which took approximately 40 minutes. This classroom observation showed that these four students exhibited active participation and that they can communicate easily with the first researcher. Thus, Baris, Erva, Azra, and Busra (pseudonyms), agreed to participate in this study voluntarily. The participants were eleventh-grade students attending a state high school in Turkey. Among the participants, Busra and Azra's success in mathematics were at a high level, while Baris and Erva's success in mathematics were above a medium level.

Research Process

Six video-recorded focus group discussions, which lasted approximately 35 minutes, were conducted with these classroom vignettes prepared for each concept. In the focus group discussions, the students also answered the questions in these classroom vignettes. During all the interviews, the students were given a pencil and paper and they were allowed to use mathematical tools like rulers and compasses. All focus group discussions were conducted at the library of the high school, and the participants sat next to each other at desks. Following the focus group discussions, individual interviews lasting approximately 10 minutes were conducted with each participant. All individual interviews were conducted in a classroom environment that was quite different from the library. Similar to the focus group interviews, the students were allowed to use a pencil and paper and sat on a desk and answered the first researcher's questions. In the video-recorded interviews, the students were asked to define and exemplify the concepts. For instance, the question provided to the participants about the translation concept in the individual interviews was, as follows:

> "Could you explain what you think/know about the translation concept in your own words? Explain the concept with examples if possible."

Data Collection Tools

Semi-structured interview forms and interview video recordings were used as data collection tools in the research. The semi-structured interview forms used as the primary data source were designed as classroom vignettes. The findings of the studies in the literature (Gulkilik, 2008, 2013) were used to form four classroom vignettes. **Table 1** presents a section of a classroom vignette designed for the "translation" concept prepared for the focus group discussions. These vignettes were the primary data source of this study.

Data Analysis

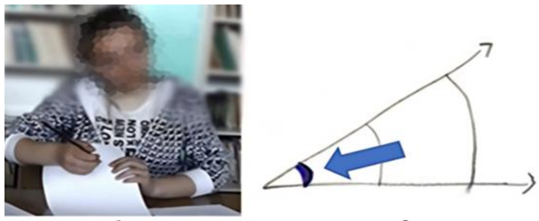
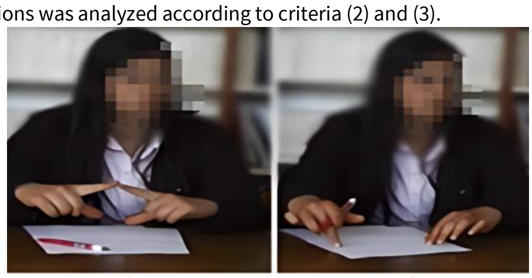
Content analysis was used to analyze the collected data (Patton, 2002). First, all video-recorded focus group interviews and individual interviews were transcribed. The gestures of the participants about their mathematical cognition were classified according to McNeill's (1992) gesture study. Thus, the embodied cognition of the students was classified into three gesture types: deictic gesture, iconic gesture, and metaphoric gesture. In some cases, the same cognition fell into more than one category.

Table 1. An example from the vignette: The following conversation took place between Mrs. Seyma, the mathematics teacher, and her student Ali on translation

Person	Conversation
Mrs. Seyma	How would your explanation be if you were to explain the translation to one of your friends?
Ali	How would it be? Errr ... I would explain it as "Changing the location of an object only, without changing its direction, shape, area, and size …"
Mrs. Seyma	How will I change the location?
Ali	We will not change anything about the object, for instance, its direction, or its area. We will change the location of the object.
Mrs. Seyma	How will I change its location?
Ali	By sliding it ...

Note. "What are your opinions about answers Ali provided about translation? How would your answer be to the same question? Please explain."

Table 2. Examples and criteria used for the analysis of gestures

Gesture	Criteria	Examples from findings
Deictic gesture	(1) Indexing the student's speech to worksheets or inscriptions in the physical environment utilizing pointing out or showing. (2) Indexing the student's speech to objects or locations in the physical environment utilizing pointing out or showing. (3) Indexing the student's speech to their body using pointing out or showing. (4) Indexing student's speech similar to an object or inscription that is not in the physical environment (this criterion is for abstract deictic gestures). (5) Indexing student's speech to the locations associated with those objects or to the spaces they set (this criterion is for abstract deictic gestures).	Erva talks about the measure of an angle concept and says, "when we draw, it belongs to the measure of each arc it corresponds." Here, when Erva says, "… the measure of each arc it corresponds," she is pointing to (**Figure 1**) her drawing on the paper (**Figure 2**). This finding in the focus group discussions was analyzed according to criterion (1).  1 2
Iconic gesture	(1) The student's gesture that is formally similar to the semantic content. (2) The student's gesture that evokes a direct form of visual representations of the concept. (3) Representation and description of semantic content in their speech by the student's gestures. (4) Student's gestures that show a semantic and formal similarity to their mental images and drawings of the concept.	Azra talks about the angle concept and says, "… when angles … are larger, we can draw a larger triangle, for instance a ratio is formed, I mean … At the side … it forms as a side not an angle…." Here, when Azra says, "… when angles…," her gesture by bringing together the tips of her fingers evokes the angle concept (**Figure 1**). Later on, she makes a drawing with her finger on the paper; she draws the "side" of the triangle she mentioned while speaking about similar triangles (**Figure 2**). This finding in the focus group discussions was analyzed according to criteria (2) and (3).  1 2
Metaphoric gesture	(1) Students display gestures to depict or explain concepts through metaphors. (2) Students describe their abstract thoughts utilizing gestures as if these abstract thoughts have a form. (3) Students use gestures for spatial expressions, such as down, up, right, left, inside, and outside. (4) Students use their bodies and parts of their bodies to explain concepts.	Erva talks about the congruence-similarity concept and says, "… these are congruent because everything about them is the same." While Erva is speaking about congruence, she moves her two hands to depict congruent geometric shapes (**Figure 1**). This finding in the individual interviews was analyzed according to criterion (4).  1

According to Arzarello and Edwards (2005), the relation between the content of speech and gestures is an important element for the analysis of gestures. In this respect, we analyzed gestures in our study while considering the synchronous and co-expressive nature of gestures with speech (McNeill, 1992) and the physical environment. This way, we tried to observe how and in what way the participants reveal their embodied geometrical ideas via gestures. To reveal students' mathematical cognition in terms of gestures, we developed a methodological tool consisting of the criteria shown in **Table 2**, using the literature, and analyzed the data accordingly.

