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Assessing the influence of primary mathematics educators' content knowledge and pedagogical content knowledge on teaching effectiveness

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ABSTRACT

The research aimed to measure the effect of educators' teaching effectiveness focusing on content knowledge (CK) and pedagogical content knowledge (PCK) as key determinants. A descriptive survey design was utilised for the study and the participating educators were selected through a simple random sampling method. Four research questions were addressed and the data obtained were analysed using correlation coefficient and multiple regression analysis. The findings indicated a lack of significant association between primary mathematics teachers' CK and teaching effectiveness ($r = 0.18$; $p > 0.05$). However, a substantial association was found between mathematics teachers' PCK and teaching effectiveness ($r = 0.35$; $p < 0.05$). Additionally, the study revealed that both teachers' content knowledge ($\beta = 0.26$; $t = 2.30$; $p < 0.05$) and pedagogical content knowledge ($\beta = 0.40$; $t = 3.58$; $p < 0.05$) are essential for effective teaching. Based on these findings, several recommendations were made.

Keywords: teaching effectiveness, content knowledge, pedagogical content knowledge, primary mathematics teachers

INTRODUCTION

Inquiry into the efficacy of mathematics teaching and learning has garnered significant attention from scholarly circles. A primary focus of this examination pertains to assessing the pivotal role of educators in influencing mathematics pedagogy and students' comprehension. Notably, research efforts underscore the significance of educators possessing specific knowledge and competencies conducive to proficient mathematics instruction (Gess-Newsome et al., 2017). Teacher content knowledge (CK) is central to this discourse, encompassing a deep comprehension of the principles and concepts integral to the subject matter (Saxena, 2015). Simultaneously, the emergence of pedagogical content knowledge (PCK), an amalgamation of pedagogical insights and content expertise has garnered considerable attention for its discernible impact on effective teaching and facilitating a comprehensive student grasp of the curriculum (Kadinga & Kapenda, 2022).

The significance of teachers' professional knowledge as an indicator of their effectiveness is underscored by various scholars. For instance, (Hafisal & Sukor, 2020), and William and Mistima (2020) highlight content knowledge (CK) and pedagogical content knowledge (PCK) as necessary categories of knowledge that contribute to effective teaching and students' comprehension of the subject matter and postulated CK as a basic fragment of teaching, shaping teacher's organization of content-specific instruction. Mathematics teachers with a strong mastery of CK can deliver their lessons more effectively, while those who struggle with CK are likely to encounter difficulties in their teaching (Izzati & Mistima, 2020). In contrast, PCK is assumed as an essential means that enables teachers to convey content knowledge to students comprehensively and effectively. PCK is the core component for a teacher in efficiently conveying knowledge that can assist in enhancing pupil achievement (Wiliam & Mistima, 2020). Thus, a teacher must be proficient in PCK as the ability of a teacher to convey content to pupils will affect the efficacy of teaching.

According to Jacob et al. (2020), PCK involves a teacher's ability to prepare materials, manage time, utilize teaching methods, and employ questioning and assessment techniques. Converging with these perspectives, it is recognized that educators adeptly communicate subject matter to students in an accessible manner, transcending mere dissemination of information to encompass

effectively fostering student understanding (Kathirveloo et al., 2014) This underscores the complex and multifaceted nature of quality teaching, necessitating a confluence of specialized knowledge areas including pedagogy, subject matter expertise, and insightful understanding of the learning process (Mishra & Koehler, 2006). Furthermore, an emerging consensus among scholarly authorities accentuates that seamlessly transmuting knowledge for students' comprehension is a hallmark skill of exemplary educators. This underscores the nuanced nature of teaching as an intricate craft that warrants a deep interweaving of diverse knowledge domains and pedagogical proficiency to realize optimal educational outcomes Kasanda and Kapenda (2015).

