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An analysis of classification skills of the 6th grade students on fractions

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ABSTRACT

The purpose of this study is to examine how the categorization skills of the 6th grade students in fractions are distributed and to determine the classification errors made by the students, along with the procedural errors of the students in fractions. The descriptive survey model, one of the quantitative research methods, was used in the study. In this study, 292 6th grade students from four middle schools in two districts of a province in Turkey participated. Developed by the researcher, the fraction operation skill test and classification skills identification test were used. The results showed that students had difficulty in distinguishing examples and characteristics of fractions and fraction types, and even though they partially succeeded in operations, they made procedural errors. The researchers recommend combining procedural knowledge with conceptual knowledge and explaining the basic characteristics of the concept by comparing examples with non-examples.

Keywords: fraction concept, classification skill, conceptual knowledge, procedural knowledge

INTRODUCTION

The concept of fractions is one of the first concepts that primary school students acquire abstractly (Celebioglu, 2014). This concept is challenging for students to understand (Behr et al., 1983; Kavuncu & Invincible, 2021; Siemon et al., 2015). However, there are several reasons for the difficulties in learning the concept of fractions beyond the epistemological structure of mathematics. One of these reasons is that the concept of fractions has different meanings, such as part-whole, ratio, division, measurement, and operator (Cetin, 2020; Kieren, 1976). Due to these different meanings, students face difficulty in learning the concept of fractions, which negatively affects their effective use of it in daily life.

The concept of fractions is a prominent learning outcome and subject in a significant part of the Ministry of National Education Curriculum (MoNE, 2018) in Turkey. It serves as a fundamental concept in the teaching of various mathematical subjects, including rational numbers, ratios, proportions, decimal notation, percentages, probability, and measurement (Fritz et al., 2019; Im & Jitendra, 2020; Pedersen & Bjerre, 2021).Since there are interconnections between the concepts within the field of mathematics, each new concept students learn is built upon the foundation of the previous ones (Cetin, 2020). Consequently, mastering the concept of fractions correctly is vital not only for understanding subsequent concepts but also for establishing the right relationships between the concept of fractions and other mathematical ideas. Moreover, a thorough understanding of fractions is essential for developing students' proportional reasoning skills (Im & Jitendra, 2020). As proportional reasoning and problem-posing skills are interconnected, they hold significant importance in mathematics courses (Karaduman, 2018). Proportional reasoning is a skill widely employed in problem-solving and is crucial for comprehending subjects such as geometry, algebra, probability, and trigonometry. Furthermore, we unconsciously apply this skill in everyday scenarios, including profit and loss calculations, interest computations, cake recipes, and making advantageous choices while shopping (Beyazit & Kirnap-Donmez, 2017; Reyna & Brainerd, 2007).

Learning mathematical concepts plays a crucial role in acquiring mathematics skills. These concepts are formed by combining our existing knowledge as the fundamental building blocks and associating them with each other (Tatli, 2020, p. 2), making a precise understanding of concepts essential for facilitating mathematics learning. Concept learning begins from birth and continues throughout life, as concepts help individuals make sense of objects, assets, or ideas they encounter (Merrill et al., 1992) and organize and classify knowledge (Birisci & Metin, 2010). This leads people to form classifications in their minds (Sezgin, 2019), learning concepts through abstraction, a mental process that allows distinguishing similarities and differences between

This study is derived from the thesis titled "An analysis of classification skills of 6th grade students on fractions" in Cukurova University.

experiences by generalizing and establishing relationships with other concepts (Yanik, 2013; Isemen, 2019). In other words, individuals categorize concepts by identifying their distinctive and non-distinctive characteristics (Layng, 2019). According to Merrill et al. (1992, p. 9), if a student can indicate the class to which this symbol, object, or event belongs when confronted with any symbol, object, or event, it means that his or her classification skill is advanced. Acquiring classification skills significantly influences the concept learning process as students distinguish between exemplary and non-exemplary ones of a concept (Kilic, 2008). This ability to distinguish exemplary from non-exemplary ones indicates that students have abstracted the distinctive and non-distinctive characteristics of the concept and its hierarchy (Akbulut-Tas & Karatas-Coskun, 2017). Conversely, students' inability to classify correctly or learn the concept accurately indicates a failure to distinguish the distinctive and non-distinctive characteristics of the concept (Kilic, 2008). Classification errors, including undergeneralization, overgeneralization, and mis-conceptualization, make learning the concept challenging (Ural, 2017, p. 13-14). Overgeneralization error occurs when students consider non-exemplary ones as examples of the concept (Coskun, 2011, p. 51; Ulusoy & Cakiroglu, 2017). On the other hand, undergeneralization error arises when students perceive exemplary ones as not belonging to the concept due to their inferences (Coskun, 2011, p. 51; Demirezen, 2011; Ulusoy & Cakiroglu, 2017; Ural, 2017, p. 14). Mis-conceptualization refers to students having difficulty in defining the concept and identifying exemplary and non-exemplary ones that belong to the concept (Demirezen, 2011). These classification errors negatively impact learning and hinder the intended learning process. When a student misconceives a concept, it becomes an obstacle to understanding other inputs like correct concepts (Luceariello et al., 2014). Hence, learning concepts accurately during the concept learning process is crucial as it enables students to acquire subsequent information by performing classification skills correctly and more easily. Reviewing relevant studies reveals that there are limited investigations on classification skills (Akbulut-Tas, Karatas-Coskun, 2017; Cokakadar, 2013; Ergin, 2014; Ergun, 2010; Fujita & Jones, 2007; Istemen, 2019; Tennyson et al., 1972; Yanik, 2013). When these studies are taken into consideration, it is seen that classification skills are mostly investigated in science and social subjects. In the mathematics course, studies predominantly concentrate on the concept of polygons and quadrilaterals. In his study, Ergin (2014) examined the images and classification strategies of 8th grade students on geometric objects. In the findings of the study, it was found that students were far away from formal definitions, described objects with examples from their surroundings, and had misconceptions that led to overgeneralizations that guided their concept images. Ergun (2010) aimed to determine the perception, definition and classification of polygons by 7th grade elementary school students. As a result of the study, it was concluded that students had difficulty in perceiving the hierarchical relationship between quadrilaterals, that they made definitions that did not include sufficient conditions when defining polygons, and that students' personal definitions of polygons were different from formal definitions. According to the findings of these studies, it was observed that students could not correctly identify the non-examples of concepts, could not make concept definitions at the formal level, and had classification errors related to concepts. However, when the literature is examined, there are many studies on the teaching of fraction concept and the difficulties and misconceptions encountered during teaching (Biber et al., 2013; Flores et al., 2020; Jiang et al., 2021; Kocaoglu & Yenilmez, 2010; Kucam & Demir, 2020; Namkung & Fuchs, 2019; Newton 2008; Ormeci, 2012; Ozaltun et al., 2020; Pesen, 2008; Soylu & Soylu, 2005; Stafylidou & Vosniadou, 2004; Sengul, 2015; Ustun, 2019; Woodward, 2017; Yetim & Alkan, 2010; Yildirim, 2019). Nevertheless, no research examining students' classification skills related to the concept of "fraction" was found among the studies conducted. The fact that the concept of fraction is an important concept that forms the basis for the teaching of many subjects and is frequently encountered in our daily lives has revealed the necessity of determining students' errors regarding the concept of fraction. This study is important because it reveals the 6th grade students' classification skills and errors about fractions and offers suggestions for students to learn the fraction concept and related concepts correctly. It is hoped that this study will provide data to curriculum developers, literature and decision makers in the context of the listed elements.