Figure 1. Busra's deictic gestures for the angle concept (Source: Authors' own elaboration)

Figure 2. Baris' deictic gestures for the angle concept (Source: Authors' own elaboration)

FINDINGS

Secondary-school students exhibited 203 gestures for the angle, a measure of an angle, congruence-similarity, and translation concepts. In total, 62 of these gestures were deictic, 55 were iconic, and 86 were metaphoric. In addition, 10 of the 62 deictic gestures were observed as abstract deictic gestures. This section presents the findings related to each type of gesture.

Deictic Gestures

In the context of this study, any pointing out of any location, or any mathematical object present or nonpresent in the physical environment by the students using their fingers, hands, or any object (e.g., a pencil) was considered a deictic gesture. Here, "physical environment" means the environment in which the data collection tools, representations, tools, instruments, and technology were used, and which contained the participant(s) and the first researcher. Gestures and expressions of Busra supported this argument, while she showed an object or a location as a representation of the angle concept in the physical environment of focus group discussions:

> "… For instance, there is such an incline on your camera (deictic gesture/**Figure 1.1**) … if we could imagine one or two lines between them too, we can call the region where the camera is (deictic gesture/**Figure 1.2**) an angle … for instance, it is now like this, just between that vertex point (deictic gesture/**Figure 1.3**) …"

Here, it is seen that Busra did not provide any verbal or formal mathematical explanation for the angle concept. However, using the indexing function of deictic gestures gave us hints about Busra's concept image of the angle. She tried to explain that openness between camera tripod's legs formed an angle with the statement "*… to the region, where the camera is* (**Figure 1.2**)."

In another example, it was observed that Baris made use of the physical environment in both the focus group discussions (**Figure 2.1**) and the individual interview (**Figure 2.2**). Baris expressed his thought that the corner of the paper formed an angle with deictic gestures such as "*for instance, here on the paper* (deictic gesture/**Figure 2.1**) *the corner is an angle too*" and "*the corners of the paper* (deictic gesture/**Figure 2.2**) *forms an angle….*"

Erva, on the other hand, faced difficulties in her explanations of both the angle and the measure of angle concepts and used to the drawing she made for the concept. Erva first started defining the angle concept as "*(something) between two half-lines* (abstract deictic gesture/**Figure 3.1**) *… I mean … I would say the angle*," and then she pointed at a location does not present in the physical environment with her index finger and said that she would call that an angle. When she faced some difficulty in her explanations, she sketched the drawing in **Figure 3.3** by saying "*… there for instance, I would show on a drawing*." This drawing became an object present in the physical environment, and Erva completed her thoughts by saying, "*I would call this section* (deictic gesture/**Figure 3.2**) *as an angle*."

Figure 3. Erva's gestures and drawing for the angle concept (Source: Authors' own elaboration)

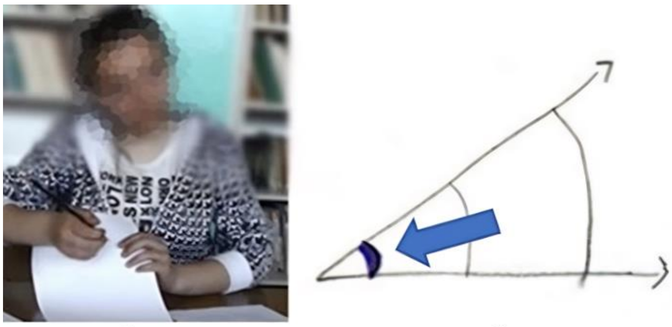


Figure 4. Erva's gestures and drawing for the measure of an angle concept (Source: Authors' own elaboration)

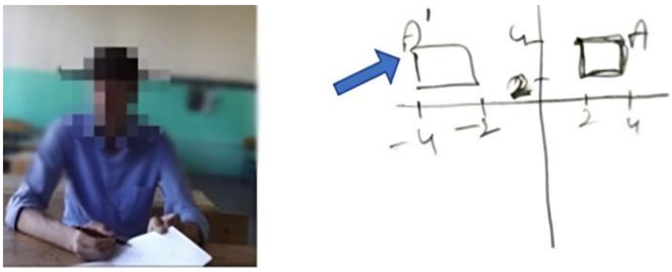


Figure 5. Baris' deictic gestures for the translation concept (Source: Authors' own elaboration)

It was observed that Erva also had some difficulty when she tried to explain the measure of an angle concept, and again she referred to her own drawing (**Figure 4.2**) and used deictic gestures with the statement, "*when we draw, it belongs to the measure of each arc it corresponds* (deictic gesture/**Figure 4.1**)," and she pointed at the object with a deictic gesture. Although Erva could provide some explanation of these concepts, she managed to convey and make her thoughts understood via the indexing function of deictic gestures.

Similarly, when exemplifying the translation concept, it was seen that Baris used his own drawings. Baris drew a quadrilateral (**Figure 5.2**) on a coordinate system and applied translation to this shape. His statement was, as follows:

> "… if we are to call this as shape A, I will translate shape A. For instance, if we translate it four units to the left, it will become a shape like this. Let's say A′ to the second form of shape A (deictic gesture/**Figure 5.1**) …."

Although Baris verbally expressed the translation that he performed, he pointed at the translated geometrical shape (A') to explain what translation means. It was found that Baris referred to a geometrical shape (circle) that was not present in the physical environment using an abstract deictic gesture when answering the question "*How do you decide whether two shapes (objects) are congruent or similar?*"

> "For instance, if we say that a circle (abstract deictic gesture/**Figure 6.1**) has a radius of two, in another circle (abstract deictic gesture/**Figure 6.2**), if we are to say congruent, it (abstract deictic gesture/**Figure 6.3**) should have a radius of two. We can say that these are congruent; but if one has a radius of two and the other four or six, then we can say they are similar."

Here, Baris sets up different spaces for the "*a circle*" and the "*another circle*" and pointed at those spaces for indexing. Although there was no image or object for the circle concept, he used abstract deictic gestures by pointing at self-created locations.