Statement of the Problem

Recently, there has been a noticeable decline in the teaching effectiveness of many primary mathematics educators. This research study was undertaken to investigate the potential reasons for this trend. The study was aimed at assessing the relationship between primary mathematics teachers' knowledge (CK and PCK) and their teaching effectiveness. Additionally, the study sought to analyze the impact of primary mathematics teachers' knowledge (CK and PCK), whether considered jointly or individually, on their effectiveness in teaching mathematics. This research was conducted in selected primary schools within the Ibadan Metropolis.

Research Questions

This study explored the following research questions:

RQ1 What relationship exists between mathematics teachers' content knowledge and their teaching effectiveness?

RQ2 What relationship exists between mathematics teachers' pedagogical content knowledge and their teaching effectiveness?

RQ3 What is the combined influence of teachers' content knowledge and pedagogical content knowledge on their teaching effectiveness?

RQ4 What is the relative contribution of teachers' content knowledge and pedagogical content knowledge on teachers' teaching effectiveness?

Significance of Study

The study holds significance because it addresses the principles of effective mathematics teaching, which is crucial in providing quality education. It investigated teachers' knowledge as a factor correlated with their effectiveness in teaching mathematics. The findings from this study will be pivotal as they aim to establish the importance of content knowledge (CK) and pedagogical content knowledge (PCK) in enhancing the effectiveness of primary mathematics teachers. This study's scientific contribution lies in its potential to bridge the gap between teachers' instructional practices and knowledge base, directly affecting their teaching and students' outcomes.

LITERATURE REVIEW

Over the last two decades, scholars in the field of education have explored several perspectives on teaching practices, particularly in mathematics education. These perspectives are based on empirical observations of teachers' practices and thought processes, aiming to describe what is observed and generalize. They offer models that outline the mechanisms and organizations of teachers' knowledge. The authors identify two main types of teacher knowledge, based on both positive and behaviorist backgrounds. Teacher knowledge is conceptualized as a collection of generalizations that can be identified through classroom research and effectively applied by practitioners, reflecting a positive perspective. In addition, teacher knowledge and proficiency are recognized through apparent capabilities, behaviors, and preferences, in line with a behaviorist perspective (Calderhead, 1996).

While these categories of knowledge are critical for the mastery of subject matter skills, it is imperative to acknowledge the significance of other forms of knowledge that may be equally important. For instance, knowledge of lesson structure is vital for planning and conducting lessons effectively, including transitioning between segments and explaining material clearly (Leinhardt & Greeno, 1986). Additionally, understanding learners' mental development and previous learning involvement is crucial for effective lesson design and skill execution (Garner et al., 2023).

According to Shulman (1986), the concept of content knowledge for teaching is described as acquisitionist, indicating that such knowledge is viewed as a cognitive resource that educators can acquire and possess. Shulman's perspective is founded on a transmission model of teaching and learning, wherein teachers are perceived as knowledgeable authorities who deeply understand a specific subject area. They employ various instructional strategies to communicate this understanding to students, such as well-designed tasks, clear explanations, and effective demonstrations. This model underscores the teacher's crucial role in facilitating student learning by making intricate concepts more accessible to learners (Calderhead, 1996). However, gaps exist in fully understanding the mathematical demands of teaching. Ball et al. (2001) emphasize the necessity of closely examining practice and the required mathematical understanding for effective teaching.

A comprehensive investigation into the prerequisite knowledge for effective teaching emphasizes the critical role of pedagogical content knowledge (PCK) alongside subject matter expertise. Shulman (1987) defines PCK as a multifaceted concept encompassing several key components. First, it requires a deep understanding of learners, and their unique characteristics, including prior knowledge, learning styles, and developmental stages. In addition, PCK involves familiarity with various educational contexts, including understanding the specific settings in which teaching and learning occur—urban, rural, or

suburban environments—and how these contexts shape educational practices. Teachers also need to be aware of the educational goals, purposes, and principles that guide their teaching, as well as their discipline's historical and philosophical foundations.

