Review Article

MODESTUM

Digital technologies and mathematics teaching: An analysis of teacher professional knowledge

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ARTICLE INFO	ABSTRACT
	Professional teaching knowledge interferes with the planning and development of classes, which also relates to integrating digital technologies in the classroom. This study aimed to understand and discuss teachers' professional knowledge for teaching mathematics with digital technologies. The methodology of this systematic literature review was composed of selecting 42 scientific articles in the ERIC, Latindex, Scopus, Scielo, and Web of Science databases and vertical and horizontal analyses of the indicated studies. The results suggest that the studies have similar structures with similar theoretical frameworks, objectives, and methodological procedures, identifying and classifying teacher professional knowledge based on knowledge achieved or that still needs to be developed. The teachers mobilize their knowledge by recognizing the potential of digital technologies for teaching mathematics critically and reflectively. **Keywords:** teacher professional development, systematic literature review, mathematical knowledge, technological knowledge

INTRODUCTION

Digital technologies are increasingly present in everyday life. In 2020, they took the leading role in all contexts, including school. However, national and international studies point out that the integration between digital technologies and the teaching of mathematics has faced barriers that hamper its implementation in classrooms (Bulut & Isiksal, 2019; Maciel et al., 2020; Niess, 2013; Polly, 2014; Rocha, 2021; Sampaio, 2016; Zambak & Tyminski, 2020). Thus, we highlight the importance of research on teachers' professional knowledge, understanding it as a determinant for the actions to be taken and influencing the teaching and students' learning.

This study aims to understand and discuss teachers' professional knowledge of and in mathematics teaching with digital technologies. Therefore, we will discuss the results based on the following investigative questions:

1. What is the research structure on teachers' professional knowledge of mathematics teaching with digital technologies?

2. How is teachers' professional knowledge explored and understood in situations of mathematics teaching with digital technologies?

3. How does integrating digital technologies in mathematics classes mobilize teachers' professional knowledge?

For this, we carried out a systematic review of scientific articles. We used the concept of systematic literature review based on research by Depaepe et al. (2013) and Stahnke et al. (2016), justifying the term based on the systematic criteria for selecting and excluding articles. The following section presents the main theoretical models that investigate teachers' knowledge. Next, we will describe the methodological procedures and the analyses performed.

TEACHER PROFESSIONAL KNOWLEDGE: THEORETICAL MODELS

The concern with understanding and researching teachers' professional knowledge is not current. Shulman's (1986, 1987) research contributed to conceptualizing a knowledge base for the teacher, among them the *pedagogical content knowledge* (PCK), characteristic of the teaching profession.

From this knowledge base, Ball et al. (2008) established domains of mathematical knowledge for teaching (MKT), modeling the previous results for mathematics. The model became known as *MKT* and highlights the distinction between content knowledge
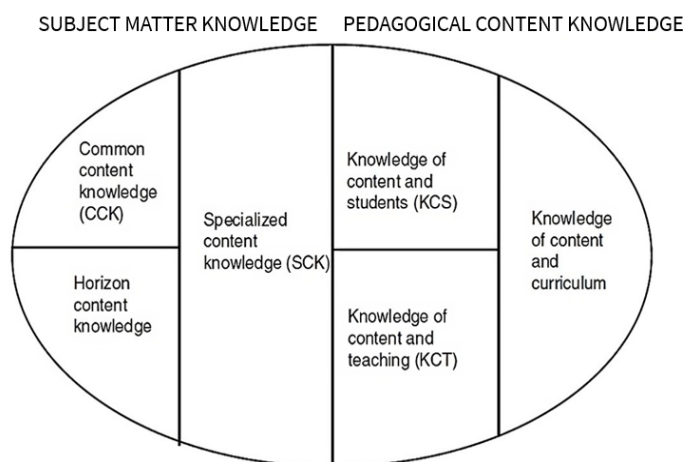
Figure 1. Domains of mathematical knowledge for teaching according to MKT (Ball et al., 2008)
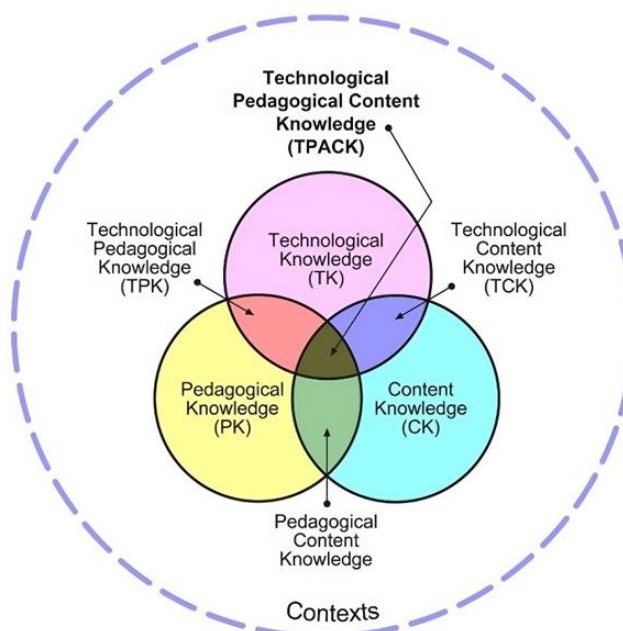

Figure 2. Representation of TPACK theoretical model (Koehler & Mishra, 2009)

and pedagogical knowledge inherent in the teaching profession. To that end, they determined the *subject matter knowledge* and the *pedagogical content knowledge*, respectively, and subdivisions that covered the students' knowledge, curriculum, and the organization of mathematical contents through schooling. A representation of the model can be found in **Figure 1**.

With the advancement of digital technologies, the studies incorporated the technological domain into the knowledge base. Koehler and Mishra (2009) developed the *technological pedagogical content knowledge* (TPACK), an expansion of Shulman's (1986) results on PCK that encompassed content knowledge, pedagogy, and technology. While MKT had two major domains and three disjoint subdivisions for each, TPACK was constituted precisely by the intersection between its three main domains. In **Figure 2**, we identify the three domains for this model (content, pedagogy, and technology), the knowledge formed by the two-by-two intersections and the union of all components that results in TPACK.