In conclusion, considering all these explanations, it is aimed to examine how the classification skills of the 6th grade students on fractions are distributed and to determine their operational errors on fractions. For this main purpose, the following sub-problems were identified:

1. How is the correct classification related to the concepts of fraction, mixed fraction, proper fraction, improper fraction, and equivalent fractions and the overgeneralization, undergeneralization, and mis-conceptualization errors that 6th grade students make in the classification skills identification test (CSIT) distributed?
2. What are the operational errors made by the 6th grade students in the fraction operation skill test (FOST) on fractions?
3. What is the relationship between the scores of the 6th grade students obtained from CSIT on fractions and FOST?

METHOD

Research Model

A descriptive survey method was used in this study. According to Creswell (2020, p. 481), the descriptive survey method, which is frequently used in education, is a quantitative research pattern in which the data are collected by surveys or one-to-one interviews of the sample group or the entire population in order to explain the attitudes, opinions, characteristics, and behaviors of the population. The survey method is a research approach that aims to describe a past or present situation as it exists, and the situation, individual, or object that is the subject of the research is tried to be defined under its own conditions and as it is (Karasar, 1995).

Study Group

The sample of the research consists of 6th grade students in four public secondary schools in two districts of a province in the south of Turkey in the 2021-2022 academic year. A total of 292 students from these schools participated in the research. In the research, the reason for choosing the 6th grade level is that the learning outcomes of fractions in 6th grade are more comprehensive than the learning outcomes of fractions in the 5th grade and that the 6th grade level is critical for the concept of rational numbers at the 7th grade level.

Since the accessibility of the researcher to the students was more convenient in the selection of the district and school in the research, the convenience sampling method was used. Convenience sampling is a non-random sampling method that works on a situation or sample that is accessible and will provide maximum savings (Buyukozturk et al., 2020, p. 95). The simple random sampling method, one of the random sampling methods, was used in student selection. Simple random sampling is a method, where each sampling unit is likely to be selected equally and is stronger than others in terms of ensuring the representativeness of sampling methods (Buyukozturk et al., 2020, p. 88).

Data Collection Tools

Classification Skills Identification Test (CSIT) and Fractional Operation Skill Test (FOST) were used as data collection tools of the research.

Classification skills identification test

With this test used in the research, it was aimed to analyze the students' skills of "correct classification", "overgeneralization", "undergeneralization," and "mis-conceptualization (misconception)" about the concepts. In the first stage of CSIT, students marked the answer for the questions, the reason for the answer they marked in the second stage, and the degree of confidence indicating how sure they were of the answer to the question in the third stage. In the research, it was accepted that the student systematically made the same mistake in similar items selected according to the characteristic to be measured of the concept shown in **Appendix A** and that there was a misconception, that is, a mis-conceptualization, about that characteristic in that student.

CSIT was presented to the opinion of four Mathematics and one Turkish teacher working in a public institution, as well as three academicians working at the university. The pilot application of the test was carried out with 268 volunteer 7th grade students studying in secondary schools. Lastly, the test analysis results of CSIT are shown in **Table 1**.

Table 1. Analysis results of classification skills identification test

Number of questions	n	Mean	Sum of squares	Median	Peak value	Average difficulty of test	KR-20
28	268	16.15	6.82	16	13	.58	.89

The average of the test applied to 268 people was 16.15, the standard deviation was 6.82, the average difficulty was .58, and the KR-20 reliability coefficient is .89. Kayis (2018, p. 405) states that if the reliability coefficient of the test is between .80 and 1.00, it means that test is highly reliable. Since the reliability coefficient of CSIT is.89 in result of test analysis, it can be said that CSIT is a highly reliable test.

The sample question of CSIT is presented in **Figure 1**, and then how the answers given to the question are coded is explained.

Which of the following is a proper fraction?

A) $\frac{3}{3}$ B) $\frac{7}{3}$ C) $\frac{9}{11}$ D) 1

( ) is option A. Because a fraction that has the equal numerator and denominator is called a proper fraction.

( ) is option B. Because a fraction whose numerator is larger than its denominator is called a proper fraction.

( ) is option C. Because a fraction whose numerator is smaller than its denominator is called a proper fraction.

( ) is option D. Because one (1) is a proper fraction since it is the smallest number.

1. Trust Degree	(1) Only guess	(2) I am not very sure	(3) I am not sure	(4) I am sure	(5) I am very sure	(6) I am completely sure

Figure 1. Example of a question measuring overgeneralization error in classification skills identification test (Güler, 2022)

The question in **Figure 1** is the first question in CSIT. This question measures the student's ability to "define the concept of proper fraction when given an example of the concept of proper fraction". In the sample question, the student makes the correct classification when he or she marks the option "C". Because the answer in option "C" is an example of a proper fraction. The students who marked this option were coded as three in the SPSS. Options "A" and "B" are examples of improper fractions, and option "D" is an example of a mixed fraction. The student who marked options A, B, and D made an overgeneralization error because they accepted the given examples as examples of the concept even though they were not examples of the concept. The students who marked these options were coded as one in the SPSS program.

The statements given below the question are sentences explaining why the student chose the option he or she marked. According to the sentence marked by the student, it was determined why the student marked that option. In the question in **Figure 1**, the answers of the students who marked the first, second, and fourth justifications were coded as zero in the SPSS program. Because it is not the right justification for the problem. The students who marked the third justification were coded as one because they marked the correct justification of the problem.