In contemporary educational research, PCK is regarded as equally important as subject matter knowledge. Shulman stresses the interdependence of PCK with subject matter and pedagogical knowledge. Furthermore, he also underscores the significance of articulating this knowledge through various instructional methods and strategies. By doing so, teachers can enhance student comprehension and create effective learning experiences across diverse educational environments. Ozdemir et al. (2017) further elaborate on PCK by positing that it constitutes a unique purview within teacher understanding, specifically tailored to individual topics within various subjects. They emphasize that, unlike general pedagogical knowledge which can be broadly applied across diverse teaching contexts, PCK is grounded in the specifics of particular subject areas. This specificity empowers educators to engage more profoundly with their content and adapt their teaching strategies to address the unique challenges of different topics.

Mishra and Koehler (2006) expand upon this notion by defining PCK as the foundational knowledge base for effectively depicting and creating educational concepts. They contend that PCK necessitates a thorough understanding of the subject matter and pedagogical strategies that enhance student learning. This dual focus equips educators with the skills to devise appropriate instructional techniques that resonate with students and foster a deeper level of comprehension. This aligns with the perspectives articulated by Shulman (1986), and De Jong and Van der Leij (1999) regarding pedagogical content knowledge. Moreover, it encompasses familiarity with procedures for evaluating students' comprehension and identifying their misconceptions, as well as strategies for bridging students' learning with their existing knowledge and rectifying any misconceptions they may harbor. Pedagogical content knowledge (PCK) is a critical component that enables teachers to effectively convey knowledge, ultimately enhancing student achievement (Wiliam & Mistima, 2020).

Mathematics teachers' pedagogical knowledge has prompted a series of questions, including inquiries into how mathematics educators can enhance their teaching skills and effectively facilitate student learning. Scholars from diverse fields, such as psychologists, mathematics educators, and curriculum experts, have contributed to various perspectives. Research indicates that trainee teachers with high PCK can identify solutions when faced with challenges in the teaching and learning process (Kugra et al., 2021). PCK is also vital for predicting and addressing student misconceptions (Ansah et al., 2020), allowing teachers to tackle difficulties that arise during lessons effectively.

It is widely recognized in the educational field that a teacher’s knowledge of mathematics, pedagogical skills, and perception of a student's mathematical development are critical factors that significantly impact students’ achievement in mathematics (Mushtaq et al., 2018). While well-defined standards, comprehensive curricula, high-quality instructional resources, and effective evaluation methods are essential components of education, they are insufficient to enhance students' mathematical learning outcomes. A holistic approach that emphasizes the interplay between these various educational elements is necessary for fostering an in-depth knowledge of mathematics.

Conceptual Review

In mathematics education, the differences among content knowledge (the understanding of subject matter), pedagogical knowledge (the strategies and techniques used in teaching), and pedagogical content knowledge (the integration of both content and pedagogical knowledge) are fundamental to conceptualizing teacher knowledge (Keller et al., 2016). Grasping these distinctions is essential for effective teacher development and practice.

The differentiation between content knowledge and pedagogical content knowledge has generated considerable attention among researchers and educators. This focus has led to extensive studies aimed at gaining insight into how teachers can effectively bridge the gap between their mathematical expertise and their ability to communicate that knowledge in an engaging and accessible manner for students. Consequently, evolving conceptualizations of teacher knowledge continue to shape practices in teacher training and professional development, underscoring the significance of proficient skills and know-how that combine strong mathematical understanding with effective teaching strategies.

The mathematical knowledge for teaching (MKT) model, developed by Ball et al. (2005), offers a comprehensive framework for understanding the different facets of a teacher’s content knowledge in mathematics. This model identifies three key components of content knowledge:

1. Common content knowledge (CCK) refers to the mathematical knowledge that is generally expected of anyone proficient in the subject. This includes basic mathematical concepts, procedures, and the ability to solve standard problems.
2. Specialized content knowledge (SCK) encompasses the unique understanding and skills that teachers need specifically to teach mathematics effectively. This involves knowing how to represent mathematical ideas, how to explain the reasoning behind specific mathematical concepts, and how to address students' misunderstandings.
3. Horizon content knowledge (HCK) involves an awareness of how mathematical concepts extend beyond the current curriculum. It includes knowledge of concepts that students will encounter in the future and the interconnections among various mathematical topics.