Although TPACK includes a technology-specific component, it was not developed strictly for mathematics. Thus, Getenet et al. (2016) developed *specialized technological and mathematics pedagogical knowledge* (STAMPK), integrating TPACK results with MKT. Considering that specific models for content would be more relevant than general models, the authors were inspired by MKT and used a structure similar to TPACK for the formation of STAMPK, having the *technological knowledge*, the *specialized pedagogical knowledge* and the *specialized mathematics knowledge* as the three domains that, together, promote STAMPK. But those models' results and structure point to a checklist based on knowledge teachers acquired (or not). We understand that establishing minimum knowledge necessary for teaching work is fundamental. We also reflect that complexity of the relationships, classification of situations, and activities that permeate teachers' practice should be analysed in professional education.

Rowland et al. (2005) developed a theoretical analysis tool to identify the knowledge mobilized by teachers, the *knowledge quartet* (KQ). The purpose of this tool was not to list the knowledge necessary for the teacher but to help teachers reflect individually or collaboratively (Gumiero & Pazuch, 2020). KQ was developed from the teacher's practice and aimed at teaching mathematics, consisting of four dimensions indicated in **Figure 3**.
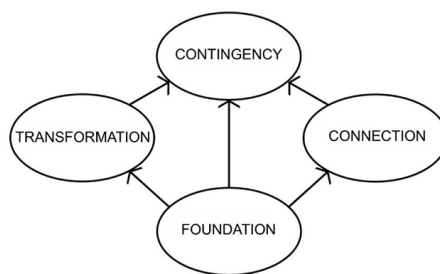
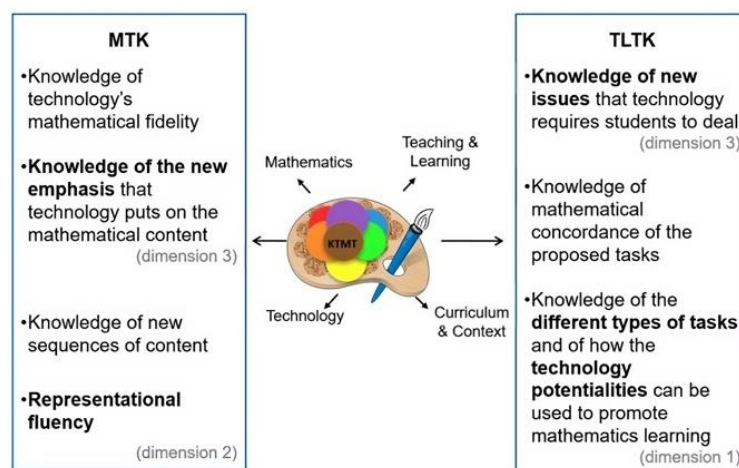**Figure 3.** KQ dimensions (Rowland & Weston, 2019)

Figure 4. Representation of theoretical model KTMT (Rocha, 2013)

Each dimension comprises codes that indicate the teachers' actions and help them reflect on their practice. As shown in **Figure 3**, *foundation* represents the starting point for the other dimensions, portraying beliefs and knowledge built by teachers throughout their education and experiences. The dimension *transformation* indicates the teacher's work to transform their knowledge into something the students can understand, similar to PCK presented by Shulman (1986). In *connection*, we have codes that describe the coherence and sequence in planning and teaching and the relationships between mathematical contents. Finally, the ability to respond to unexpected events convincingly relates to *contingency*. However, technology was not considered, and this may be an expansion in KQ codes indicated by Gumiero and Pazuch (2021).

Inspired by MKT, but wary of the questions raised by KQ about just listing the necessary knowledge for the teacher, Rocha (2013) developed the theoretical model entitled *knowledge for teaching mathematics with technology* (KTMT) to unite research results on teacher professional knowledge and the integration of digital technologies for mathematics teaching. With a structure like MKT and attentive to professional practice like KQ, KTMT can be identified in practice and informed by it. It is composed of three basic knowledge domains and their intersections.

Figure 4 illustrates the model from a metaphor with the painter's working tool. At KTMT, the curriculum and context in which the teacher works permeate the process and all knowledge, represented by the painter's palette. The primary colors–red, blue, and yellow–indicate the three basic knowledge domains: *mathematics*, *teaching and learning,* and *technology*, respectively. The intersections between the technology and other domains result in interdomains, i.e., new knowledge represented by secondary colors. Purple, although not indicated in **Figure 4**, characterizes math-specific PCK and is not classified as an interdomain. Orange and green symbolize the interdomains of *mathematics and technology knowledge* (MTK) and *teaching and learning and technology knowledge* (TLTK), respectively (Rocha, 2020).

The author presents representative knowledge for each interdomain, highlighting how technology integration changes mathematics, teaching, and learning. Although it is similar to the structure of intersections of TPACK, Rocha (2020) highlights that the metaphor of the palette expresses a significant difference in the idea of intersection for KTMT. Secondary colors are those created by merging two primary ones. However, they can take on different shades depending on the number of primary colors mixed. Therefore, increasing a single primary color may or may not change the hue of the secondary color. Considering colors as knowledges, the increase in knowledge of a basic domain may not directly increase interdomain, while for TPACK this would occur directly. Therefore, after a brief description of the theoretical models referring to teachers' professional knowledge, we will present the methodological procedures that guided the systematic literature review.

RESEARCH METHODOLOGY

A systematic literature review differs from other review and/or mapping research by establishing strategies and inclusion and exclusion criteria in a critical and transparent manner (Vosgerau & Romanowski, 2014).

Table 1. Result of survey on Scielo based on descriptors used

New descriptors	Articles found	Articles selected after reading
"teachers' knowledge" AND "mathematics"	10	0
"conhecimento docente" AND "matemática"	3	0
"conhecimento do professor" AND "matemática"	2	0
"conhecimento" AND "ICT" AND "matemática"	4	2
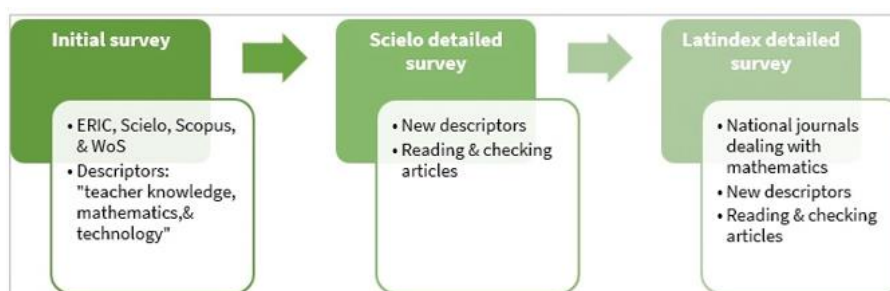
Figure 5. Description of processes carried out for selection of articles (Source: Authors' own elaboration)

It performs a deep analysis to answer specific research questions (Mandujano & Juárez, 2022; Reis et al., 2018). We present all the steps in detail for the readers to understand our choice and analysis of the articles that compose this study.