Finally, the scale that determines the degree of trust at the last stage allows us to determine how confident the student was in answering the question. These explanations were coded as zero because one, two, and three scores showed uncertainty and indecision in the SPSS program, and four, five, and six scores were coded as one because they showed certainty.

The student's achievement score for each question is obtained by multiplying the scores obtained from the three stages coded in SPSS. Another question that measures similar characteristics is the 21st question of CSIT. The coding of this question are made in the same way, and the achievement score of the student is obtained. In both questions, it was accepted that a "mis-conceptualization" error related to the characteristic of "being able to define the concept of proper fraction when given an example" was identified in students who scored zero. If any of the questions were answered correctly, overgeneralization and undergeneralization errors by the student were detected. The distribution of questions in CSIT, corresponding to the concept and its content items to be measured, is provided in **Appendix A**.

Fraction operation skill test

The questions in fraction operation knowledge test (FOKT) were organized according to the learning outcomes of the 6th grade fraction unit in the primary and secondary school MoNE curriculum (MoNE, 2018). A multiple-choice test was prepared according to the learning outcomes in the 6th grade fraction unit, considering the mistakes made by the students and the difficulties that they have in the relevant research (Biber et al., 2013; Kocaoglu & Yenilmez, 2010; Ormeci, 2012; Pesen, 2008; Soylu & Soylu, 2005; Ustun, 2019; Yetim & Alkan, 2010; Yildirim, 2019). Then, it was presented for the opinions of four mathematics teachers, one Turkish teacher working in a public institution, and three academicians working at the university. The pilot application was carried out with the participation of 268 volunteer 7th grade students studying in secondary schools. After the pilot application, item analyses of the test were performed, and items with low difficulty and reliability were excluded from the test.

The test analysis results of FOST obtained by the researcher after the item analysis are given in **Table 2**.

Table 2. Analysis results of fraction operation skill test

Number of questions	n	Mean	Sum of squares	Median	Peak value	Average difficulty of test	KR-20
20	268	11.52	5.89	12	6	.58	.91

According to **Table 2**, the average of FOST is 11.52, the standard deviation is 5.89, the mean difficulty of the test is .58, and the KR-20 reliability coefficient is .91. Kayis (2018, p. 405) stated that if the reliability coefficient of the test is between .80 and 1.00, the test is highly reliable. Since the reliability coefficient of FOST as a result of test analysis is .91, it can be said that FOST is a highly reliable test.

The sample question of FOKT is shown in **Figure 2**, and then how the answers given to the question are coded is explained.

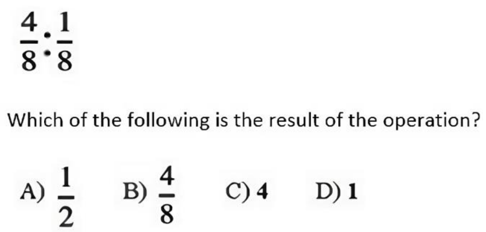

Figure 2. Question example of fraction operation skill test (Güler, 2022)

As in the achievement test, the correct answers were coded as one and the wrong answers as zero in SPSS. Which option for the students marked in the question they made a mistake was also analyzed. In the sample question in **Figure 2**, when the student marks option C, he or she answers the question correctly, and it is coded as "1" in SPSS. When the student marks other options, they are coded as "0" in SPSS. Considering the errors in the literature, it was accepted that the students who marked option B among those who answered this question incorrectly applied the rule of addition in fractions while dividing in fractions in the research. In this way, the operational errors made by the students were determined for all the questions in the test.

Data Collection

In the research, four schools were determined to collect data, and the tests were applied by the researcher to 292 6th grade students studying in the chosen schools. In practice, the fact that the test was based on voluntariness was conveyed to students, and the tests were performed only on students who volunteered. The application process of each test was planned for one lesson hour, and the application process time of the two tests was planned for two lesson hours on average.

Data Analysis

The total scores obtained by the students in FOST and CSIT, as well as the total scores obtained from the explanations they provided for the classification and the classification of the samples given in the test with respect to the concept's name, were analyzed using quantitative analysis methods. Kolmogorov-Smirnov normality analyses were performed to determine whether the students' scores in FOST and CSIT showed a normal distribution, and it was determined that the scores obtained from both tests did not meet the normality condition (for CSIT $z=.12$, $p<.01$ and for FOST $z=.14$, $p<.01$). Since the scores obtained did not show a normal distribution, Spearman Correlation Analysis among nonparametric tests was performed to determine the relationship between the scores obtained in FOST and CSIT.

FINDINGS

In this section, the data collected for the purposes of the research were analyzed, and the findings obtained as a result of the analysis and the interpretations of the findings were included.

Table 3. Frequency & percentage distribution of total scores obtained by students in CSIT & FOST

CSIT grade points	Frequency (n)	Percentage (%)	Cumulative percentage (%)	FOST grade points	Frequency (n)	Percentage (%)	Cumulative percentage (%)
2	2	.7	.7	1	8	2.7	2.7
3	2	.7	1.4	2	9	3.1	5.8
4	7	2.4	3.8	3	19	6.5	12.3
5	14	4.8	8.6	4	21	7.2	19.5
6	15	5.1	13.7	5	27	9.2	28.8
7	16	5.5	19.2	6	21	7.2	36.0
8	24	8.2	27.4	7	19	6.5	42.5
9	21	7.2	34.6	8	8	2.7	45.2
10	17	5.8	40.4	9	10	3.4	48.6
11	13	4.5	44.9	10	9	3.1	51.7
12	15	5.1	50.0	11	8	2.7	54.5
13	8	2.7	52.7	12	16	5.5	59.9
14	9	3.1	55.8	13	11	3.8	63.7
15	11	3.8	59.6	14	13	4.5	68.2
16	13	4.5	64.0	15	13	4.5	72.6
17	10	3.4	67.5	16	13	4.5	77.1
18	13	4.5	71.9	17	16	5.5	82.5
19	9	3.1	75.0	18	18	6.2	88.7
20	17	5.8	80.8	19	19	6.5	95.2
21	10	3.4	84.2	20	14	4.8	100
22	10	3.4	87.7				
23	8	2.7	90.4				
24	7	2.4	92.8				
25	8	2.7	95.5				
26	6	2.1	97.6				
27	4	1.4	99.0				
28	3	1	100				
Total	**292**	**100**			**292**	**100**	

The percentage and frequency distribution of students' scores in CSIT and FOST is shown in **Table 3**. The highest score achievable in CSIT is 28, and 1.0% of the students achieved this score. Additionally, 52.7% of the students scored less than 14 in CSIT, while 47.3% of the students scored above 14. In FOST, the highest achievable score is 20, and 4.8% of the students attained this score. Moreover, 48.6% of the students scored below 10, while 51.4% of the students scored 10 or higher.