In addition to these components of content knowledge, the MKT model also highlights the significance of pedagogical content knowledge (PCK), which is essential for effective teaching. This category is further divided into three areas:

- Knowledge of content and students (KCS) focuses on understanding how different students comprehend mathematical concepts, what common misconceptions they may have, and how their prior knowledge influences their learning.

- Knowledge of content and teaching (KCT) examines the strategies and methods that teachers can employ to effectively convey mathematical ideas, including the use of appropriate teaching materials and techniques that enhance student learning.
- Knowledge of content and curriculum (KCC) involves an understanding of how mathematical content is structured within the curriculum, allowing teachers to align their instruction with educational standards and learning goals.

Research has demonstrated the critical role that these components of the MKT model play in fostering effective teaching practices (Jakobsen et al., 2013). This underscores the importance of the model in informing educational research and enhancing teacher training programs aimed at improving mathematics instruction. Teachers must possess a nuanced understanding of the mathematical ideas relevant to their curriculum, enabling them to go beyond the surface-level content that their students are expected to master. This depth of understanding should ideally exceed that of their students (Aguilar, 2021), thereby empowering teachers to facilitate a more meaningful and effective learning experience in the classroom.

Mathematics is recognized as a synthesis of logic and mental skills (Hannula et al., 2004). Research examining the influence of teachers' subject knowledge on student performance in South African schools indicates that, while a profound understanding of the subject matter is essential, the capability to convey this knowledge to learners effectively—utilizing suitable skills and strategies—holds greater importance. This has led to further investigation into the knowledge, approaches, and skills employed by mathematics educators in their pedagogical methodologies (Voogt et al., 2012).

The concept of "effective teaching" and its assessment criteria have generated considerable debate on a global scale, largely due to the absence of a universally accepted definition, which emerges from the varied perspectives among scholars. Effective teaching is often closely associated with teacher effectiveness, which can be understood through three primary dimensions identified by Evans and Green (2006):

(i) the unique personalities and characteristics of teachers,
(ii) the nature of interactions between teachers and students, and
(iii) the impact teachers have on student behavior and learning outcomes.

An effective educator is not merely someone who follows established methods; rather, they are those who can adapt their knowledge and skills to respond to the diverse and dynamic demands of various educational contexts. They demonstrate a commitment to meeting their objectives through flexible teaching strategies and an openness to different approaches, showing a willingness to pivot as necessary to enhance student understanding.

Research has highlighted several critical factors that contribute to successful mathematics instruction, allowing us to conceptualize pedagogical content knowledge, particularly within the realm of mathematics. Central to effective instructional practice are mathematical tasks, which not only occupy a significant amount of lesson time but also serve as the main vehicles through which students engage with and construct knowledge (Clarke & Roche, 2018). When chosen and implemented with care, these tasks become more than mere exercises; they form the basis for students' intellectual engagement and provide meaningful opportunities for deeper learning (Verschaffel et al., 2002). Effective pedagogical knowledge is essential for delivering knowledge to students (Nurmelda & Roslinda, 2020). Thus, to facilitate an effective teaching and learning process, teachers must not only be familiar with CK but also be proficient in PCK.

Moreover, effective educators must be attuned to their students' existing knowledge and prior understandings. Recognizing that students will inevitably bring misconceptions and errors to the learning environment offers valuable insights into their intrinsic thought processes. Understanding such misconceptions and comprehension challenges is paramount for any educator. With the right instructional aid—such as varied explanations, visual representations, or concrete examples—educators can help students work through their misunderstandings and ultimately lead them to successfully construct knowledge (Mayer, 2004). Consequently, mastering effective mathematics-specific instructional techniques becomes a fundamental and indispensable element of pedagogical content knowledge.

