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Examination of the relationship between mathematical and critical thinking skills and academic achievement

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ABSTRACT

In today's world, it is important for individuals to have critical thinking and mathematical thinking skills, which are high-level thinking skills. In advanced education systems, it is among the important priorities that students not only learn knowledge but also acquire high-level thinking skills. This study aims to examine the relationship between mathematical and critical thinking skills and academic achievements of secondary school students. In the study, relational screening model, one of the quantitative research models, was used. The research was carried out in a province located in the southern region of Turkey in the 2022-2023 academic year with students studying in secondary school (n=346) who were determined by maximum variation method selected from the types of purposeful sampling. Mathematical thinking scale and critical thinking scale was used as a data collection tool. In the process of analyzing the data, one-way analysis of variance and regression analysis techniques were used. According to the research findings, it was observed that there was a relationship between the critical thinking and mathematical thinking skills of the students, both the critical and mathematical thinking skills of the students did not differ significantly according to gender, and the mathematical thinking skills differed significantly according to the grade level. In addition, it was concluded that both mathematical and critical thinking skills of the students increased according to their grade levels. In addition, students' critical skills and academic achievement together explained 25.0% of the total variance of mathematical thinking skills. In other words, as students' critical thinking skills and academic achievement increase, their mathematical thinking skills also increase.

Keywords: critical thinking, mathematical thinking, academic achievement

INTRODUCTION

In the present era of rapid transformation, national education systems prioritize empowering students to master technology and play a pivotal role in steering global change. Success in this educational direction hinges on cultivating individuals capable of providing innovative and well-founded solutions to everyday challenges, leveraging contemporary opportunities with optimal effectiveness and efficiency. Consequently, curricula are designed to integrate the cultivation of 21st century foundational skills, underscoring the significance of learning processes geared towards achieving this goal. In order for the education to be permanent, appropriate education and training programs should be organized. It is important that the education and training programs that need to be organized are oriented to the needs of the students. At the stage of determining the needs of the students, it should be considered that the students can use their full capacities. Educators have a great duty in providing students with high-level thinking skills such as scientific, creative, democratic, multidimensional, mathematical and critical thinking. With the implementation of educational programs centered on these skills, individuals with the desired characteristics can be trained.

Thinking is a set of deliberate mental behaviors carried out to eliminate the events that disrupt the physical and psychological balance of the individual (Kazanci, 1989). The ability to think is one of the basic characteristics of human beings. In the course of their lives, individuals solve problems, evaluate events, and perform actions such as prediction and discovery (Hughes & Lavery, 2004). In the process of the individual, individuals form hypotheses, identify the causes, and reach a solution.

According to Umay (2003), mathematics is one of the tools that develop thinking and constitutes the most important building blocks of basic education. Because mathematics education has a function beyond teaching numbers and operations, gaining calculation skills that are an indispensable part of daily life, and provides important support such as thinking, making connections between events, making predictions, reasoning and solving problems. This way of thinking is called mathematical thinking (Yildirim, 2017). Stacey (2006) indicated that mathematical thinking, which can be defined as a natural tool in solving real-life problems, is an important requirement for learning and teaching mathematics, and therefore it is one of the most important goals of mathematics education. With the development of mathematical thinking, new information can be reached by abstracting,

predicting, generalizing, hypothesizing, testing, proving, reasoning and describing by using mathematical information and concepts in the mind (Alkan & Bukova-Guzel, 2005). Yildirim (2017) stated that individuals with advanced mathematical thinking power are more successful in analyzing the events they encounter and systematically reaching the correct and shortest solution, and they try to reach generalizations.

The development of mathematical thinking and its processes (customization, generalization, assumption, and persuasion) is carried out through problem-solving activities. With the problem-solving activities, students' skills in using mathematical knowledge, generating and testing hypotheses, checking/proving the accuracy of the obtained result, producing different solutions, inductive/deductive thinking, abstraction, persuasion skills and critical thinking skills develop (Milli Eğitim Bakanlığı (MEB), 2009).