Results Obtained in Classification Skills Identification Test in Research

The first sub-objective question of this research is, "How is the correct classification related to the concepts of fraction, mixed fraction, proper fraction, improper fraction, and equivalent fractions and the overgeneralization, undergeneralization, and mis-conceptualization errors that 6th grade students make in CSIT distributed?" The findings related to this question are presented using tables.

Table 4. Classification skills of students belonging to concept of fraction

Characteristic of concept	QN	CC		OG		UG		MC	
		F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Distinguishing between those that have an example & those that do not when given name of concept of fraction	Q23	52	17.8	-	-	18	6.2	222	76
	Q27	51	17.5	-	-	19	6.5	222	76.0
Being able to determine characteristics of concept of fraction	Q17	112	38.4	23	7.8	-	-	157	53.8
	Q28	91	31.1	-	-	44	15.1	157	53.8
Being able to find representation of fraction when given fraction with its reading	Q2	180	61.6	55	18.9	-	-	57	19.5
	Q25	187	64.1	48	16.4	-	-	57	19.5
Being able to find representation of fraction of shaded portion shown on figures	Q6	239	81.9	1	.3	-	-	52	17.8
	Q15	41	14.0	199	68.2	-	-	52	17.8
Being able to show fraction on number line	Q7	117	40.1	51	17.5	-	-	124	42.5
	Q11	148	50.7	20	6.8	-	-	124	42.5
Being able to show fraction shown on shape that is not divided into equal parts	Q12	78	26.7	9	3.1	-	-	205	70.2
	Q20	61	20.8	26	9.0	-	-	205	70.2
Being able to find verbally expressed fraction	Q14	123	42.1	-	-	29	10.0	140	47.9
	Q18	104	35.6	48	16.5	-	-	140	47.9

Note. QN: Question number; CC: Correct classification; OG: Overgeneralization; UG: Undergeneralization; MC: Mis-conceptualization; F: Frequency; & P: Percentage

When **Table 4** is examined, there are two questions in CSIT, Q23 and Q27, according to the characteristic of distinguishing the examples of the concept of fraction and the ones that are not examples by being given the name of the concept of fraction. 17.8% of the students made the correct classification in Q23, and 17.5% of the students made the correct classification in Q27. 82.2% of the students made a misclassification in Q23, and 82.5% made a misclassification in Q27. 76.0% of these misclassifications were mis-conceptualizations, and 6.2% of the students made undergeneralization errors in Q23 and 6.5% of the students made undergeneralization errors in Q27. When the students were given the name of the concept of fraction, it was determined that they made a misclassification in distinguishing between ones that are examples of the concept of fraction and ones that are not examples.

In CSIT, there are two questions, Q17 and Q28, according to the ability to determine the characteristics of the concept of fraction. 38.5% of the students made the correct classification in Q17, and 31.1% made the correct classification in Q28. 61.6% of the students made a misclassification in Q17, and 68.9% of the students made a misclassification in Q28. 53.8% of the misclassifications were mis-conceptualizations, and 7.8% of the students made overgeneralization errors in Q17 and 15.1% of the students made undergeneralization errors in Q28. It was observed that most of the students made misclassifications in determining the characteristics of the concept of fraction.

The fraction has multiple different forms of representation. Among these forms of representation, the form in which students make correct classification the most is being able to find the representation of a fraction when given the fraction with its reading, with 61.5% and 64.1% correct classification rates. In this representation, 19.5% of the students made a mis-conceptualization error, 18.9% made an overgeneralization error in Q2, and 16.4% made an overgeneralization error in Q25. The form of representation in which the students made misclassification the most was to indicate the fraction shown on the figure that is not divided into equal parts, with a 73.3% misclassification rate in Q12 and a 79.2% misclassification rate in Q20.

In this form of representation, 70.2% of the students made mis-conceptualization errors, 3.1% of the students made overgeneralization errors in Q12, and 9.0% made overgeneralization errors in Q20. In the study, it was concluded that students made an undergeneralization error because they could not distinguish the different representation forms of fractions and thought that fractions were only numbers that could be written in the form of $\frac{a}{b}$ and they did not accept $\frac{0}{3}$ and $\frac{0}{1}$ as fractions; they made an overgeneralization error by accepting $\frac{1}{0}$ as a fraction because they could write it in the form of $\frac{a}{b}$.

Table 5. Classification skills of students belonging to concept of proper fraction

Characteristic of concept	QN	CC		OG		UG		MC	
		F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Being able to determine its characteristics when given a proper fraction example	Q13	128	43.8	-	-	29	9.9	135	46.2
	Q19	138	47.3	19	6.5	-	-	135	46.2
Being able to define concept when given an example of a proper fraction	Q1	177	60.6	19	6.5			96	32.9
	Q21	158	54.1	-	-	38	13.0	96	32.9

Note. QN: Question number; CC: Correct classification; OG: Overgeneralization; UG: Undergeneralization; MC: Mis-conceptualization; F: Frequency; & P: Percentage

In the examination of **Table 5**, when the example of the proper fraction is given, 43.8% of the students made the correct classification in Q13 and 47.3% of the students made the correct classification in Q19 in determining the characteristic of the proper fraction concept. 56.1% of the students made a misclassification in Q13, and 52.7% of the students made a misclassification in Q19. Considering these data, it is seen that the rates of correct classification and misclassification of the students are close to each other, but more than half of the students cannot determine their characteristics when given an example of a proper fraction. In addition, 46.2% of the misclassifications made by the students are mis-conceptualizations. Besides, it was determined that 9.9% of the students made undergeneralization errors in Q13 and 6.5% made overgeneralization errors in Q19.