Nonetheless, the interpretations of what constitutes comprehensive subject knowledge vary significantly. This variability is particularly pronounced when examining the complexities surrounding content knowledge (CK) and pedagogical content knowledge (PCK) within the distinct field of mathematics (Baumert et al., 2010). Furthermore, an individual's success in mathematics is influenced by their understanding, attention, and confidence—qualities that heavily rely on a teacher's ability to effectively guide and connect students' cognitive processes with critical reasoning skills through logical and engaging presentations. This includes the combination of diverse perspectives and the fostering of higher-order thinking abilities within students. The multifaceted nature of teaching and learning processes prompts ongoing examination and analysis of general teaching methods. This scrutiny often leads to reflective practices that, while enlightening, may yield inconclusive outcomes regarding the best approaches to enhance educational effectiveness (Jagals & Van der Walt, 2018).

Empirical Review

The assumed disparity between content knowledge (CK) and pedagogical content knowledge (PCK) presents significant challenges due to the lack of empirical certainty surrounding these categories. Previous studies suggest that the level of PCK among mathematics teachers, including secondary school teachers, pre-service teachers, and those specializing in mathematics, is relatively low (Aksu, 2019; Kugra et al., 2021; Moh'd et al., 2021; Nurmelda & Roslinda, 2020). Research findings also indicate that trainee teachers possess insufficient PCK (Aksu, 2019; Kugra et al., 2021). Trainee teachers often struggle with comprehending subject matter, developing goals and objectives, and selecting effective strategies (Kugra et al., 2021). Additionally, they face challenges in creating relevant and effective teaching aids, as they often do not consider PCK when designing their lessons (Fiangga et al., 2021).

Table 1. Number of schools and participants per LGA

LGAs	Number of schools	Number of participants (Teachers)
A – Akinyele	13	68
B - Ibadan North East	3	10
C - Ibadan North	5	20
D – Lagelu	4	16
Total	25	114

A limited number of studies have empirically examined these two forms of knowledge, and some researchers have intentionally refrained from assessing mathematics teachers' content knowledge. This avoidance is driven by a desire to mitigate the perception that teachers are being "tested," which could lead to negative associations with the assessment process. In a notable study conducted by Moh'd et al. (2021), it was discovered that primary mathematics teachers often lack sufficient mastery of the subjects they are assigned to teach. This deficiency indicates substantial gaps not only in common content knowledge—an essential understanding of the subject matter—but also in pedagogical content knowledge, which pertains to the effective teaching of that subject, and in subject content knowledge, which encompasses a deeper comprehension of mathematics itself.

Some studies suggest a considerable overlap between the two categories of knowledge, indicating that there may be a common underlying factor related to effective mathematics instruction (Kahan et al., 2003). However, contrasting findings from Phelps and Schilling (2004) in the domain of reading identified clear separations between categories of knowledge, without evidence of a shared factor. These conflicting outcomes may be attributed to variations in knowledge structures across different teacher populations. They imply that the degree to which PCK and CK are separable might depend on teachers' varying stages of proficiency. Notably, prior experiential efforts to develop models of teacher knowledge have predominantly focused on elementary educators, who are often perceived as having lesser expertise in their subject area.

Ball et al. (2005) assert that achieving meaningful progress in education is unlikely without a straight focus on teaching practices themselves. They emphasize that teachers' mastery of mathematics is pivotal to fostering educational advancement. This assertion has catalyzed extensive discussions and research on teachers' subject matter knowledge, particularly pedagogical content knowledge, and mathematical knowledge for teaching. Some scholars argue that the effective teaching of mathematics necessitates an understanding that transcends basic subject matter knowledge, which they refer to as mathematical knowledge for teaching (Ball & Bass, 2000). Despite this understanding, earlier studies have often overlooked the mathematical challenges that arise in everyday learning scenarios when assessing teachers' pedagogical content knowledge. On the other hand, some research has shown that teachers with strong and extensive mathematical knowledge are uniquely positioned to facilitate student learning and improve their problem-solving skills (Carpenter et al., 1996). In this regard, a teacher's comprehension of mathematical concepts is fundamental to effectively supporting students in their academic pursuit (Mushtaq et al., 2018).