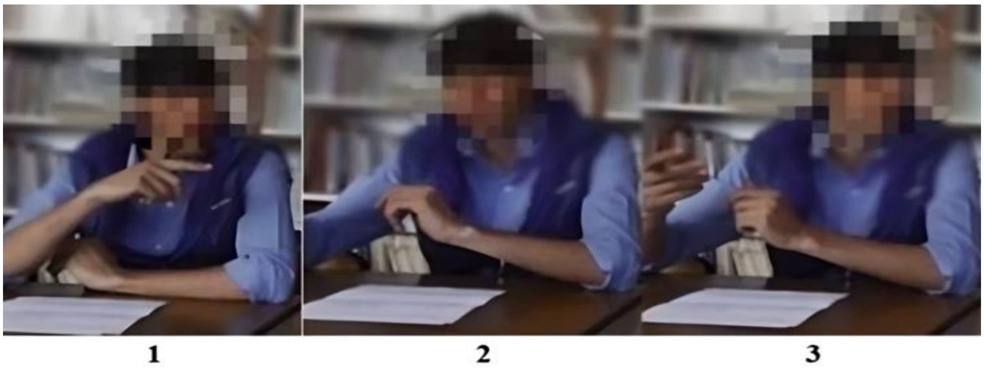


Figure 6. Baris' abstract deictic gestures for the congruence-similarity concept (Source: Authors' own elaboration)

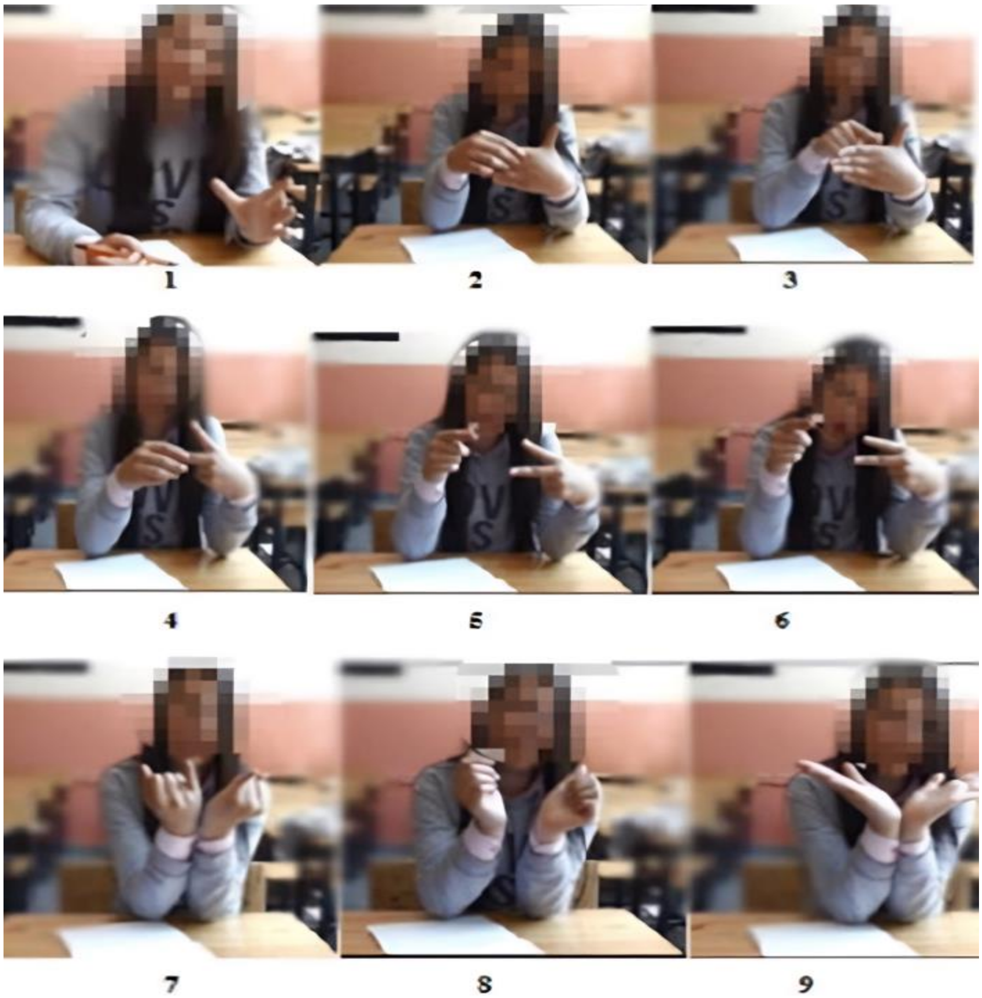


Figure 7. Erva's iconic gestures for the angle and the measure of an angle concepts (Source: Authors' own elaboration)

Iconic Gestures

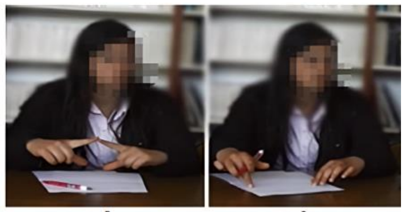
In this study, the hand and arm movements of the students were regarded as iconic gestures that evoke the exact form of the visual representations of the geometrical concepts and resemble the form of the semantic content of the mathematical discourse. For example, Erva revealed her mental image of the angle concept by configuring with the inspiration of a pair of scissors, which was present in the physical environment. In this process, it was observed that Erva used iconic gestures using her fingers to evoke the concepts of angle, acute angle, obtuse angle, and right angle:

> "I would call the gap between two half lines an angle (iconic gesture/**Figure 7.1**) … Our fingers are the most appropriate example. For instance, when we make this shape (iconic gesture/**Figure 7.2**) and visualize it here 90-degree angle (iconic gesture/**Figure 7.3**) is formed. When we make like this shape (iconic gesture/**Figure 7.4**) an angle is formed (iconic gesture/**Figure 7.5**). Such an acute angle is formed (iconic gesture/**Figure 7.6**). Or if we, do it like this (iconic gesture/**Figure 7.7**), just like we open to much (iconic gesture/**Figure 7.8**), such an obtuse angle (iconic gesture/**Figure 7.9**) is formed."

1 2

Figure 8. Busra's gestures and drawing for the angle concept (Source: Authors' own elaboration)

Table 3. Students' cognition and iconic gestures

Students' cognition	Iconic gestures
Erva: "The arcs of the entire circle (iconic gesture/**Figure a**) are 360°." She produces a gesture similar to a circle while speaking about the measure of an angle concept.	 *a*
Azra: "… when angles (iconic gesture/**Figure b.1**) … are larger, we can draw a larger triangle, for instance a ratio is formed, I mean … at the side (iconic gesture/**Figure b.2**) … it forms as a side not an angle …." She makes a drawing with her finger on the paper, and she draws a side of the triangle she mentioned while speaking about similar triangles.	 1 2 *b*
Büşra: "… I would look at its direction, angle, length, (iconic gesture/**Figure c**) … We would call them congruent if there is not any difference." She describes the length concept by moving the fingers of her hands away from each other and creating a distance while speaking about the identity concept in the focus group discussions.	 *c*
Erva: "Moving a shape without changing its direction, length (iconic gesture/**Figure d**) I mean size and form are called translation." She describes the length concept by moving her hands away from each other and creating a distance while speaking about the translation concept.	 *d*

Although Erva could not provide a formal explanation of the concepts, her use of iconic gestures provided clues about her mental images. This situation is a good example of the argument that cognitive processes are rooted in the interactions between the human body and the physical world.