When the example of the proper fraction was given, in the skill of being able to define the proper fraction, 60.6% of the students made the correct classification in Q1 and 54.1% made the correct classification in Q21; 39.4% of the students made a misclassification in Q1 and 45.9% made a misclassification in Q21. 32.9% of the misclassifications made by the students were mis-conceptualizations; 6.5% of the students made overgeneralization in Q1, and 13.0% of the students made undergeneralization errors in Q21.Considering these results, it was concluded that more than half of the students were able to define the proper fraction concept when given an example of the proper fraction concept.

Table 6. Classification skills of students belonging to concept of improper fraction

Characteristic of concept	QN	CC		OG		UG		MC	
		F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Being able to define concept of improper fraction	Q3	136	46.6	31	10.6	-	-	125	42.8
	Q22	133	45.5	-	-	34	11.6	125	42.8
Being able to determine characteristics of improper fraction concept	Q5	108	37.0	50	17.1	-	-	134	45.9
	Q16	119	40.8	39	13.3	-	-	134	45.9

Note. QN: Question number; CC: Correct classification; OG: Overgeneralization; UG: Undergeneralization; MC: Mis-conceptualization; F: Frequency; & P: Percentage

In the analysis of **Table 6**, when given the name of the improper fraction concept, in the learning outcome of being able to define the improper fraction concept, 46.6% of the students made the correct classification in Q3, 45.5% made the correct classification in Q22, 53.4% of the students made a misclassification in Q3, and 54.4% made a misclassification in Q22. Upon examining these data, although the rate of correct classification and misclassification made by students is close to each other, more than half of the students made misclassifications. 42.8% of the misclassification errors made by the students are mis-conceptualization errors. 11.6% of the students made an undergeneralization error in Q22, and 10.6% made an overgeneralization error in Q3.

In the ability to determine the characteristics of the improper fraction concept, 37.0% of the students made the correct classification in Q5 and 40.8% made the correct classification in Q16;63% of the students made a misclassification in Q5 and 59.2% made a misclassification in Q16. In line with these results, it can be said that students had difficulty in acquiring this skill because more than half of the students showed misclassification. Of the misclassifications made by the students, 45.9% were mis-conceptualization errors, 17.1% were overgeneralization errors in Q5, and 13.3% were overgeneralization errors in Q16.

Table 7. Classification skills of students belonging to concept of mixed fraction

Characteristic of concept	QN	CC		OG		UG		MC	
		F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Being able to determine characteristics of a mixed fraction	Q4	208	71.2	7	2.4	-	-	77	26.4
	Q24	120	41.1	-	-	95	32.5	77	26.4

Note. QN: Question number; CC: Correct classification; OG: Overgeneralization; UG: Undergeneralization; MC: Mis-conceptualization; F: Frequency; & P: Percentage

When **Table 7** is examined, it is seen that 71.2% of the students made the correct classification in Q4 about distinguishing characteristics of the mixed fraction, and 41.1% of the students made the correct classification in Q24; 28.8% of the students made a misclassification in Q4 and 58.9% made a misclassification in Q24. While most of the students made the correct classification in Q4, more than half made a misclassification in Q24. The reason for this is that the example of the mixed fraction concept gives clues to the students while determining the characteristics of the mixed fraction concept in Q4. In Q24, it was directly asked to know the characteristics of the mixed fraction concept. As a result, while students can determine the characteristic of the mixed fraction concept because finding an example of it, they have difficulty in determining the characteristic of the concept directly without seeing an example of the mixed fraction.

Table 8. Classification skills of students belonging to concept of equivalent fraction

Characteristic of concept	QN	CC		OG		UG		MC	
		F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Being able to define concept of equivalent fraction	Q8	159	54.5	17	5.8	-	-	116	39.7
	Q26	154	52.7	22	7.5	-	-	116	39.7
Finding equivalent fraction examples of fraction represented on figure	Q9	106	36.3	-	-	23	7.9	163	55.8
	Q10	82	28.1	-	-	47	16.1	163	55.8

Note. QN: Question number; CC: Correct classification; OG: Overgeneralization; UG: Undergeneralization; MC: Mis-conceptualization; F: Frequency; & P: Percentage

In the analysis of **Table 8**, when the equivalent fraction concept is given, in ability to make the definition of the concept, it is observed that 54.5% of the students made the correct classification in Q8 and 52.7% of the students made the correct classification in Q26, and 45.5% of the students made a misclassification in Q8 and 47.2% of the students made a misclassification in Q26 . The analysis revealed that more than half of the students were able to define the equivalent fraction concept accurately. Among the misclassifications made by the students, 39.7% were identified as mis-conceptualization errors, while 5.8% were overgeneralization errors in Q8, and 7.5% were overgeneralization errors in Q26.

In the ability to find the equivalent fraction examples of the fraction represented on the figure, 36.3% of the students made the correct classification in Q9 and 28.1% made the correct classification in Q10; 63.7% of the students made a misclassification in Q9 and 61.9% made a misclassification in Q10. It was determined that students could not acquire these characteristics because more than half of them made misclassifications. 55.8% of the students showed mis-conceptualization errors, 7.9% showed undergeneralization errors in Q9, and 16.1% showed undergeneralization errors in Q10.

Results Obtained in Fraction Operation Knowledge Test in Research

The second sub-objective question of the research is "What are the operational errors made by 6th grade students in FOST on fractions?" **Table 9** indicates the type and content of mistakes made by students based on the options they marked in FOST questions. FOST was prepared for the purpose of detecting the operational errors made by the students on fractions. Among the learning outcomes in the 6th grade fraction in the MoNE curriculum (MoNE, 2018), there is "M.6.1.5.1. compares the fractions, orders them, and shows them on the number line" as a learning outcome. Considering the content of this learning outcome and the research in the literature, it was concluded that the mistake made in this learning outcome was due to the student's inability to develop an understanding of a part-whole relationship. It was detected that other learning outcomes arose from operational errors. For this reason, only one conceptual error related to a learning outcome has been detected in FOST.