A wealth of studies supports the notion that teachers with deep-rooted knowledge of their subjects are particularly effective in their roles (Kousar et al., 2023). Robust evidence indicates a strong relationship between teachers' content knowledge and student performance in mathematics. Research has consistently shown that the quality of a teacher—reflected in both their content knowledge and pedagogical skills—exerts a more substantial influence on student performance than factors such as students' previous educational records or the overall quality of the school they attend. Furthermore, findings suggest that the performance gap between students receiving instruction from highly effective teachers and those teaching less effective teachers tend to expand yearly (Odumosu et al., 2018). This pattern underscores the notion that substantial improvements in student performance are most probable to occur when students enjoy continuous, high-quality tutoring from effective teachers over successive periods. This factor is particularly important given the collective nature of mathematics education.

METHODOLOGY

The study utilized survey and observation design through a quantitative process to gather information from primary mathematics teachers in some selected schools in Ibadan Metropolis. A survey design was used for the CK and PCK questionnaires, while an observation design was used to rate teachers' teaching effectiveness. The target population of this study included all mathematics teachers in primary schools in Ibadan, Oyo State, Nigeria. The researchers randomly selected four of the eleven local government areas (LGAs) in Ibadan metropolis, Oyo State, Nigeria. A proportionate sampling technique was used to select twenty-five primary schools from the four selected LGAs. Various targeted sample schools were drawn randomly. The numbers of schools and participants involved in the study are indicated in **Table 1**.

Out of the 150 teachers originally selected for the study, 114 (76%) agreed to participate. Notably, 10 teachers in LGA (A), 4 in two schools in LGA (B), 10 in one of the schools in LGA (C), and 4 in two different schools in LGA (D) declined to be part of the study, totaling 36 teachers who refused participation.

The study employed a structured questionnaire comprising three sections: Teachers' content knowledge (CK), teachers' pedagogical content knowledge (PCK), and an observation checklist on teaching effectiveness (TEC). Each participant independently completed the questionnaire containing sections A, B, and C, gathering biographic information, general content knowledge questions, and pedagogical content knowledge inquiries, respectively.

To ensure validity, experts in mathematics education independently reviewed the instruments. Reliability was assessed using the Cronbach alpha method, yielding reliability coefficients of 0.73, 0.87, and 0.89 for CK, PCK, and TEC, respectively. The

Table 2. Correlation matrix showing the relationship between teachers' content knowledge and teachers' effectiveness

Variables	Teachers' Effectiveness	Content Knowledge
Teachers' Effectiveness	1	.178
Content Knowledge	.178	1

Table 3. Correlation matrix showing the relationship between teachers' pedagogical content knowledge and teachers' effectiveness

Variables	Teachers' Effectiveness	Content Knowledge
Teachers' Effectiveness	1	.141
Pedagogical Content Knowledge	.141	1

Table 4. Multiple regression analysis showing the combined influence of teachers' content knowledge and pedagogical content knowledge on their teaching effectiveness

Sources of Variance	Sum of Squares	Df	Mean Square	F	Significant
Regression	417.008	2	208.504	7. 693	.001*
Residual	1815.997	67	27.104		
Total	2232.986	69			
R = .432					
R Square = .162					
Adjusted R Square = .162					
Std. Error of the Estimate = 5.20616					
*denotes significance at p<0.05					

researcher administered the instruments to the respondents in all participating schools. Subsequently, the data was analysed using a correlation coefficient and multiple regression analysis for comprehensive evaluation.