Until 1960, scientists such as J. Dewey, E. Dimnet, K. Duncker, H. Hazlitt, V. Noll, J. Jastrow, J. Rossman, J. Piaget, E. Thorndike, J. Wertheimer, and G. Wallas had made great contributions to the field of critical thinking. Critical thinking, which was first discussed by Perry in the 1970s from an intellectual point of view, started to be defined differently after the 1980s with the effect of behaviorist and cognitive theories looking at the subject from different sides (Dutoglu & Tuncel, 2016). Terenzini and Pascarelle (1991) defined critical thinking as a concept that includes individual skills that will accomplish all or some of these, such as identifying basic issues and assumptions, recognizing relationships between subjects, making correct inferences from data and drawing conclusions from data that are or are obtained, testing whether those conclusions can be drawn from data, evaluating evidence or authority.

Critical thinking should be considered as being aware that it does not only mean criticizing facts, findings, events, finding mistakes, or that there is no opposite of the concept of "criticism", which is a negative judgmental evaluation, and that there may be different dimensions of these (Yesilyurt, 2021). Critical thinking means that there may be alternative explanations rather than drawing an absolute conclusion about the information read, said or found. The critical thinking process, which differs from other thinking skills by being evaluative, can be expressed by bringing an acceptable objective criterion to beliefs and behaviors (Akpinar, 2012; Kalayci, 2001; Kokdemir, 2003).

Today, social development depends on individual development. Therefore, the education of the individual is extremely important. For a productive society, there is a need for individuals who ask, question, investigate, discuss, open to communication and change, who can produce information and discover new information. To realize all these skills, the thinking skills of individuals must develop (Kurnaz, 2019). Within the thinking skills, mathematical thinking and critical thinking skills have an important place.

Examining the field literature, there are studies available with aim of revealing the mathematical thinking processes of the students within the resources available (Aygun et al., 2021; Baltaci, 2017; Cezikturk & Hangul, 2022; Kamarulzaman et al., 2022; Sahin, 2019). In addition, the concept of mathematical thinking and reasoning, philosophy of mathematics, geometric thinking, problem-solving, mathematical self-efficacy concepts are considered together with the studies (Aulia & Fitriyani, 2019; Bulbul & Guven, 2019; Cetin, 2021; Kocaman, 2017; Ratnasari, 2016; Tuzun, 2019). When the research in the literature on critical thinking is examined, it is seen that research in this field has increased, especially in recent years (Alsancak & Aybek, 2023). There are studies examining the critical thinking skills of prospective teachers and examining the relationship between critical thinking and academic achievement of primary and secondary school students (Akbiyik & Seferoglu, 2006; Alsancak & Aybek, 2023; Can & Kaymakci, 2015; Gok & Erdogan, 2011). In addition, there are studies on the teaching of critical thinking and its relationship with different variables (Basmaz & Kutlu, 2021; Caliskan, 2019). Within the available resources, no study has been found that examines the relationship between students' critical thinking, mathematical thinking and academic achievement together. Unlike other studies, the critical and mathematical thinking skills of the students were examined together, and their critical and mathematical skills were examined in terms of various variables. In addition, this study also focuses on the relationship between students' mathematical thinking and their critical thinking and academic achievement. Individuals with critical thinking skills look at the world from an open-minded and tolerant perspective and can think analytically and systematically in the face of events and problems. Individuals with mathematical thinking skills are more successful in analyzing the events they encounter and systematically reaching solutions in the correct and shortest way. For a productive society, it is important to raise individuals with these two important skills. In this context, it is thought that examining the relationship between students' critical thinking, mathematical thinking and academic achievement in terms of various variables will make a significant contribution to the literature. In this context, the following research questions were formed in this research.

1. Do students' mathematical and critical thinking skill scores differ significantly according to gender?
2. Do mathematical and critical thinking skill scores differ significantly according to grade levels?
3. Is there a significant relationship between the mathematical thinking of their students and their critical thinking and academic achievements?
4. Do the critical thinking skills of their students predict their mathematical thinking skills?
5. Does the academic achievement of their students predict mathematical thinking skills?
6. Do students' critical thinking skills and academic achievements together predict mathematical thinking skills?