Table 9. Distribution of errors detected in fraction operation skill test by type & content

Type of error	Content of error	Item no & option containing error
Conceptual error	Failing to establish part-whole relationship while showing fractions on number line	Q16 (B)
Operational error	When ordering fractions, confusing rule of ordering fractions with equal numerators with the rule of ordering fractions with equal denominators	Q1(B), Q6 (A), & Q11(A)
	Students expanding only number in the denominator	Q3 (B)
	Incorrect application of rule of addition	Q2 (B), Q12 (C), & Q8(A)
	Incorrect application of rule of subtraction	Q13 (A), Q14 (A), & Q4 (C)
	Applying rule of addition in division	Q5 (B)
	Not being able to predict what natural number a fraction is close to	Q7 (B)
	Incorrect application of rule of multiplying a natural number by a fraction	Q6 (A) & Q9 (A, C, & D)
	Ignoring whole number when doing operations on mixed fractions	Q10 (A) & Q20 (A)
	When doing operations on mixed fractions, doing operation on whole parts by whole parts & on fractions by fractions	Q17 (D) & Q19 (B)
	Making a mistake in dividing a fraction by a natural number	Q15 (B) & Q18 (B)

Table 9 presents the options that contain errors, along with the type and content of errors that students can make in the options signified in FOST. The mistakes made by the students were identified by looking through previous studies in the literature. According to **Table 9**, it was determined that the students who marked option "B" in Q16 made a mistake in establishing the part-whole relationship while showing the fractions on the number line. Therefore, according to the options signified in **Table 9**, the option marked by the students served our purpose of detecting the mistakes made by the students in FOST.

Table 10. Distribution of students' answers to questions for options

Items	Option A		Option B		Option C		Option D		Total	
	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)	F (n)	P (%)
Q1	68	23.3	68	23.3**	119	40.8**	37	12.7	292	100
Q2	233	79.8*	52	17.8**	6	2.1	1	.3	292	100
Q3	59	20.2	41	14.0**	53	18.2	139	47.6*	292	100
Q4	163	55.8*	19	6.5	92	31.5**	18	6.2	292	100
Q5	15	5.1	103	35.3**	164	56.2*	10	3.4	292	100
Q6	157	53.8**	25	8.6	15	5.1	95	32.5*	292	100
Q7	168	57.5*	44	15.1**	49	16.8	31	10.6	292	100
Q8	99	33.9**	26	8.9	148	50.7*	19	6.5	292	100
Q9	32	11.0**	234	80.1*	14	4.8**	12	4.1**	292	100
Q10	67	22.9**	23	7.9	37	12.7	165	56.5*	292	100
Q11	59	20.2**	51	17.5	46	15.8	136	46.6*	292	100
Q12	174	59.6*	10	3.4	98	33.6**	10	3.4	292	100
Q13	90	30.8**	7	2.4	171	58.6*	24	8.2	292	100
Q14	117	40.1**	120	41.1*	9	3.1	46	15.8	292	100
Q15	158*	54.1*	83	28.4**	26	8.9	25	8.6	292	100
Q16	48	16.4	93	31.8**	114	39*	37	12.7	292	100
Q17	18	6.2	130	44.5*	27	9.2	117	40.1**	292	100
Q18	134	45.9*	63	21.6**	42	14.4	53	18.2	292	100
Q19	18	6.2	99	33.9**	155	53.1*	20	6.8	292	100
Q20	114	39.0**	130	44.5*	24	8.2	24	8.2	292	100

Note. F: Frequency; P: Percentage; *Shows percentage of correct answer that students gave to question; & **Shows percentage of procedural or conceptual errors students made in question

When examining **Table 10**, we observe that the question that was answered correctly by students the most is Q9, with a rate of 80.0%. The majority of the students achieve the learning outcome of "multiplying a natural number by a fraction and making sense of it". However, in this question, it was seen that 11.0% of the students marked option A. Here, these students did an incorrect operation by thinking that the number in the denominator of the natural number was the same as the number in the denominator of the other fraction. 4.8% of the students marked option C and found an incorrect result by multiplying the natural number by the number in the denominator of the other fraction. 4.1% of the students marked option D and found an incorrect result by multiplying the natural number with both the numerator and the denominator of the other fraction. Another question that measures the same learning outcome is Q6. 32.5% of the students answered this question correctly.

In Q9, the result of the multiplication of a fraction with a natural number was directly asked, and in Q6, the students were asked which given fraction multiplied by a natural number would give the largest result. As can be seen, although the learning outcome was the same, the rate of students doing it correctly decreased. This is due to the fact that Q6 requires a different mental process than Q9. While directly being asked for the multiplication in Q9, in addition to the multiplication, the student is also expected to order the results found as a result of the multiplication in Q6. Therefore, based on these data, we can say that students made errors in the ordering of fractions.

The question on which students made mistakes the most was Q16, with a rate of 61.0%. It was revealed that the majority of the students could not achieve the learning outcome of "comparing fractions and orders and show them on a number line". While 31.8% of the students marked option B and showed the fractions on the number line, they could not establish a part-whole relationship. In other words, they had difficulty in writing the fraction given on the number line. While the students were dividing the two integers into equal parts, they confused the number of points between the two integers, found the value of the fractions incorrectly, and made an ordering error. 40.8% of the students answered Q1 correctly, and 46.6% answered Q11 correctly. According to these rates, it was observed that more than half of the students had difficulty in ordering fractions. It was observed that 23.3% of the students answered option B for Q1 and 20.2% answered option A for Q11 and applied the rule that "the fractions with a larger numerator are larger while ordering the fractions with the same numerator". 47.6% of the students answered Q3, which was the ordering question in fractions, correctly. In this question, since the numerator and denominator of the fractions are different, students are expected to order by expanding the fractions. However, it was determined that 14.0% of the students made an incorrect ordering in fractions by expanding only the denominator part of fractions.

Q2, Q8, and Q12 are questions that measure whether students can apply the rule of addition correctly in fractions. 79.8% of the students answered Q2 correctly, 50.7% answered Q8 correctly, and 59.6% answered Q12 correctly. Considering these data, we can say that more than half of the students applied the rule of addition correctly in fractions. 17.8% of the students marked option B in Q2, and 33.6% marked option C in Q12, and they applied the rule, as applied in the numerator, to the denominator by writing the sum of the numerators to the numerator and the sum of the denominators to the denominator in fractions. In Q8, 33.9% of the students marked option A, and while adding the natural number to the fraction, they directly added the natural number to the numerator of the fraction and wrote the result. As it is seen, when adding a fraction to a natural number, it was found that the students obtained incorrect results because they did not equalize the denominators.

Q4, Q13, and Q14 are questions that measure whether students can apply the rule of subtraction correctly in fractions. 55.8% of the students answered Q4 correctly, 58.6% answered Q13 correctly, and 41.1% answered Q14 correctly. When these three questions are evaluated, it can be said that approximately half of the students correctly applied the rule of subtraction in fractions. 31.5% of the students applied the rule, as applied in the numerator, to the denominator by writing the differences of the numerators in the numerator and the differences of the denominators in the denominator in fractions. They marked option C in Q4. Similarly, 30.8% of the students marked option A in Q13, and 41.1% marked option A in Q14.In this context, it was observed that the students found an incorrect result due to the fact that they did not write the common denominator to the result while subtracting fractions.