RESULTS AND DISCUSSIONS

RQ1 What is the relationship between mathematics teachers' content knowledge and their effectiveness in teaching?

Table 2 presents findings that indicate a positive yet weak correlation between teachers' content knowledge and their effectiveness in the classroom, with a correlation coefficient of r = .18. However, it's important to note that this correlation is not statistically significant, as indicated by a p-value greater than 0.05. This means that although there is a slight positive relationship between the level of content knowledge possessed by teachers and their overall effectiveness in teaching, this relationship is not strong enough to be considered reliable or meaningful in a statistical context.

The study underscores that while teachers may benefit from strong content knowledge, the correlation suggests that other factors may also play significant roles in influencing teaching effectiveness. This finding aligns with the theoretical framework proposed by Ball et al. (2005), which argues that possessing robust content knowledge is a prerequisite for delivering effective instruction. The nuances of this relationship highlight the complexity of teaching effectiveness and suggest that enhancing teachers' content knowledge could potentially support their effectiveness, even though the current evidence reflects a weak correlation.

RQ2 What relationship exists between mathematics teachers pedagogical content knowledge and their teaching effectiveness?

Table 3 illustrates a positive, albeit weak, correlation between teachers' pedagogical content knowledge (PCK) and their effectiveness in the classroom, with a correlation coefficient of r = .14. Importantly, this correlation is statistically significant ($p < .05$), suggesting that there is a meaningful relationship between the two variables.

Although the strength of the correlation is modest, its significance underscores the crucial role that teachers' PCK plays in enhancing their overall effectiveness. This finding is consistent with the perspective put forth by Shulman (1987), who argued that PCK represents one of the most essential knowledge bases necessary for effective teaching. Shulman's work emphasizes that possessing a deep understanding of subject matter alone is not enough for successful teaching; educators must also be adept at conveying that knowledge in a way that is accessible and engaging for their students.

Further reinforcing this idea, Ball and Bass (2000) contended that effective mathematics instruction demands a level of knowledge that extends well beyond basic subject matter expertise. This suggests that teachers must acquire additional competencies on how to teach mathematical concepts effectively, thereby illustrating the multifaceted nature of pedagogical effectiveness. The findings from **Table 3**, therefore, highlight the importance of developing teachers' PCK as a vital component in fostering effective educational practices.

Table 4 presents compelling evidence that the combined influence of teachers' content knowledge and pedagogical content knowledge plays a crucial role in their overall effectiveness as educators. The statistical analysis yields a significant result ($F(2, 67) = 7.69$; Adj $R^2 = 0.16$; $p < .01$), indicating that the relationship is not due to chance. Specifically, the adjusted R-squared value of 0.16 reveals that these two types of knowledge—content knowledge (understanding of the subject matter) and

Table 5. Multiple regression analysis showing relative contributions of teachers' content knowledge and pedagogical content knowledge in terms of their impact on teaching effectiveness

Model	Unstandardized Coefficients		Standardized Coefficient	T	Sig.
	B	Std. Error	Beta (β)		
S(Constant)	29.980	2. 742		10.933	.000
Content Knowledge	.619	.269	.259	2.304	.024*
Pedagogical Content Knowledge	1.027	.287	.402	3.576	.001*
*denotes significance at p<.05					

pedagogical content knowledge (the ability to convey that knowledge effectively)—together account for 16% of the variability observed in teaching effectiveness among the studied teachers.

This finding underscores the idea that effective teaching is not solely reliant on mastery of the subject. Instead, it highlights the importance of integrating different domains of knowledge. This notion is supported by the research conducted by Kasanda and Kapenda (2015), which asserts that a synthesis of various forms of knowledge is vital for achieving positive educational outcomes. Furthermore, the work of Keller et al. (2016) reinforces this perspective, emphasizing that the meaningful integration of both content knowledge and pedagogical knowledge is essential for a comprehensive understanding of what constitutes effective teaching practices. By acknowledging the critical interplay between these knowledge domains, educators can better enhance their instructional strategies and ultimately improve student learning outcomes.