It was observed that the students also use iconic gestures to describe and embody their mental images about the concepts. This way, the students facilitated the conveying of meaning by representing and describing the semantic content of their speech with these gestures. The iconic gestures (**Figure 8.1**) accompanying Busra's definition, trying to explain her mental image about the measure of an angle concept, are an example of this situation. With her expression, that is, "… the combination of the invisible lines in the angle between two half lines form the measure of the angle," Busra used iconic gestures that exhibit a formal similarity to angle and the lines that we could not see in the angle and evoking these concepts. The formal similarity of these iconic gestures to Busra's drawing about the measure of an angle (**Figure 8.2**) is remarkable, and this drawing provides clues about her mental image. The gestures that she used along with her speech are a concrete representation of the concept and play an important role in revealing her mental image of this concept. Busra's iconic gestures presented a description that clarifies the complexity of her expressions and, thus, enabled her to convey the semantic content of her speech.

Figure 9. Erva's metaphoric gestures for the translation concept (Source: Authors' own elaboration)

Figure 10. Azra's metaphoric gestures for the measure of an angle concept (Source: Authors' own elaboration)

It was seen that the students exhibited iconic gestures (**Table 3**) while speaking about geometric objects and their properties (angle, side, length, etc.). These gestures arose as concrete representations showing formal congruence or similarity with the concepts.

Metaphoric Gestures

In this study, the students' gestures that describe their abstract thoughts, reflect their body-based conceptual metaphors, picture the visual image that they feel about the concept, and assert the directional (left, right, up, and down) metaphors are regarded as metaphoric gestures. When the students spoke about the concepts, they used metaphoric gestures frequently, representing their oral explanations visually. In the individual interviews, Erva's picturing her thoughts about the translation concept using metaphoric gestures is an example of this situation:

> "Translation is moving a shape from one location to another (metaphoric gesture/**Figure 9.3**, **Figure 9.4**, and **Figure 9.5**) without changing its direction, length, I mean size (metaphoric gesture/**Figure 9.1**) and form (metaphoric gesture/**Figure 9.2**)."

Erva tried to describe the size and form of the shape that she translated in her mind using metaphoric gestures (**Figure 9.1** and **Figure 9.2**). While she was animating the translation movement that she applied to this shape in her mind with metaphoric gestures, she moved her fingers from where they were (**Figure 9.3** and **Figure 9.4**) to another position (**Figure 9.5**). In another example, Azra, while trying to define the angle concept, used metaphoric gestures that represent her thoughts visually. This way, she tried to embody her thoughts and clarify the complexity of her expressions in a more understandable manner:

> "… For instance, here, it could be 45 degrees or 90, in other words, its measure, well … the expansion of the angle (metaphoric gesture/**Figure 10**) like obtuse angle and acute angle …."

Here, Azra used gestures that embodied her abstract thoughts about the angle concept, instead of a representational gesture directly evoking the angle concept. However, these gestures also functioned to describe the mental image of Azra about the concept.

Some metaphoric gestures produced by the students revealed their mathematical cognition that they configured in their body. These examples (**Table 4**) provided clues about the body-based conceptual metaphors of the participants about the concepts. At the same time, they provided insights into the role that the body plays in shaping cognition.

It was observed in the study that the participants used metaphoric gestures, which are the gestural equivalent of spatial expressions (up, down, left, right, in, and out). Azra's gestures pointing at directions with a pencil in her hand while speaking about the translation concept are an example of this situation:

> "For us to call a movement as a translation, in my opinion there should be … I suppose an expression like two units to the left (metaphoric gesture/**Figure 11.1**), three units up (metaphoric gesture/**Figure 11.2**) … the same holds for the down (metaphoric gesture/**Figure 11.3**) and up (translation)."

DISCUSSION AND CONCLUSIONS

In this study, we investigated how secondary-school students describe concepts with their bodies or with the physical environment using gestures and classified these different gestures. We observed that students frequently use gestures while

Table 4. Students' cognition and metaphoric gestures

Students' cognition	Metaphoric gestures
Erva: "… these are congruent because everything about them is the same (metaphoric gesture/**Figure a**)." While speaking about congruence, Erva moves her hands depicting an example of congruent geometric shapes.	 *a*
Büşra: "I do not feel any difference in congruence (metaphoric gesture/**Figure b.1**). If everything, length, size, direction, if everything is the same I would call it congruent (metaphorical gesture/**Figure b.2**)." In an individual interview, she brought her hands or two fingers (vis-à-vis) together while speaking about the congruence concept.	 1 2 *b*

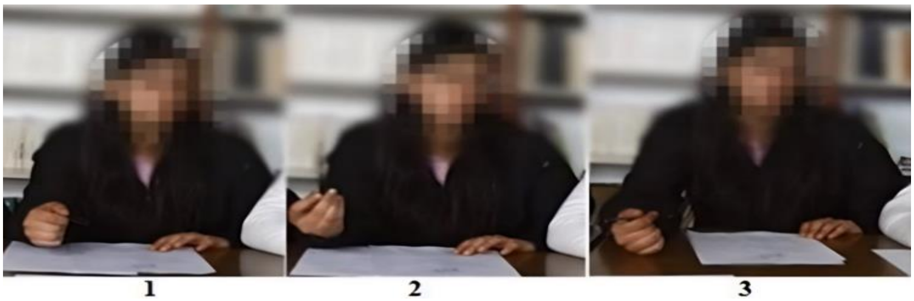


Figure 11. Azra's (spatial) metaphoric gestures for the translation concept (Source: Authors' own elaboration)

expressing their mathematical cognition about the angle, measure of an angle, congruence-similarity, and translation concepts. This situation supports the claim that gestures play an active role in mathematical thinking (Akcakoca, 2018; Alibali & Nathan, 2012; Edwards, 2019; Nemirovsky et al., 1998; Radford, 2003).

The gestures of the students mostly accompanied their speech and, thus, played an integral role in the mathematical content (Akinci & Arikan, 2017; Edwards, 2009, 2019). In addition, they facilitated the conveying of mathematical meaning and made the students' mathematical cognition more perceptible and understandable. Students use gestures as an alternative way to embody and organize their knowledge that they cannot express verbally or formally (Akcakoca, 2018; Alibali et al., 2019; Arzarello et al., 2009; Kim et al., 2011; Gurefe, 2022). These gestures bring out implicit knowledge not expressed in their speech (Broaders et al., 2007). They also included their gestures and bodily movements in their cognitive processes during the interviews and exhibited these as evidence of the embodied nature of mathematical knowledge (Alibali & Nathan, 2012; Alibali et al., 2014b; Kim et al., 2011). This shows that gestures can emphasize the active and original elements of thinking and make cognition embodied (McNeill, 2005). All the deictic and representational gestures of the students supported the idea that cognitive processes are rooted in the interaction between the human body and the physical environment (Wilson, 2002; Kita et al., 2017).