For the systematic literature review, we decided that the articles should be peer-reviewed and published in national and international journals in English or Portuguese. Initially, we searched on ERIC, Scielo, Scopus, and Web of Science (WoS) databases between November 2021 and December 2021. Scopus and WoS were chosen because they are leaders in global citations and are widely used in systematic reviews and meta-analyses (Zhu & Liu, 2020). We added the ERIC database as it covers the educational research domain (Bagger et al., 2020) and, finally, we included the Scielo because it is a national database and covers Brazilian journals.

We stipulated 10 years (from 2012 to 2022) as a selection criterion. Then, we defined the descriptors that had to be included in the abstract, title, and/or keywords of the articles. The descriptors were "*teacher knowledge*" (in quotes to ensure a unique search term), *mathematics*, and *technology*. This was the inclusion criterion established for this systematic review.

As a result, we found 44 articles in *Scopus*, six in WoS, and 259 in ERIC. There were no registers in the Scielo; however, this would be a significant database for searching national articles. Therefore, we chose to

(a) perform a search with other descriptors,

(b) read the articles found, and

(c) verify their suitability for the subject of the literature review.

The results of this new selection are shown in **Table 1**.

We excluded the terms related to technology because we imagined some authors could use other terms, such as information and communication technologies (ICT). However, the articles found without a descriptor associated with technology also did not address this topic and were discarded. When using the term ICT, we obtained four results. Although they addressed technology, two of them did not focus on teacher knowledge. The other two were added to the analysis corpus (Bairral & Powell, 2013; Sampaio, 2016).

However, the comparison with international results revealed a yet small sample of national publications. Thus, we incorporated journals indexed to the Latindex in this review. Latindex is an indexer of journals from Latin America, Portugal, and Spain with a more flexible evaluation system and a greater number of journals than Scielo (Shinaku et al., 2014). In the search per country, we found 7,143 Brazilian journals of which 3,737 were online. This was the first criterion stipulated to facilitate the search for articles. Afterwards, we selected the journals identified in the mathematics subtopic and covered teaching and/or education areas, a total of 36 scientific journals.

For each, we repeated steps (a), (b), and (c) described earlier in the detailed research for the Scielo. The descriptors used were only "conhecimento docente" (teaching knowledge) or "conhecimento do professor" (teachers' knowledge), considering that we selected journals that already covered mathematics. As a result, we found nine articles on the desired topic. **Figure 5** summarises the sequence defined for the search for articles.

We surveyed the repeated articles, i.e., the ones identified in more than one database. The results are shown in **Figure 6**, in the Venn Diagram. In total (without repetitions), we obtained 292 articles.

As the articles selected on ERIC, Scopus, and WoS had not been read yet, we individually checked whether the descriptors were in the title, abstract, and/or keywords. After the verification, we read the abstract to confirm the inclusion of the article in the corpus, according to the theme addressed in the text (**Figure 7**).

Most articles whose descriptors were not confirmed were in the ERIC database, as the platform searches for terms in other fields, such as the journal's name and automatic descriptors from the database itself. Not all texts were within the desired topic, even though they presented the indicated descriptors. Thus, of the 281 studies found in ERIC, Scopus, and WoS, 186 articles were excluded because they did not present the descriptors in the stipulated fields (title, abstract and/or keyword), leaving 95 articles.
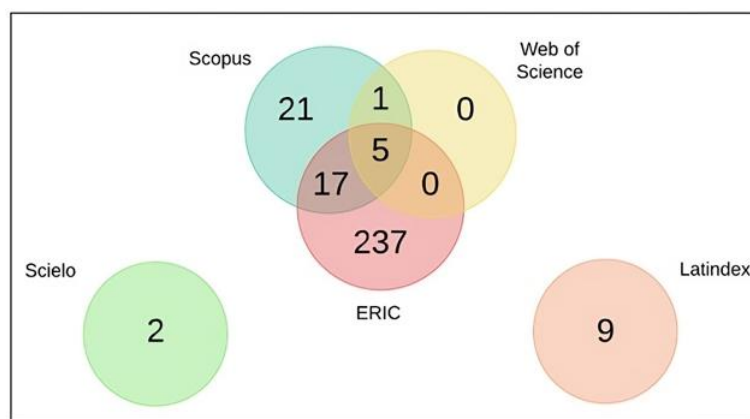
Figure 6. Number of articles obtained represented in a Venn diagram (Source: Authors' own elaboration)
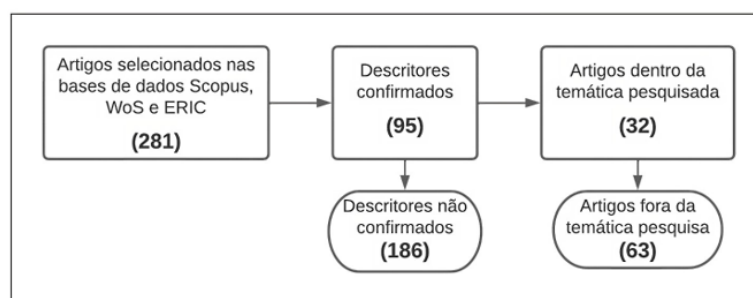

Figure 7. Processes of verification & confirmation of articles (Source: Authors' own elaboration)

We read the abstracts and stipulated, as an exclusion criterion, articles that did not focus on the teacher's knowledge and the use of technology in mathematics classes. As a result, studies that addressed other topics, such as STEAM or student learning, were removed, which left us with only 32 articles. We could not find one of the articles, so we removed it from the corpus of analysis. In this way, we finished this search for this systematic literature review with 31 articles selected from the databases plus the 11 previously checked articles (on Latindex and the Scielo), i.e., 42 scientific articles (identified with an * in the reference list).