Table 5 presents data indicating that both teachers' content knowledge and pedagogical content knowledge have significant contributions to predicting teachers' effectiveness in the classroom. Specifically, the analysis reveals that the coefficient for teachers' content knowledge is β = .26, with a t-value of 2.30 and a p-value of .02, which is below the threshold of .05, indicating statistical significance. Conversely, pedagogical content knowledge shows an even stronger impact, with a coefficient of β = .40, a t-value of 3.58, and a p-value of .00, also indicating strong statistical significance.

These findings suggest that both types of knowledge—content knowledge regarding the subject matter and pedagogical content knowledge regarding how to teach that subject—are essential for enhancing teachers' overall effectiveness. Mushtaq et al. (2018) underscores this importance in his research, pointing out that a teacher's deep understanding of mathematics, combined with effective pedagogical skills, and an accurate perception of students' mathematical development are critical elements that significantly influence educational outcomes. This highlights the necessity for teacher training programs to focus on both content mastery and teaching strategies to foster better learning environments.

CONCLUSION

The findings of this study indicate that there is no definitive connection between primary mathematics teachers' content knowledge (CK) and their overall teaching effectiveness. This suggests that simply having a strong understanding of mathematical concepts may not be sufficient to enhance student learning outcomes. Conversely, the study highlights a significant relationship between teachers' pedagogical content knowledge (PCK) and teaching effectiveness. This implies that how well teachers understand and can convey mathematical concepts plays a crucial role in their ability to engage students and facilitate learning.

Consequently, it can be concluded that both content knowledge and pedagogical content knowledge are essential components of effective mathematics teaching. Teachers are central to the educational process, particularly in the field of mathematics, where the complexity of concepts can challenge students' understanding. Their effectiveness in the classroom is largely dependent on their capability to blend substantial content knowledge with well-developed pedagogical skills. Moreover, this integration is key to creating a learning environment that captivates students' interest and encourages a deeper comprehension of mathematical principles. By doing so, teachers enhance their instructional practices and foster a more meaningful learning experience for their students. Hence, investing in the development of both CK and PCK can greatly improve the quality of mathematics education.

Recommendations

These recommendations were proposed to enhance the professional development of mathematics teachers following the outcomes of the study:

1. The National Teachers Institute (NTI), in partnership with the Universal Basic Education Commission (UBEC), should prioritize the organization of comprehensive in-service training programs aimed specifically at primary mathematics teachers. These programs should focus on building a strong foundation in both mathematical content and pedagogical content knowledge, with particular emphasis on teaching strategies that effectively engage students and enhance their learning experiences.
2. To stay current with educational advancements and innovative teaching methodologies, professional development courses in mathematics must be offered regularly. These courses should encompass contemporary mathematical theories, instructional strategies, and the integration of technology in teaching, ensuring that teachers are equipped with the latest knowledge and practices.
3. School principals and unit heads have to actively promote and support their teaching staff in attending seminars and professional development courses. Creating a supportive environment where teachers feel encouraged to pursue further

education and training is critical. Additionally, schools could facilitate access to relevant resources, funding, and flexible schedules that allow for professional growth with minimal disruption to teaching responsibilities.

Limitations of the Study

Most teachers were unwilling to participate, citing the time-consuming nature of the study as their primary concern. Relying solely on self-reported data also presents a limitation. Initially, both the school authorities and the teachers were hesitant to engage in the study, as they believed it would consume too much of their time. The researcher had to spend time persuading them to grant their consent and cooperation.

Suggestions for Further Study

The findings of this study could potentially be expanded to other states in Nigeria. Future researchers may consider conducting similar studies on various subjects offered in secondary schools. Investigating the impact of school location and type could also provide valuable insights, as these factors play a significant role in determining the integration of ICT among Mathematics teachers.

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