In this study, the deictic gestures of the students revealed the geometrical cognition that they grounded in the physical environment. The students expressed their cognition using deictic gestures by pointing at locations, objects, and instructions that might be present (instructional environment, interview questions, drawings by the student, etc.) or nonpresent (e.g., geometrical objects). This supported the claim that the physical environment is part of the cognitive system (Wilson, 2002) the deictic gestures reflect the grounding of cognition in the physical environment (Akcakoca, 2018; Alibali & Nathan, 2012; Alibali et al., 2014b; Krause & Salle, 2018; Novack & Goldin-Meadow, 2017; Salle & Krause, 2016). Cases in the study are examples of the roles of deictic gestures in conveying information by pointing out or showing locations and objects that are present or nonpresent in the physical environment. It was seen that the gestures exhibited by the participants reveal their cognition configured in the physical environment. It was also found that by using the indexing function of deictic gestures, the students made their geometrical thoughts more perceivable and understandable. According to the findings of the study, for 62 deictic gestures, the participants used 20 gestures for the angle concept, three gestures for the measure of an angle concept, 21 gestures for the congruence-similarity concept, and 18 gestures for the translation concept in the interviews. These gestures of the students provided us with clues about how they include the physical environment in the grounding process of geometrical concepts (e.g., the corner of a paper defining an angle). According to the findings, the deictic gestures of the participants revealed their embodied cognition, which they configured about the concepts in the physical environment. The students used the indexical function of the deictic gestures via pointing out or showing (Glenberg & Robertson, 2000) to convey the "meaning"; this way, they manifested the grounding of their cognition about concepts in the physical environment.

In this process, the students tried to make their geometrical ideas more comprehensible by showing visual representations of the concepts that were present or nonpresent in the physical environment. It was observed that the students first drew shapes to explain the ideas that they had difficulty expressing verbally, and then they pointed at these drawings and tried to explain their cognition. The students' use of abstract deictic gestures to point out objects and locations that were not present in the physical environment supported the following argument: abstract ideas also had a physical location (Akcakoca, 2018; McNeill, 2005; Edwards, 2009). According to the findings, while six of the abstract deictic gestures belong to the congruence-similarity concept, two of them were related to the angle concept and the remaining two were related to the translation concept. It was found that the participants use deictic gestures frequently when the visual representations of the concepts are present in the physical environment or in the interview questions. As in the examples of Busra in **Figure 1** and Baris in **Figure 2**, the presence of visual representations of the angle concept made the students exhibit deictic gestures. Similarly, it was found that the students pointed at the visual representations of the geometrical shapes (even when they are not present in the physical environment) with deictic gestures when talking about the congruence-similarity and translation concepts (as in the example of Baris in **Figure 5** and **Figure 6**). In addition, it was seen that the students made use of deictic gestures when they could draw an illustration of a concept (Erva's example in **Figure 3.2** and **Figure 4** and Baris' example in **Figure 5**). The students also used deictic gestures when they wanted to express locations (Erva's example in **Figure 3.1**) and (geometrical) objects (Baris' example in **Figure 6**) that were present in their minds but not in the physical environment. This kind of gesture usage is classified as abstract deictic gesture. It was seen that the students used these gestures frequently for the congruence-similarity concept to mention the geometrical shapes in their minds (Baris' example in **Figure 6**). In summary, the findings suggest that the presence of a visual representation of a concept (either present in the physical environment or not) promotes the use of deictic gestures.

By examining the deictic gestures produced, what our study contributed to the literature is that the students "[…] physically linked speech and associated mental processes to the physical environment" (Alibali & Nathan, 2012, p. 253). Students mostly use deictic gestures to indicate the visual representations of figural concepts when expressing their cognition about the concepts. Fischbein (1993) defines *geometrical figural concepts* in the following way: "a geometrical figure may, then, be described as having intrinsically conceptual properties. Nevertheless, a geometrical figure is not a mere concept. It is an image, a visual image. It possesses a property which usual concepts do not possess, namely, it includes the mental representation of space property" (p. 141). In this study, deictic gestures are evidence that cognition about figural concepts (angle, geometrical shapes, etc.) is grounded in the physical environment and that the physical environment is included in cognitive processes. This study also emphasized the role of deictic gestures in teaching figural concepts with visual representations, with regard to both the physical environment and the concept instruction. In this respect, it is suggested that deictic gestures be used in enriching the figural concept with regard to visual representations and that these representations be used in concept instruction. On the other hand, our findings show that students index their thoughts to the space that they set up by means of abstract deictic gestures (e.g., when Baris sets up locations for circles in his mind).

On the other hand, the representational gestures (iconic and metaphoric gestures) of the students emerged as concrete signs reflecting their mental images about concepts (Akcakoca, 2018; Alibali & Nathan, 2012, Edwards, 2019; Ping & Goldin-Meadow, 2008). Some metaphoric gestures became concrete metaphors revealing their cognition, which they configured in their bodies. This supported the claims that representational gestures explicitly manifest the simulations of action and perception and that some metaphoric gestures reflect body-based conceptual metaphors (Akcakoca, 2018; Alibali & Nathan, 2012; Kim et al., 2011). The students used iconic gestures directly evoking the generic models of the figural concepts (e.g., angles, geometrical shapes, like a circle) used in textbooks or in instructions. In addition, they tried to describe semantic content using simulations (i.e., using "move" or "carry" instead of "translate") or some metaphors (using their body parts as an example of some concepts) while expressing their mathematical cognition about the concepts. This supported the claim in the literature that representational gestures are gestures that convey and reflect conceptual knowledge (Boyatzis & Watson, 1993; Edwards, 2009; Ping & Goldin-Meadow, 2008).

According to the findings, the participants used iconic gestures that evoke or describe the exact forms of the visual representations of the concepts. These gestures played a conceptual information-conveying role by representing and describing the semantic content of speech during oral explanations (Akinci & Arikan, 2017; Boyatzis & Watson, 1993; Edwards, 2009; Novack & Goldin-Meadow, 2017; Ping & Goldin-Meadow, 2008). Students who revealed their mental simulations of the concepts via iconic gestures provided evidence supporting the idea that cognition is rooted in our actions (Akcakoca, 2018; Alibali & Nathan, 2012; Alibali et al., 2014a). Iconic gestures related to geometrical concepts and their features revealed the mental images of the students, and these gestures became concrete representations of their mathematical thoughts. In addition, the formal similarity of the gestures that they used for the length concept supported the claim that certain aspects of mathematical thinking are inclined to become embodied, with the gestures clearly revealing this situation (Alibali & Nathan, 2012). The students produced 30 of the 55 iconic gestures in the interviews for the angle concept, 15 for the measure of an angle concept, seven for the congruence-similarity concept, and three for the translation concept.