56.2% of the students correctly answered Q5 and correctly applied the rule of division in fractions. More than half of the students did division correctly in fractions. 35.3% of the students marked option B and also applied the rule of addition and subtraction in fractions in this question while dividing fractions.

Q7 is a question that involves the learning outcome of being able to predict which natural number a fraction is close to. This question was answered correctly by 57.5% of the students, but it was seen that 15.1% of the students did not pay attention to the decimal part by marking option B in the question and only marked by considering the whole part.

54.1% of the students answered correctly Q15, and 45.9% of the students answered correctly Q18. Approximately half of the students found the correct result by applying the rule correctly while dividing a natural number by a fraction. However, it was discovered that 28.4% of the students in Q15 and 21.6% of the students in Q18 marked option B and that they tended to divide the numbers in the numerator and denominator of the fraction that are directly divisible by the natural number when dividing a natural number by a fraction. For example, in the operation $4{:}\frac{3}{4}$, the student divided the number four by four and found the result as $\frac{3}{1}$. As a result, while the students divided the fraction by the natural number, they divided the numbers that they can divide, and the students found the incorrect result in this way.

It was determined that 56.5% of the students answered Q10 correctly and 44.5% answered Q20 correctly, and that they do operations correctly, considering the whole number part of the fraction when doing operations in mixed fractions. 22.9% of the students marked option A in Q10 and 39.0% marked option A in Q20. As a result of these markings, it was seen that the students found incorrect results by doing operations without considering the whole part in mixed fractions. For example, in the operation $\frac{1}{4}{:}3\,\frac{1}{2}$, students found $3\frac{1}{2}$ as the result by dividing fractions directly without considering the whole number "3".

It was observed that 44.5% of the students answered Q17 correctly and 53.1% answered Q19 correctly, and that they applied the rules correctly while doing operations in mixed fractions and found the correct result. It was determined that 40.1% of the students marked option D in Q17 and 33.9% of them marked option B in Q19, and that they found incorrect results by doing operations on the whole parts by the whole parts and on the fractions by the fractions in mixed fractions. For example, in the operation $1\frac{1}{2} \times 1\frac{1}{2}$, the students found one by multiplying the whole numbers by themselves, and by multiplying the fractions by themselves, they found $\frac{1}{4}$, and finally they got the result as $1\frac{1}{4}$.

Relationship Between Students' Classification Skills Identification Test & Fraction Operation Skill Test

The third sub-objective question of our research is: "What is the relationship between the scores of 6th grade students obtained from CSIT on fractiions and FOST?"

Spearman Correlation analysis, which is a nonparametric test, was performed to determine the relationship between the scores that the students obtained in CSIT and FOST. A strong, positive, and statistically significant relationship was found between the scores obtained from in classification skills test and the FOST in the Spearman correlation analysis ($r=.720>.70$, $p<.01$). The calculated correlation coefficient ($r=.720>.70$) shows that there is a high level of relationship between the two tests (Kalayci, 2016, p. 116).

DISCUSSION & CONCLUSIONS

In this study, it was aimed to examine the classification skills of the 6th grade students regarding the concepts of fraction, fraction types, and equivalent fractions and to determine the operational errors made by the students in the fraction.

It was determined that more than half of the students could not correctly classify the definitions, characteristics, examples, and non-examples of the concepts of fractions, fraction types, and equivalent fractions. The skill that the majority of the students can perform regarding the concept of fraction is the ability to identify the fraction representation when provided with its reading. On the other hand, the characteristics for which the students made mis-conceptualization (misconception) errors the most were distinguishing between those that have an example and those that do not when given the name of the concept of fraction, determining the characteristics of the concept of fraction, and showing the fraction on the shape that is not divided into equal parts from different representations of the fraction, as well. Students made undergeneralization errors the most in their ability to determine the characteristics of the concept of fraction, and overgeneralization errors in their ability to show the representation of the fraction shaded portion shown on the figures. The reason why the concept of fraction is difficult to comprehend is that the counting principles in natural numbers do not comply with the counting principles in fractions (Stafylidou & Vosniadou, 2004). For this reason, it is thought that the reason for the errors made by the students is that the students generalized the counting principles in natural numbers to the counting principles in the fraction concept, as a result of this, the students handled the numerator and denominator independently and could not understand the meaning of the fraction concept. Rodrigues et al. (2017) and Siegler and Pyke (2013) reported that many students have difficulty understanding basic concepts including the relationship between numerator and denominator and equivalent fractions. When the studies in the literature are examined, it is seen that the concept of fraction is difficult for the students to comprehend. While Peck and Jenks (1981) observed in their study that almost all of the students determined how many pieces the denominator divided the whole into, less than half of them showed that the pieces should be of equal size. In related studies, it was found that students ignored the rule of dividing the whole into equal pieces or could not divide it into equal pieces in representations of fractions (Demiri, 2013; Haser & Ubuz, 2002; Macit, 2019; Okur & Gursel, 2016; Pesen, 2008). In addition, it was observed that the students could not interpret the concept of fraction, could not establish a part-whole relationship of the concept of fraction, considered the numerator and denominator parts separately, and found that the result of a number divided by zero is zero, and when the zero was divided by a natural number, it was undefined or indeterminate (Copur Gencturk, 2021; Demiri, 2013; Deringol, 2019**;** Ersoy & Ardahan, 2003; Karaagaç & Kose, 2015; Kocaoglu & Yenilmez, 2010; Macit, 2019; Okur & Gurel, 2016; Siap & Duru, 2004; Yetim & Alkan, 2010).Besides, it was observed that students had difficulty in showing and explaining fractions on the number line, could not show the points being equal to the mixed fraction on the number line, could not divide the whole into equal parts on the number line, and could not determine between which two integers the fractions were (Demiri, 2013; Ersoy & Ardahan, 2003; Ozaltun et al., 2020; Pesen, 2008).