We registered each text on Google Forms. Then, we built a form to fill in the main information about the article. With this, we carried out a vertical analysis (Depaepe et al., 2013), i.e., an individual analysis of each article contemplating the following aspects:

(1) year of publication,

(2) country, where the research was developed,

(3) research focus,

(4) theoretical framework,

(5) context and participants,

(6) nature of research,

(7) data production,

(8) analysis procedures, and

(9) results, conclusions, and implications.

The nine aspects mentioned represent the analysis parameters used when individually examining the articles (vertical analysis). Subsequently, we analyzed the articles simultaneously, identifying characteristics, similarities, and/or differences that could contribute to answering the research questions. By this horizontal analysis (Depaepe et al., 2013), we compared the studies in each aspect. Aspects (1) to (8) contributed to the discussion of the first inquiry question. As for the second, we emphasize aspects (4), (7), (8), and (9). Finally, the third inquiry question was discussed based on all analyses, emphasizing aspect (9). Next, we present the horizontal analyses reported descriptively, correlating the studies that made up this article.

DATA ANALYSIS

Year of Publication

Most articles (13) were published in 2020. That year, we experienced the coronavirus pandemic, which led all educators to rethink the role of technology and its insertion in the classroom. Videoconferences and remote classes also stimulated the emergence of new research from the immediate contact with digital technologies in the educational area. However, publications dropped severely in 2021, as shown in **Figure 8**.
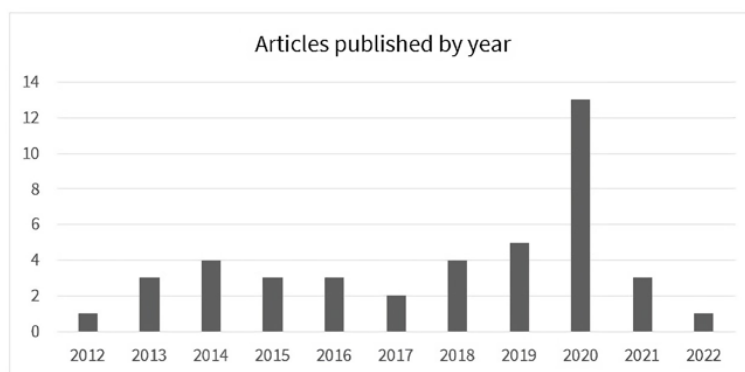
Figure 8. Number of articles per year of publication (Source: Authors' own elaboration)

Figure 9. Infographic distribution of the articles by continent (Source: Authors' own elaboration)

Figure 10. Number of articles that present objective and/or investigative question (Source: Authors' own elaboration)

Country Where Research was Developed

The United States, Brazil, and Portugal are the leaders in publications on the researched topic. In another article, we also verified the extensive participation of the United States studies (Gumiero & Pazuch, 2019).

We understand that the number of national articles is because the selection of articles approached a national database (Scielo), and Latindex indexed the consultation of national journals. On the other hand, this research proved appropriate because it revealed an expressive number of articles, showing the importance of studies like ours and encouraging Brazilian research in digital technologies and mathematics teaching areas.

Four of the articles found were consequences of international collaborative work (including intercontinental ones), exemplifying partnerships between researchers from Brazil and the United States (Bairral & Powell, 2013), Brazil and Portugal (Silva et al., 2019), Indonesia and China (Mailizar & Fan, 2020), and Tanzania and the Netherlands (Kafyulilo et al., 2015). In **Figure 9**, we consider the articles on both continents. Thus, the sum of the results is greater than the total number of articles in this study.

The infographic in **Figure 9** shows that all continents are represented in this systematic literature review. Although there is a more outstanding contribution from the northern hemisphere, the participation of different locations produces significant results that symbolize global scenarios.

Research Focus

In this section, we analyze whether the articles explicitly presented (or not) the objective(s) and the investigative question(s). Approximately 17% of the articles did not explicitly present the study objective. This fact stands out negatively, because it makes it harder for the reader to interpret the information, as it is unclear. Regarding the inquiry question, the number is even higher (approximately 40%). However, we supposed that articles could present either the objective or the inquiry question. Thus, we developed the Venn diagram shown in **Figure 10** to explore more objective results.

Table 2. Classification of articles according to focuses of analysis

Article	(i)	(ii)	(iii)	(iv)	(v)	Other
Anat et al. (2020)			x		x	
Bairral and Powell (2013)		x		x		
Bate et al. (2013)				x		
Bulut and Isiksal (2019)						x
Chinellato and Javaroni (2020)	x			x		
Filho and Gitirana (2022)			x			
Hill and Uribe-Florez (2020)						x
Joubert et al. (2020)				x		
Kafyulilo et al. (2015)			x			
Karatas et al. (2017)			x			
Kim and Park (2018)		x				
Koh (2019)				x		
Lin et al. (2015)			x			
Maciel et al. (2020)			x			
Mailizar and Fan (2020)		x				
Niess (2013)				x		
Niess and Gillow-Wiles (2014)				x		
Nogueira et al. (2020)	x		x		x	
Nonato and Costa (2021)						x
Orlando and Attard (2016)		x				
Ozudogru and Ozudogru (2019)				x		
Polly (2014)				x	x	
Polly (2015)			x			
Polly and Orrill (2012)				x		
Rocha (2016)		x				
Rocha (2019)		x				
Rocha (2020)			x			
Rocha (2021)						x
Rocha and Prado (2018)				x	x	
Rocha et al. (2020)		x		x		
Sampaio (2016)		x		x		
Santos and Vasconcelos (2018)		x				
Setyawan et al. (2018)				x	x	
Silva et al. (2019)		x				
Souza and Martins (2020)				x		
Taley and Adusei (2020)		x				
Tournaki and Lyublinskaya (2014)				x		
Yigit (2014)			x			
Yildiz and Baltaci (2017)				x		
You et al. (2021)				x	x	
Za'ba et al. (2020)				x		
Zambak and Tyminski (2020)	x			x		

In **Figure 10**, we can observe that 41 articles had an explicit objective and/or an inquiry question. Therefore, only one of the studies analyzed (Za'ba et al., 2020) did not clearly describe the purpose of their work. We understood the authors' purpose when preparing the article as we read it. Therefore, this information must be explicit so the study can be better understood.