The findings of the study showed that iconic gestures are frequently used in expressing concepts that are rich in terms of visual representation, such as the angle and the measure of an angle concepts. It was seen that the presence of a visual form of the concepts caused the students to use iconic gestures (Erva's example in **Figure 7**) while speaking about that concept. Again, it is thought that the presence of a visual form of their mental images (if there is one) triggers the production of these gestures (Busra's example in Figure 8). It was also seen that iconic gestures, which are prominent in their formal similarity with the concepts, were used to describe geometrical shapes (Erva's example in **Table 3 Figure a**) and their properties (as in the examples in **Table 3 Figure b**, **Figure c**, and **Figure d**). It was found that metaphoric gestures are frequently used to explain a mathematical situation or to express an abstract idea. It was also seen that students frequently use these gestures in their explanations for geometrical concepts, even for concepts like translation, which can be expressed by analogy (in Erva's example in **Figure 9**, this is simulated by the verb "move") or metaphor (Erva's example in **Table 4 Figure a**). The participants produced this type of metaphor few in number regarding the angle and measure of an angle concepts. Despite using fewer gestures for these concepts, they produced metaphoric gestures to describe their abstract thoughts (as in Azra's example in **Figure 10**) instead of representations directly evoking the concept. It was also observed that if a bodily feature plays a role in the configuration of a concept, this affects the production of metaphoric gestures. As in Erva's example in **Table 4.a** and Busra's example in **Table 4.b**, it was seen how the students configured their body-based metaphors and how they revealed them with gestures while speaking about the congruence-similarity concept. Hence, it was concluded that the production of these gestures is usual for concepts containing spatial expressions (as in Azra's example in **Figure 11**).

In addition, the students revealed their abstract thoughts about the concepts via metaphoric gestures that they used as if their abstract thoughts had some kind of form or shape. The students related their abstract thoughts about the concepts with mental simulations and metaphors via these gestures, which supported the idea that cognition is rooted in perceptions (Alibali et al., 2014a) and those metaphoric gestures reflect body-based conceptual metaphors (Akcakoca, 2018; Alibali & Nathan, 2012; Kim et al., 2011; Walkington et al., 2021). Some of the participants' metaphoric gestures in this study became prominent for being concrete visual representations describing concepts, whereas some others became prominent for reflecting body-based conceptual metaphors. The students produced 10 of the 86 metaphoric gestures in the interviews for the angle concept, nine for the measure of the angle concept, 25 for the congruence-similarity concept, and 42 for the translation concept. In total, 14 of the 42 metaphorical gestures for the translation concept were metaphoric gestures, which are the gestural equivalent of directions, also called spatial gestures.

Another significant contribution of this study to the literature on gestures is that some differences occur in the use of representational gestures depending on the structure of the concepts. It was observed that iconic gestures are more frequent in expressing cognition about a concept if there is an image of the figural concept (angle, geometrical shapes, etc.) or a representation in the drawing of the student (e.g., the image of the measure of an angle by Busra in Figure 8). These gestures became an alternative sign for the representation of a concept and provided clues about the mental images of the students. In no figural concepts (e.g., translation, congruence-similarity), the students used simulation or metaphor, and metaphoric gestures accompanied their speech, the students used simulation or metaphor, and metaphoric gestures accompanied their speech. These gestures, which pictorially described the abstract thoughts of the students, revealed their mental images about the concepts. In addition, it was seen that the students showed their cognition, which they had configured in their bodies, about some concepts (e.g., congruence-similarity) using these gestures. Again, in the presence of spatial expressions (e.g., left, right, up, down, in, and out) in the structure of a concept (e.g., translation), these gestures were observed to be frequently used. All representational gestures, emerging as iconic or metaphoric depending on the structure of the concept, ultimately became concrete representations of the students' mental images. This suggests that representational gestures can be used as an alternative representation form (Goldin, 1998; Lesh et al., 1987, Novack & Goldin-Meadow, 2017; Ping & Goldin-Meadow, 2008) in the instruction of mathematical concepts. In this respect, experts designing curricula can consider gestures as an alternative data source in their teaching/learning methods and techniques, as well as the assessment and evaluation sections of the curricula.

In the context of mathematics learning environments, gesture has become an increasingly object of study as it is both a window to the mental representations of the gesturer and a teaching tool that guides students to focus and think with representations (Akcakoca & Yazgan-Sag, 2021). Thus, researchers willing to conduct studies on the use of gestures in mathematics teaching can design their research to observe the in-class processes of mathematical concepts. This would enable the investigation of whether gestures are used in classrooms in the instruction process of concepts by the students and, especially, by the teachers and, if so, how they are used. Therefore, investigating the reasons underlying the change in gestures depending on the concept by comparing the in-class processes with the teacher and student interviews would contribute to the literature. Besides, changes in the students' use of gestures in algebraic concepts can also be investigated in mathematics learning environments.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Acknowledgements: The authors would like to thank the participants in the study.

Ethical statement: Authors stated that at the time of the study, Gazi University Graduate School of Educational Sciences did not require an ethics committee approval to conduct the study. Ethical considerations were strictly followed during this study. Authors further stated that the participants in this study were not exposed to any risk or potential harm. Formal permission was obtained from the principal of the school that the study to be conducted. Additionally, informed consent was obtained from all participants of the study.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Abrahamson, D. (2004). Embodied spatial articulation: A gesture perspective on student negotiation between kinesthetic schemas and epistemic forms in learning mathematics. In D. E. McDougall, & J. A. Ross (Eds.), *Proceedings of the 26th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 791-797). Preney.

Akcakoca, T. (2018). *The investigation of secondary school students' embodied cognitions of some mathematical concepts through gestures* [Master's thesis, Gazi University].