Another result of the research is that the students' mis-conceptualization rate is low in terms of finding the representation of the fraction when given the fraction with its reading, finding the representation of the fraction of the shaded region shown on the figures, defining the improper fraction concept, determining the characteristics of the improper fraction concept, determining the characteristics of the mixed fraction, and defining the equivalent fraction concept. Therefore, we can say that students have superficial knowledge of the concepts of fraction and fraction types, that the concept of fraction image does not deeply exist, and that the concept of fraction exists only symbolically. In the findings of the research, it can be observed that students associated the concepts with only the examples or only the characteristics of the concepts, without fully considering the distinctive and non-distinctive characteristics of the concept and distinguishing between examples and non-examples that belong to the concept. The reason for this may be the early transition to operations with number and symbolic representation without teaching students concepts such as the division of the whole into equal parts and the determination of the unit, which are involved in the formation of the fraction concept (Wijaya, 2017). It is important to emphasize the definitions of concepts and their types, the properties of concepts, examples and non-examples, and the use of concepts in different contexts before moving on to number and symbolic operations during teaching to students, in order to learn the concept correctly and to use the concepts correctly in symbolic operations.

The results of the study in FOST showed that while students were finding fractions equivalent to a fraction, they added or subtracted the same numbers from/ to the numerator and denominator. In addition, it was found that they made mistakes when comparing, ordering, and showing fractions on the number line. In particular, the students were confused about whether the number in the denominator of the fraction was the number of equal parts that should be divided between two integers or the number of points that should be placed between two integers. In the relevant studies, it was found that when comparing fractions, students made decisions solely based on the numerator or solely on the denominator. They also considered that the fraction with a larger sum of numbers was larger and believed that the fraction with a larger difference between the denominator and the numerator was also larger. Furthermore, students attempted to place fractions on the number line by keeping the numerator constant and increasing the denominator, which resulted in mistakes. Lastly, they also made errors while dividing the whole into equal parts on the number line (Alacaci, 2015; Chambers, 2020; Demiri, 2013; Hannula, 2003; Hansen, 2014; Okur & Gurel, 2016; Ozaltun et al., 2020; Stafylidou & Vasniadou, 2004).

More than half of the students are able to do the operations correctly in fractions. However, some errors were detected in most of the students. These errors are: doing operations by assuming that the number in the denominator of the natural number is the number in the denominator of the fraction multiplied; multiplying the fractions diagonally while multiplying two fractions and finding results; applying the rule of addition while dividing and multiplying the fractions; doing operations on the fractions without considering the whole number part of the fractions; finding results by doing operations on the fractions by fractions and the whole number by the whole number; finding the result by adding the numerators and denominators among themselves while adding the two fractions, adding the natural number and the numerator of the fraction directly while adding the natural number to the fraction, writing the difference of the numerators in the numerator and the difference of denominators in the denominator while finding the difference of the two fractions, and tending to divide the numbers that can be divided directly instead of applying the rule of division. However, while the students guessed which number the fractions were close to, it was seen that they did not pay attention to the decimal part of the numbers and headed for the answer only by considering the whole number part. Studies in the literature have also found that students make similar mistakes (Baidoo, 2019; Biber et al., 2013; Demiri, 2013; Ercan, 2020; Jarrah et al., 2022; Newton, 2008, Ojese, 2015; Okur & Gurel, 2016; Onal & Yorulmaz, 2017; Ozaltun et al., 2020; Soylu & Soylu, 2005; Ubah & Bansilal, 2018). The reason for these errors may be that the counting principles in natural numbers do not match the counting principles in fractions (Stafylidou & Vosniadou, 2004). Students may have made procedural errors by generalizing the procedural knowledge they learned in natural numbers to fractions. It is thought that students' inability to make sense of procedural knowledge after conceptual knowledge about fractions means that students tend to apply the operations they perform with natural numbers when performing operations with fractions.

As the final finding of our study, a positive relationship was found between CSIT and FOST. In this context, although mathematics is considered to contain more procedural knowledge, it is thought that meaningful and accurate learning cannot occur without solid conceptual knowledge. Silver (1986, p. 181-198) stated that incorrect operational information supports misconceptual information. In order for students to develop a full understanding of fractions, they need to master all five meanings of the fraction (Charalambous & Pitta-Pantazi, 2007; Hanson, 2001; Lemon, 2012). Therefore, in order tfor students to understand and develop their understanding of fractions, the conceptual understanding of fractions should be developed before the operations (Makhubele, 2021; National Council of Teachers of Mathematics [NCTM], 2014; Sharp et al., 2002, p. 18-28). Desta (2019) also defended that it is important for students to develop operational fluency in operations with fractions, but that the operational dimension alone is not sufficient in fraction operations and interpretation, and that it is equally important for students to develop solid conceptual understanding to deepen their understanding of fractions.

As a result, the study revealed that students had difficulty in distinguishing the examples and properties of fractions and fraction types, and that they made procedural errors even though they were partially successful in the operations related to the fraction subject. Considering the results of this study, it is recommended that clear and precise definitions with distinctive features should be made while teaching the concepts of fraction and fraction types to students, the most distant examples and non-examples of these concepts should be presented to the students starting from the most obvious examples of these concepts, the concepts should be explained by paying attention to the species-genus distinction in the teaching of concepts, and the differences and similarities of each type should be presented to the students in detail. The study is limited to 6th grade students and data collection tools. Researchers can conduct more in-depth studies on fractions to determine the causes of classification errors.

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APPENDIX A

Table A1. Concept & question distributions of classification skills identification test

Concept	Classification of Substances	Items
Concept of fraction	Being able to name concept of fraction & distinguish between those who have an example & those who do not	S23 & S27
	Being able to determine the characteristics of the concept of fraction	S17 & S28
	Being able to find the fraction representation when given the fraction with its reading	S2 & S25
	Beings able to find the fractional representation of the shaded portion shown on the figures	S6 & S15
	Being able to show fraction on number line	S7 & S11
	Being able to show the fraction shown on the shape that is not divided into equal parts	S12 & S20
	Being able to find the verbally expressed fraction	S14 & S18
Proper fraction	Being able to determine the characteristic of a proper fraction when given an example	S13 & S19
	Being able to define the concept when given an example of a proper fraction	S1 & S21
Improper fraction:	Being able to define the concept of improper fraction	S3 & S22
	Being able to determine the characteristics of the improper fraction concept	S5 & S16
Mixed fraction	Being able to determine the characteristics of a mixed fraction	S24 & S4
Equivalent fraction	Being able to define the concept of equivalent fraction	S8 & S26
	Finding the equivalent fraction examples of the fraction represented on the figure	S9 & S10