Initially, we defined the focuses of analysis to group the articles based on the objectives described by the authors. This list was expanded and modified after we read the works. The focuses of analysis were the

(i)   use of dynamic geometry software,

(ii) use of technological resources (except for dynamic geometry software),

(iii) teacher development in initial education,

(iv) teacher development in continuing education, and

(v) continuing education courses.

We chose an exclusive category for dynamic geometry software, as we expected to find many articles within this theme, especially using GeoGebra. We understand that the categories were not completely disjointed and that some studies fit into multiple options. Therefore, we worked with checkboxes in the form sheet to allow multiple choices. The classification is presented in **Table 2**.

The expectation of many articles in category (i) was not achieved. On the contrary, it was the category with the fewest studies identified, which indicates a gap in research on digital technologies and mathematics teaching. Firstly, we have the category (iv) on teacher development in continuing education (20 articles), which, combined with the third classification, adds up to 29 articles, representing approximately 69% of the corpus of analysis of this literature review.

Although we have not found many studies on dynamic geometry software, the second category was also expressive in the results (11 articles). We noticed that many works do not indicate and/or do not limit the type of technological resource used, which may also involve the dynamic geometry software.

In category (v)–continuing education courses–we identified that all studies were also covered by other categories of teacher development, whether in initial or continuing education. Thus, the articles that propose to analyze a specific course eventually engage in a discussion about teacher professional development, since the motivation for carrying out a course is to help the teacher in this development process.

Not all articles were related to the previously defined categories. For those cases, which we identified as "other" in **Table 1**, we determined different analysis focuses. The studies by Bulut and Isiksal (2019), Hill and Uribe-Florez (2020), and Rocha (2021) aim at teaching knowledge. Unlike (v), which is dedicated to examining a specific course, and (iii) and (iv), intended for an education process that led to teacher development (object of analysis), the articles focused on teaching knowledge aimed to identify what teachers knew or needed to know to teach with digital technologies.

Even though they have a specific course or space for collaboration, those studies wanted to explore knowledge as if it was a list of goals teachers should achieve. Thus, it was possible to point out the constituted knowledge and gaps in teacher education. In turn, Nonato and Costa (2021) conducted a curriculum analysis.

Theoretical References

Based on the articles we read, we identified the theoretical references used to analyze and/or characterize the professional teaching knowledge of the research participants. Not all theoretical models discussed in the initial sections were found in the selected articles. We did not determine categories a priori nor expect to identify all models since some did not contemplate the technological domain, such as MKT (Ball et al., 2008) and KQ (Rowland et al., 2005).

Approximately 64% of the articles analyzed used TPACK as a theoretical model for research discussions, reinforcing the wide dissemination of this reference to analyse the integration of digital technologies in the school context. Furthermore, Koehler and Mishra's (2009) study was cited in most works, representing a relevant reference in digital technology and teaching.

In the literature review carried out by Yigit (2014), the author investigates the development of TPACK in pre-service teachers and concludes that "the researchers did not explicitly describe the instruments they used to measure PSMTs' development of TPACK" (Yigit, 2014, p . 33), indicating that, although it is a widely known theoretical model, its use is limited in the research carried out. Among the 27 articles analyzed that used TPACK as a theoretical framework, seven cited it as an analysis instrument but did not specify how this would be carried out. Another 10 studies developed a questionnaire or used an existing questionnaire model whose questions were related to TPACK knowledge domains. In these surveys, we found gradations on different scales, such as Likert, which led to a quantitative analysis of the results and a measurement of the knowledge levels of the participating teachers. The others presented analysis categories related to TPACK domains, as in Polly (2014), or a re-elaboration of the design, as in Niess (2013) and Niess and Gillow-Wiles (2014).

Based on the analyses carried out, we list strengths and limitations in the use of TPACK as a theoretical framework. Its general structure–without specifying an area–contributes to its dissemination and use in different contexts. Moreover, as previously seen, TPACK is a flexible analysis tool used in research with qualitative and quantitative approaches, allowing researchers to model it according to the proposed objectives. In Filho and Gitirana (2022), the results obtained in the questionnaires based on TPACK were also used as a criterion for forming groups among the participating teachers. With the advancement of digital technologies in the classroom, TPACK can "rebuild the basis of professional teaching knowledge for the digital era" (Rocha & Prado, 2018, p. 208) and provide "a theoretical vocabulary to understand the different kinds of pedagogical considerations involved" (Koh, 2019, p. 15).

On the other hand, Kafyulilo et al. (2015) note that "despite the extensive use of and research on TPACK, TPACK as a theoretical construct is not yet very well understood and this probably has its influence on measuring TPACK development too" (Kafyulilo et al., 2015, p. 384), which adds to Rocha et al. (2020) when they say that "the process of knowledge integration from TPACK perspective is not simple to occur" (Rocha et al., 2020, p. 34). Thus, the authors indicate limitations regarding the understanding and integration of knowledge in this theoretical model. When considering TPACK as the most advanced knowledge domain in measuring professional teaching knowledge and the effective integration between content, pedagogy, and technology, we note that studies rarely show teachers who have reached this level, which makes it utopian and far from reality in the classroom. As an analysis tool, TPACK could be used by teacher educators and teachers themselves in self-assessment. However, when we set levels that are never reached, this may frustrate participants. Finally, we highlight as a limitation the fact that it is not a specific theoretical framework for mathematics, which, in a way, allows its use in different contexts but makes adaptation necessary in specific cases, such as in Koh (2019).

Although mathematics was one of the descriptors used to select the articles, the authors did not report a specific theoretical model in mathematics, mostly choosing TPACK. We can interpret those data from two perspectives. The first considers that research on professional teacher knowledge for teaching mathematics with digital technologies prioritizes investigating technological knowledge to the detriment of mathematical knowledge. The second concludes that the existing theoretical models that unite mathematics and technology are not being widely disseminated or are not achieving and/or considering the teacher's needs.

We understand that technology should not stand out from mathematics since it represents a tool for classroom use based on planned choices and defined purposes. Its integration in mathematics classes causes structural, methodological, and pedagogical changes; however, the objective remains the students' mathematical development.