Akcakoca, T., & Yazgan Sag, G. (2021). The importance and role of gesture in construncting mathematical meaning. *Ahi Evran University Journal of Kirsehir Education Faculty, 22*(2), 678-703. https://dergipark.org.tr/tr/pub/kefad/issue/62829/776349

Akinci, M., & Arikan, A. (2017). The investigation of the pre-service mathematics teachers' gestures of some geometric concepts. *Elementary Education Online, 16*(4), 1357-1383. https://doi.org/10.17051/ilkonline.2017.342960

Alibali, M. W., & DiRusso, A. A. (1999). The function of gesture in learning to count: More than keeping track. *Cognitive Development, 14*(1), 37-56. https://doi.org/10.1016/S0885-2014(99)80017-3

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: evidence from learners' and teachers' gestures. *The Journal of the Learning Sciences, 21*, 247-286. https://doi.org/10.1080/10508406.2011.611446

Alibali, M. W., Boncoddo, R., & Hostetter, A. B. (2014a). Gesture in reasoning: An embodied perspective. In L. Shapiro (Ed.), *The Routledge handbook of embodied cognition* (pp. 150-160). Routledge.

Alibali, M. W., Nathan, M. J., Boncoddo, R., & Pier, E. (2019). Managing common ground in the classroom: Teachers use gestures to support students' contributions to classroom discourse. *ZDM, 51*(2), 347-360. https://doi.org/10.1007/s11858-019-01043-x

Alibali, M. W., Nathan, M. J., Wolfgram, M. S., Church, R. B., Jacobs, S. A., Johnson, Martinez, C., & Knuth, E. J. (2014b). How teachers link ideas in mathematics instruction using speech and gesture: A corpus analysis. *Cognition and Instruction, 32*(1), 65-100. https://doi.org/10.1080/07370008.2013.858161

Arzarello, F. (2006). Semiosis as a multimodal process. *Revista Latinoamericana de Investigación en Matemática Educativa* [*Latin American Journal of Research in Educational Mathematics*]*, 9*(1), 267-299.

Arzarello, F., & Edwards, L. D. (2005). Gestures and the construction of mathematical meaning. In *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp. 123-154).

Arzarello, F., & Paola, D. (2007). Semiotic games: The role of the teacher. In *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (pp.17-24). PME.

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*, 97-109. https://doi.org/10.1007/s10649-008-9163-z

Barsalou, L. W. (2008). Grounded cognition. *Annual Review Psychology, 59*, 617-645. https://doi.org/10.1146/annurev.psych.59.103006.093639

Boyatzis, C. J., & Watson, M. W. (1993). Preschool children's symbolic representation of objects through gestures. *Child Development, 64*(3), 729-735. https://doi.org/10.2307/1131214

Broaders, S. C., Cook, S. W., Mitchell, Z., & Goldin-Meadow, S. (2007). Making children gesture brings out implicit knowledge and leads to learning. *Journal of Experimental Psychology: General, 136*(4), 539-550. https://doi.org/10.1037/0096-3445.136.4.539

Butcher, C., Mylander, C., & Goldin-Meadow, S. (1991). Displaced communication in a self-styled gesture system: Pointing at the non-present. *Cognitive Development, 6*(3), 315-342. https://doi.org/10.1016/0885-2014(91)90042-C

Church, R. B., & Goldin-Meadow, S. (1986). The mismatch between gesture and speech as an index of transitional knowledge. *Cognition, 23*(1), 43-71. https://doi.org/10.1016/0010-0277(86)90053-3

Cook, S. W., Mitchell, Z., & Goldin-Meadow, S. (2008). Gesturing makes learning last. *Cognition, 106*(2), 1047-1058. https://doi.org/10.1016/j.cognition.2007.04.010

Dove, G. (Eds). (2016). *Beyond the body? The future of embodied cognition*. Frontiers Media. https://doi.org/10.3389/978-2-88919-797-2

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk., *Educational Studies in Mathematics, 70*, 127-141. https://doi.org/10.1007/s10649-008-9124-6

Edwards, L. D. (2019). The body of/in proof: Evidence from gesture. In *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (pp. 201-208). PME.

Ehrich, L. C. (1996). The difficulties of using phenomenology: A novice researcher's experience. In P. Willis, & B. Neville (Eds.), Qualitative research practice in adult education (pp. 197-214). David Lovell Publishing.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*(2), 139-162. https://doi.org/10.1007/BF01273689

Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *Wiley Interdisciplinary Reviews: Cognitive Science, 1*(4), 586-596. https://doi.org/10.1002/wcs.55

Glenberg, A. M., & Robertson, D. A. (2000). Symbol grounding and meaning: A comparison of high-dimensional and embodied theories of meaning. *Journal of Memory & Language, 43*, 379-401. https://doi.org/10.1006/jmla.2000.2714

Goldin, G. A. (1998). Representations and the psychology of mathematics education: Part II. *Journal of Mathematical Behaviour, 17*(2), 135-165. https://doi.org/10.1016/S0364-0213(99)80055-X

Goldin-Meadow, S. (1999). The role of gesture in communication and thinking. *Trends in Cognitive Sciences, 11*, 419-429. https://doi.org/10.1016/S1364-6613(99)01397-2

Goldin-Meadow, S. (2000). Beyond words: The importance of gesture to researchers and learners. *Child Development, 71*(1), 231-239. https://doi.org/10.1111/1467-8624.00138

Goldin-Meadow, S. (2003). *How our hands help us think*. Chicago University Press. https://doi.org/10.1037/e413812005-377

Goodwin, C. (2007). Environmentally coupled gestures. In S. Duncan, J. Cassell, & E. Levy (Eds.), *Gesture and the dynamic dimension of language* (pp. 195-212). John Benjamins. https://doi.org/10.1075/gs.1.18goo

Gulkilik, H. (2008). *A phenomenographic study on the concept images and image development of prospective secondary mathematics teachers related with some geometric concepts* [Master's thesis, Gazi University].

Gulkilik, H. (2013). *The role of representations in mathematical understanding: Virtual and physical manipulatives* [Doctoral dissertation, Gazi University].

Gurefe, N. (2022). How must a polygon be according to hard of hearing students? An investigation with a semiotic approach. *REDIMAT, 11*(2), 180-213. https://doi.org/10.17583/redimat.6097

Healy, L., & Fernandes, S. H. A. A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*(2-3), 157-174. https://doi.org/10.1007/s10649-010-9290-1

Hostetter, A. B., & Alibali, M. W. (2008). Visible embodiment: Gestures as simulated action. *Psychonomic Bulletin & Review, 15*(3), 495-514. https://doi.org/10.3758/PBR.15.3.495

Jasper, M. A. (1994). Issues in phenomenology for researchers of nursing. *Journal of Advanced Nursing, 19*(2), 309-314. https://doi.org/10.1111/j.1365-2648.1994.tb01085.x

Kendon, A. (2004). *Gesture: Visible action as utterance*. Cambridge University Press. https://doi.org/10.1017/CBO9780511807572