METHOD

This study sought to investigate the correlation between critical and mathematical thinking skills and the academic performance of students enrolled in secondary schools. Consequently, the research adopts a relational screening model,

Table 1. Descriptive statistics-I

Variables		Study group	
		n	%
Gender	Female	184	53.2
	Male	162	46.8
Grade level	5	86	24.9
	6	126	36.4
	7	97	28.0
	8	37	10.7
Academic achievement	Very good	259	12.4
	Good	44	12.7
	Average	43	74.9
Total		346	100

Table 2. Cronbach's alpha of CTS total score & scores of sub-factors

Sub-factors	Cronbach's alpha
F1	.870
F2	.807
F3	.798
Total score	.913

Table 3. Cronbach's alpha of MTS total score & scores of sub-factors

Sub-factors	Cronbach's alpha
F1	.892
F2	.912
F3	.812
F4	.725
Total score	.802

characterized as a descriptive study. Screening models represent a research approach designed to depict a situation that has occurred in the past or is currently existing (Köse, 2019).

Study Group

The study group of the study consists of students in 5th, 6th, 7th, and 8th grades (n=346) studying in secondary school. In the selection of the sample, the maximum diversity method was adopted from the types of purposive sampling, and in this context, considering the representation ability of the universe, students studying in schools with upper, middle and lower socio-economic levels were tried to be selected for sampling. In this type of sampling method, it can be said that different situations related to the problem will give important clues about the universe values by sampling (Büyüköztürk et al., 2016). The personal information of the students in the study group participating in the research is shown in **Table 1**.

Data Collection Tool

In the study,

(1) critical thinking scale (CTS) and

(2) mathematical thinking scale (MTS) were used to measure the critical and mathematical thinking skills of students studying in secondary schools.

These tools are briefly explained below.

Critical Thinking Scale

CTS developed by the researcher was used to determine the critical thinking skills of the students. The scale, which consists of 18 questions, has three sub-dimensions. These are the "evaluation", "interpretation", and "reliability" sub-factor dimensions. The internal consistency coefficients found for the sub-dimensions in this study are shown in **Table 2**.

The reliability coefficient found for the entire scale is .913. The reliability coefficient for the sub-dimensions was found as .870 for F1, .802 for F2, and .798 for F3.These reliability coefficients found for this research show that the scores related to the scale and sub-dimensions are sufficiently reliable and can be used for research purposes (Tavsancil, 2002).

Mathematical Thinking Scale

MTS developed by Er et al. (2023) was used to determine the mathematical thinking skills of the students. The scale, which consists of sixteen questions, has four sub-dimensions. These include "inductive and deductive thinking", "utilitarian thinking", "planned thinking", and "problem-solving based thinking" are the sub-factor dimensions (Er et al., 2023). In this study, the internal consistency coefficients for the sub-dimensions are shown in **Table 3**.

The reliability coefficient found for the entire scale is .802. The reliability coefficient was found as .892 for F1 sub-dimension, .912 for F2 sub-dimension, .812 for F3 sub-dimension, and .725 for F4 sub-dimension. These reliability coefficients found for this

Table 4. Descriptive statistics-II

Variables	Mean	Standard deviation	Skewness	Kurtosis
MTS	55.93	.546	-.744	.698
CTS	72.35	.649	-.777	1.082
Academic achievement	4.62	.037	-1.567	.883

Table 5. Change of MTS & CTS scores by gender

Scales	Gender	Mean	Standard deviation	t	Sd	p
MTS	Female	55.72	9.80	-.411	344	.681
	Male	56.17	10.56			
Total		55.93	10.15			
CTS	Female	72.84	11.66	.798	344	.425
	Male	71.80	12.54			
Total		72.15	12.08			

research show that the scores related to the scale and sub-dimensions are sufficiently reliable and can be used for research purposes (Tavsancil, 2002).