Table 3. Classification of articles according to mathematical content

Article	Numbers	Algebra	Geometry	Magnitudes & measures	Probability & statistics
Anat et al. (2020)				x	
Bairral and Powell (2013)			x		
Bulut and Isiksal (2019)			x		
Chinellato and Javaroni (2020)			x		
Filho and Gitirana (2022)		x			
Kim and Park (2018)			x		
Koh (2019)					x
Lin et al. (2015)		x			
Polly and Orrill (2012)	x	x	x		x
Rocha (2016)	x	x	x		
Rocha (2019)		x			
Rocha (2020)		x	x		
Rocha (2021)		x			
Souza and Martins (2020)			x		
Yildiz and Baltaci (2017)			x		
Za'ba et al. (2020)		x	x	x	
Zambak and Tyminski (2020)			x		

Rocha's (2016, 2019, 2020, 2021) works conceptualized and exemplified the use of KTMT. However, we did not find other authors who used the model, despite being a structure that combines the technological and mathematical aspects. STAMPK not was indicated in any of the analyzed articles. MKT was not used as a theoretical reference for the analyses in the selected studies. However, it was cited in several articles as it represents the teacher's mathematical knowledge. In the works that used TPACK, we noticed that Ball et al. (2008) represented mathematical knowledge in the specific theoretical model for the technology. None of the articles indicated KQ. We already expected it because it is not as well-known as MKT and does not address technological knowledge. 10 studies in this systematic review did not explain a theoretical framework and/or carry out descriptive analyses based on other theoretical approaches, such as historical-critical pedagogy (Souza & Martins, 2020).

Context & Participants

In this topic, we will only address empirical research, excluding the four theoretical studies that will be explored later. Therefore, we will work with a corpus of 38 empirical scientific articles. Not all works analyzed indicated the active teaching level of the participating teachers. We understand that this information was irrelevant, which is why it was not cited in Bairral and Powell (2013). 10 of the articles analyzed addressed teachers in initial education. Here, we will not focus on identifying teachers' future performance levels (initial years, final years, or high school). Tournaki and Lyublinskaya (2014) analyzed teachers in special education training without detailing the education level. Among the 26 remaining articles, 13 contemplated a heterogeneous group with teachers of different levels. Two studies were conducted with primary school teachers (initial years of elementary school), three with middle school teachers (final years of elementary school), and eight involved high school teachers.

Regarding the mathematical content addressed, most articles do not specifically mention any thematic axis (25 articles). The others, on the other hand, illustrate heterogeneous results. **Table 3** presents the articles and the respective thematic axes of the other 17 articles. With this, we found few studies on numbers, quantities and measures and probability and statistics. Algebra and geometry are the main thematic axes addressed in the articles.

Nature of Research

Among the studies analyzed, empirical articles (38) stood out compared to theoretical articles (four). Regarding the methodological approach, we identified another distribution between qualitative, quantitative, and mixed research. **Table 4** presents an overview of the results found. Although qualitative research represents the largest share, we see an advance in mixed studies. Six works that relate to this approach were published as of 2019. In other words, it is a recently explored approach.

Data Production

To analyze the data production, we formulated a priori options that represented the main data production instruments used in research on teaching, education, and/or teacher education. The options were

(a) interview,

(b) questionnaire,

(c) field diary,

(d) report, narrative, and/or memory,

(e) observation and class registers,

(f) protocol and/or form for data collection,

(g) collective discussion (study group, collaborative group, etc.),

(h) video or audio recording, and

(i) others.

Table 4. Classification of articles according to nature of the research

Article	Qualitative approach	Quantitative approach	Mixed approach
Anat et al. (2020)	x		
Bairral and Powell (2013)	x		
Bate et al. (2013)			x
Bulut and Isiksal (2019)			x
Chinellato and Javaroni (2020)	x		
Filho and Gitirana (2022)	x		
Hill and Uribe-Florez (2020)			x
Joubert et al. (2020)	x		
Kafyulilo et al. (2015)		x	
Karatas et al. (2017)		x	
Kim and Park (2018)	x		
Koh (2019)			x
Lin et al. (2015)		x	
Maciel et al. (2020)	x		
Mailizar and Fan (2020)		x	
Niess (2013)	x		
Niess and Gillow-Wiles (2014)	x		
Nogueira et al. (2020)			x
Nonato and Costa (2021)	x		
Orlando and Attard (2016)	x		
Ozudogru and Ozudogru (2019)		x	
Polly (2014)	x		
Polly (2015)	x		
Polly and Orrill (2012)	x		
Rocha (2016)	x		
Rocha (2019)	x		
Rocha (2020)	x		
Rocha (2021)	x		
Rocha and Prado (2018)	x		
Rocha et al. (2020)	x		
Sampaio (2016)		x	
Santos and Vasconcelos (2018)	x		
Setyawan et al. (2018)	x		
Silva et al. (2019)	x		
Souza and Martins (2020)	x		
Taley and Adusei (2020)		x	
Tournaki and Lyublinskaya (2014)		x	
Yigit (2014)	x		
Yildiz and Baltaci (2017)	x		
You et al. (2021)			x
Za'ba et al. (2020)	x		
Zambak and Tyminski (2020)			x
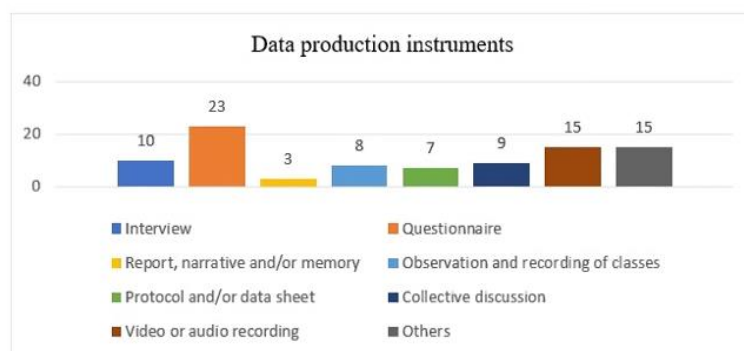
Figure 11. Number of articles per data production instrument (Source: Authors' own elaboration)

In **Figure 11**, we bring a graph that compiles the number of articles that used each data production instrument. Through the graph, we noticed that the questionnaire was the main instrument used, followed by the field diary. We also noticed a tendency to carry out questionnaires following the scale Likert to verify what knowledge teachers have or have acquired after an education course. In fact, this information dialogues with the focus of investigation and the theoretical framework used in the research, as they understand and seek analysis in a checklist of teaching knowledge.