Kim, M., Roth, W. M., & Thom, J. (2011). Children's gestures and the embodied knowledge of geometry. *International Journal of Science and Mathematics Education, 9*(1), 207-238. https://doi.org/10.1007/s10763-010-9240-5

Kita, S., Alibali, M. W., & Chu, M. (2017). How do gestures influence thinking and speaking? The gesture-for-conceptualization hypothesis. *Psychological Review, 124*(3), 245-266. https://doi.org/10.1037/rev0000059

Krause, C. M., & Salle, A. (2018). On the role of gestures for the descriptive analysis of 'grundvorstellungen': A case of linear functions. In N. Presmeg, L. Radford, W.-M. Roth, & G. Kadunz (Eds.), *Signs of signification: Semiotics in mathematics education research* (pp. 293-313). Springer International Publishing. https://doi.org/10.1007/978-3-319-70287-2_16

Krause, C. M., & Salle, A. (2019). Towards cognitive functions of gestures–a case of mathematics. In *Proceedings of the 43rd Conference of the International Group for the Psychology in Mathematics Education* (pp. 496-503). PME.

Krause, C. M., & Wille, A. M. (2021). Sign language in light of mathematics education: An exploration within semiotic and embodiment theories of learning mathematics. *American Annals of the Deaf, 166*(3), 352-377. https://doi.org/10.1353/aad.2021.0025

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. University of Chicago Press.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum Associates.

McNeill, D. (1992). Hand and mind: What gestures reveal about thought. University of Chicago Press.

McNeill, D. (2005). *Gesture and thought*. University of Chicago Press. https://doi.org/10.7208/chicago/9780226514642.001.0001

McNeill, D. (2012). *How language began: Gesture and speech in human evolution.* Cambridge University Press. https://doi.org/10.1017/CBO9781139108669

Nathan, M. J. (2008). An embodied cognition perspective on symbols, gesture, and grounding instruction. *Symbols and Embodiment: Debates on Meaning and Cognition, 18*, 375-396. https://doi.org/10.1093/acprof:oso/9780199217274.003.0018

Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics, 70*(2), 159-174. https://doi.org/10.1007/s10649-008-9150-4

Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction, 16*(2), 119-172. https://doi.org/10.1207/s1532690xci1602_1

Novack, M. A., & Goldin-Meadow, S. (2017). Gesture as representational action: A paper about function. *Psychonomic Bulletin & Review, 24*(3), 652-665. https://doi.org/10.3758/s13423-016-1145-z

Núñez, R. (2006). Do real numbers really move? Language, thought, and gesture: The embodied cognitive foundations of mathematics. In R. Hersh (Ed.) *18 unconventional essays on the nature of mathematics* (pp. 160-181). Springer. https://doi.org/10.1007/0-387-29831-2_9

Patton, M. Q. (2002). *Qualitative research and evaluation methods.* SAGE.

Pier, E., Walkington, C., Williams, C., Boncoddo, R., Waala, J., Alibali, M. W., & Nathan, M. J. (2014). Hear what they say and watch what they do: predicting valid mathematical proofs using speech and gesture. In W. Penuel, S. A. Jurow, & K. O'Connor (Eds.), *Proceedings of the 11th International Conference of the Learning Sciences* (pp. 649-656). University of Colorado.

Ping, R. M., & Goldin-Meadow, S. (2008). Hands in the air: Using ungrounded iconic gestures to teach children conservation of quantity. *Developmental Psychology, 44*(5), 1277-1287. https://doi.org/10.1037/0012-1649.44.5.1277

Radford, L. (2003). Gestures, speech, and the sprouting of signs. *Mathematical Thinking and Learning, 5*(1), 37-70. https://doi.org/10.1207/S15327833MTL0501_02

Salle, A., & Krause, C. M. (2016). On the role of gestures for the descriptive analysis of 'grundvorstellungen': A case of linear functions. In N. Presmeg, L. Radford, W. M. Roth, & G. Kadunz (Eds.), *Signs of signification* (pp. 293-313). Springer. https://doi.org/10.1007/978-3-319-70287-2_16

Seitz, J. A. (2000). The bodily basis of thought. *New Ideas in Psychology, 18*(1), 23-40. https://doi.org/10.1016/S0732-118X(99)00035-5

Shapiro, L. (Eds.). (2014). *The Routledge handbook of embodied cognition.* Routledge. https://doi.org/10.4324/9781315775845

Singer, M. A., & Goldin-Meadow, S. (2005). Children learn when their teacher's gestures and speech differ. *Psychological Science, 16*(2), 85-89. https://doi.org/10.1111/j.0956-7976.2005.00786.x

Thomas, M. O., Yoon, C., & Dreyfus, T. (2009). Multimodal use of semiotic resources in the construction of antiderivative. In *Proceedings of the 32nd Conference of the Mathematics Education Research Group of Australasia* (pp. 539-546). MERGA.

Valenzeno, L., Alibali, M. W., & Klatzky, R. (2003). Teachers' gestures facilitate students' learning: A lesson in symmetry. *Contemporary Educational Psychology, 28*(2), 187-204. https://doi.org/10.1016/S0361-476X(02)00007-3

Walkington, C., Wang, M., & Nathan, M. J. (2021). Collaborative gestures among high school students conjointly proving geometric conjectures In *Proceedings of the 14th International Congress on Mathematics Education*.

Weinberg, A., Fukawa-Connelly, T., & Wiesner, E. (2015). Characterizing instructor gestures in a lecture in a proof-based mathematics class. *Educational Studies in Mathematics, 90*(3), 233-258. https://doi.org/10.1007/s10649-015-9623-1

Williams-Pierce, C.C, Pier, E. L., Walkington, C., Boncoddo, R., Clinton, V., Alibali, M. W., & Nathan, M. J. (2017). What we say and how we do: Action, gesture, and language in proving. *Journal for Research in Mathematics Education, 48*(3), 248-260. https://doi.org/10.5951/jresematheduc.48.3.0248

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review, 9*(4), 625-636. https://doi.org/10.3758/BF03196322

Yoon, C., Thomas, M. O., & Dreyfus, T. (2011). Grounded blends and mathematical gesture spaces: Developing mathematical understandings via gestures. *Educational Studies in Mathematics, 78*(3), 371-393. https://doi.org/10.1007/s10649-011-9329-y

Yu, Y., & Uttal, D. H. (2021). Gestures, embodiment, and learning the rate of change. *Mathematical Thinking and Learning, 24*(3), 203-229. https://doi.org/10.1080/10986065.2020.1864574

Zurina, H., & Williams, J. (2011). Gesturing for oneself. *Educational Studies in Mathematics, 77*(2-3), 175-188. https://doi.org/10.1007/s10649-010-9294-x