Table 5. Classification of articles according to data production instruments

Article	I	Q	FD	R	OR	PF	D	V	O
Anat et al. (2020)		x	x						x
Bairral and Powell (2013)									x
Bate et al. 2013)		x	x				x	x	x
Bulut and Isiksal (2019)		x							
Chinellato and Javaroni (2020)	x	x	x					x	x
Filho and Gitirana (2022)			x				x	x	
Hill and Uribe-Florez (2020)		x							
Joubert et al. 2020)		x				x			x
Kafyulilo et al. 2015)		x			x	x			
Karatas et al. 2017)		x							
Kim and Park (2018)			x					x	
Koh (2019)			x				x	x	
Lin et al. 2015)		x							
Maciel et al. (2020)		x							x
Mailizar and Fan (2020)		x							
Niess (2013)	x	x	x	x	x	x	x	x	
Niess and Gillow-Wiles (2014)				x		x	x	x	
Nogueira et al. (2020)		x							
Nonato and Costa (2021)									x
Orlando and Attard (2016)									x
Ozudogru and Ozudogru (2019)		x							
Polly (2014)	x		x		x		x		x
Polly (2015)									x
Polly and Orrill (2012)	x								
Rocha (2016)	x		x		x			x	
Rocha (2019)	x		x		x			x	
Rocha (2020)			x				x		x
Rocha (2021)	x				x			x	
Rocha and Prado (2018)		x	x			x		x	
Rocha et al. (2020)		x	x			x		x	
Sampaio (2016)		x							
Santos and Vasconcelos (2018)									X
Setyawan et al. (2018)		x							
Silva et al. (2019)					x				
Souza and Martins (2020)	x	x	x					x	
Taley and Adusei (2020)		x							x
Tournaki and Lyublinskaya (2014)		x							
Yigit (2014)									x
Yildiz and Baltaci (2017)	x		x				x	x	
You et al. (2021)	x	x							
Za'ba et al. (2020)		x	x	x	x		x		
Zambak and Tyminski (2020)			x			x		x	x

Note. I: Interview; Q: Questionnaire; FD: Field diary; R: Report, narrative, and/or memory; OR: Observation & recording of lessons; PF: Protocol and/or record form; D: Collective discussion; V: Video or audio recording; & O: others

For each article, we could select more than one option since it is usual to use different instruments throughout the research, including for triangulation and/or higher data reliability. **Table 5** brings the data production instruments used in each article. In **Table 5**, we found that 18 articles used only one data production instrument. 10 articles applied questionnaires; four are theoretical studies (Nonato & Costa, 2021; Orlando & Attard, 2016; Santos & Vasconcelos, 2018; Yigit, 2014).

For articles that had as a data production instrument the option *others*, we prepared **Table 6** with the description of the strategy used. This analysis also agrees with what we pointed out in the previous paragraph, since six of those studies carried out tasks with the participating teachers for a later evaluation. Another instrument we identified and had not included in the initial list was the teachers' plans analysis.

Analysis Procedures

Unlike previous sections, we chose to establish a posteriori categories. Thus, we describe the analytical process of each article through the form used and perform a detailed reading to find possible similar ones and, consequently, group results, defining categories.

Not all works presented clear analysis procedures. However, we identified some frequent strategies. Most studies (14 articles) identified knowledge and/or domains discussed along the theoretical framework. This information aligns with the "focus of the investigation" section since most articles analyzed teachers' development. A recurring strategy for this identification was the coding and/or classification system of the results, which was cited in 10 studies.

The amount of mixed research is also noteworthy when we discuss the nature of the studies, which impacts the analysis procedures used. 10 of the articles investigated described methods of descriptive and inferential statistics.

Table 6. Description of instrument used, categorized as others

Article	Description of the instrument–Others
Anat et al. (2020)	Creation of MOOC and peer-review
Bairral and Powell (2013)	Interactions in VMT
Bate et al. (2013)	Task carried out by the participants
Chinellato and Javaroni (2020)	Task carried out by the participants
Joubert et al. (2020)	Lesson planning
Maciel et al. (2020)	Courses/disciplines syllabuses
Nonato and Costa (2021)	*
Orlando and Attard (2016)	*
Polly (2014)	Task carried out by the participants and lesson planning
Polly (2015)	Lesson planning
Rocha (2020)	Task carried out by the participants
Santos and Vasconcelos (2018)	*
Taley and Adusei (2020)	Task and tests carried out by the participants
Yigit (2014)	*
Zambak and Tyminski (2020)	Task and tests carried out by the participants

Note. *Theoretical articles

Another persistent approach was using transcripts of interviews and filming and the possibility of repeating the recordings. Eight articles described processes of identification of episodes through transcripts. Finally, we found articles that explored the document and textual analysis methods.

Results, Conclusions, & Implications

The horizontal analysis of this section followed the classification of "focus of analysis" because we could investigate whether the objectives had been achieved and what the implications were. In the first group–using dynamic geometry software–we identified that research (Chinellato & Javaroni, 2020; Nogueira et al., 2020; Zambak & Tyminski, 2020) recognize the potential of dynamic geometry software for mathematics classes. The articles highlight that the interaction between the physical and digital environment benefits teachers' development. However, we must be attentive to the constructions carried out in those environments, as they eventually disregard the mathematical rigor and do not preserve characteristics, which reveals a gap in professional teaching knowledge for teaching with digital technologies. Zambak and Tyminski (2020) conclude that poor mathematical knowledge makes integrating digital technologies into mathematics classes difficult. As a suggestion, the authors suggest the resolution of problems that cannot be solved on paper, encouraging the use of the virtual environment.

Expanding the debate to (ii) the use of technological resources, i.e., contemplating other tools in addition to dynamic geometry software, some speeches remain recurrent, such as the qualities and gains of digital technology in the classroom. Mailizar and Fan's (2020) article indicated that teachers have more knowledge of computers than portable devices, which is also greater than their knowledge of graphing calculators. Although this fact is related to the experience and coexistence with the equipment, Orlando and Attard (2016) have already highlighted that teaching with digital technologies differs from the (operational) use of technology. Thus, studies point out that for effective use of technological resources in the classroom, it is necessary to know them and understand how they will be used for teaching. Time (Santos & Vasconcelos, 2018) and teachers' willingness are needed to integrate digital technologies into the activities (Sampaio, 2016).

In topics (iii) and (iv), which focused on teachers' development, the texts indicate positive results for this development as teachers participate in projects, courses, and disciplines, especially in collaborative environments (Bairral & Powell, 2013; Filho & Gitirana, 2022). Particularly for initial education, research reveals that many subjects are technical and do not include didactics (Maciel et al., 2020), so they encourage different ways of approaching the theme of teaching with digital technology for prospective teachers (Anat et al., 2020). As for continuing education, Bate et al. (2013) describe that the opportunity for teachers to place themselves as students favors teacher development. However, mere participation in activities with digital technologies does not guarantee this development because it requires the teacher to live the experience of teaching integrated with technology (Polly, 2014). Karatas et al. (2017) add that this development is vital to increase teachers' self-confidence, affecting the teaching and learning environment. As discussed in the analysis focus section, articles classified in (v) are the same as in categories (iii) and (iv). Therefore, the results are similar, but we will discuss them from the aspect of the education course itself. Research bespeaks that courses are valuable for teacher development. No articles state that the course was not necessary. However, they suggest improvement and consideration, such as encouraging collaboration and awareness of the physical or virtual environment used to provide moments in which participants play the role of students and moments in which they must monitor and reflect on their

DISCUSSION

This section resumes the inquiry questions based on the analyzed data.

What is the structure of research on professional teaching knowledge for mathematics teaching with digital technologies?

While analyzing this research excerpt, we realized that the number of publications about the theme has been constant, except for 2020, when they increased significantly.

Mailizar and Fan (2020) indicate that, previously, the research focused on students and how they used technology in the classroom. Currently, the concern has turned to the teacher, considering the impact of their decisions on teaching, which, in turn, has boosted research focused on professional teaching knowledge.

We reiterate this finding when we analyze the focuses of the articles, which, for the most part, are on teacher development. For this, the studies used TPACK as the main theoretical framework, even though it is not designed exclusively for mathematics.

There is a trend toward empirical and qualitative approach research, although mixed methods have been more present in articles in recent years. Questionnaires, field diaries, and recordings were the most used data production instruments, along with coding and classification methods, to identify knowledge and/or domains of knowledge in data analysis. Thus, observing the research focuses, the theoretical framework, the production, and data analysis, we identified that most studies intended to measure teacher professional knowledge and classify it into pre-defined groups.

How is professional teaching knowledge explored and understood in situations of mathematics teaching with digital technologies?

As pointed out earlier, professional teaching knowledge has been explored through questionnaires, field diaries, and recordings and was later classified into domains of knowledge established by the theoretical framework, as is the case with TPACK. Its proximity to Shulman's (1986) model indicates the significant number of studies that consider it to understand teaching knowledge. On the one hand, the research results indicate that the teacher must experience integrating digital technologies as a student and teacher to appropriate and develop it. On the other, the understanding of knowledge has been limited to moments, instant situations, and photographs of the teachers without considering their influences, their experiences, and the complexity of pedagogical practice.

The articles validate the potential of technological tools; however, they indicate the importance of collaborative groups and moments of monitoring and reflection to contribute to professional knowledge development. Therefore, even though this knowledge is understood as essential and of great influence on the teacher's decision-making, which impacts the effective integration of digital technologies, we need to broaden the debate and the understanding of the teacher's knowledge starting from their practice.

How does integrating digital technologies in mathematics classes mobilize professional teaching knowledge?

The teacher is responsible for the choices that determine the course of the class. Therefore, their knowledge interferes with using and integrating digital technologies in mathematics classes. It is essential to emphasize that, although they use technological tools, the focus of mathematics classes should not deviate from mathematics, i.e., they must use technology as a means, not as an end (Kim & Park, 2018). Thus, the teacher mobilizes their knowledge when planning and interacting with students, encouraging mathematical reasoning with the support of digital technologies without disregarding mathematical rigour. The teaching practice leads teachers to learn about digital technologies, their use in the operational sense, and their pedagogical use, analyzing the moment and the most appropriate way to promote the students' development. Therefore, teachers mobilize their knowledge by recognizing the potential of digital technologies for teaching mathematics but critically and reflectively, understanding that using media and/or digital tools does not ensure students' progress if they are decontextualized or distant from the objectives of learning recommended in the teachers' planning.

CONCLUSIONS

This article aimed to understand and discuss teachers' professional knowledge of mathematics teaching with digital technologies. Thus, we conducted vertical and horizontal analyses on 42 articles comprising this systematic literature review. We note that the studies have similar structures with similar theoretical frameworks, objectives, and methodological procedures, which seek to identify and classify professional teaching knowledge based on existing knowledge or knowledge that still needs to be developed.

The results point to valuing collaborative experiences that allow experiences and teachers' reflections on the integration of digital technologies in mathematics classes. Moreover, they consider that teachers' knowledge demands more than the operational use of technological tools, considering their pedagogical purpose in the school context.

Therefore, teachers' professional knowledge for a successful integration between digital technologies and mathematics teaching encompasses the teachers' practice, experiences, context, curriculum, and knowledge of technology and mathematics.

We understand that the search and selection criteria of the articles limit the results. Therefore, we prioritized processes that covered heterogeneous studies within the same theme. However, other databases, descriptors, and analytical processes could point to divergent results.

Besides understanding teaching knowledge, we must focus on teachers' professional development and, consequently, education processes. As stated by Rocha et al. (2021), a professional development program that aims to use technology to teach mathematics must

(i) focus on mathematics with technology rather than just technology,

(ii) meet the needs of teachers,

(iii) connect the professional development and teaching practice, and

(iv) develop communities of practice.

We identified those four aspects in the discussions raised in the article, which drives us to broaden the debate and investigate how to structure those formative processes. In this way, we suggest future research on the contributions of each theoretical model of teaching professional knowledge to analyse mathematics classes with digital technologies.

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