Academic Achievement

In this study, the academic achievements of the students in the 2022-2023 academic year were discussed through the five-item system and included in the research analysis process.

Data Collection Process

To implement the data collection tools, first of all, official permission and ethics committee approval were obtained from the relevant institutions. Before the applications, the students were informed about the subject to be examined and how the applications would be made, and it was stated that the participation in this application was voluntary. It was also stated that in practice, not all of the answers to the data collection tools would be seen by anyone other than the researcher. Data collection tools were applied to 346 middle school students within one class hour in the 2022-2023 school year.

Data Analysis

Before analyzing the research data, the descriptive values of the scores obtained from MTS and CTS were examined, and it was decided which analyzes would be performed accordingly. **Table 4** provides descriptive statistics for MTS and CTS. In **Table 4**, according to the results of descriptive statistics, it is accepted that the data show a normal distribution. It is seen that the values of Skewness and Kurtosis vary in the range of -/+2. Since the number of samples was more than 300, the obtained Skewness and Kurtosis were found to be sufficient for normal distribution (Kim, 2013). For this reason, it was decided to perform parametric tests to answer the research questions for the data obtained from the research. Data was analyzed by independent groups t-test, one-way analysis of variance (LSD test from post-hoc tests), correlation analysis and regression analysis with the help of SPSS 22 software. The significance level in the analysis results was taken as $p<.05$. The independent variable of this research was determined as mathematical thinking ability, dependent variables as critical thinking ability and academic achievement.

FINDINGS

In this section, the findings, related results and comments obtained as a result of the analyzes made for the answers to the questions discussed in the study are included.

Findings Related to the First Sub-Problem

The findings obtained as a result of the t-test performed to determine whether the scores obtained from MTS and CTS in line with the first research problem differ significantly according to gender are given in **Table 5**. Analyzing **Table 5**, it was evident that MTS total score averages did not exhibit a statistically significant difference based on the gender variable ($t[344]=-.411$, $p>0.05$). Notably, male students' average score from MTS (mean [M]=56.17) was slightly higher than that of female students (M=55.72). Similarly, for CTS, the total score averages of the students did not indicate a statistically significant difference concerning the gender variable ($t[344]=.798$, $p>0.05$). The average score for female students from CTS (M=72.84) was observed to be higher than that of male students (M=71.80). Furthermore, considering that the maximum score achievable for the entire MTS is 80, it was noted that the students' MTS scores fell within the moderate range. Likewise, with the maximum attainable score for CTS being 90, the students' CTS scores were also observed to be at a moderate level.

Findings Related to the Second Sub-Problem

In line with the second research problem, the results of the ANOVA test to determine whether the scores obtained from MTS and CTS differ significantly according to grade level are given in **Table 6**. According to **Table 6**, there was a notable variance in the mean scores of students from MTS based on their grade level ($F[3.346]=3.423$, $p<.05$). The average total scores for MTS students ranked in ascending order were, as follows: 6^{th} grade (M=4.10), 5^{th} grade (M=55.87), 8^{th} grade (M=68.47), and 7^{th} grade (M=74.42). LSD test from multiple comparison tests was employed to ascertain the direction of the significant difference. According to the

Table 6. Changes of MTS & CTS scores by grade level ANOVA table

Students	Grade	n	Mean	Standard deviation		Sum of squares	Sd	Squares mean	F	p
MTS	5	86	58.60	9.89	Ga	1037.966	3	345.989	3.423	.017*
	6	126	54.10	10.48	Gi	3,4563.759	342	101.064		
	7	97	55.87	10.40	Total	3,5601.725	345			
	8	37	56.16	7.62						
CTS	5	86	74.87	12.30	Ga	873.862	3	291.287	2.013	.112
	6	126	70.75	12.71	Gi	4,9487.413	342	144.700		
	7	97	72.13	11.18	Total	5,0361.275	345			
	8	37	72.54	11.03						

Note. Ga: Intergroup & Gi: In-group

Table 7. Correlation analysis between MTS, CTS, & academic achievement of students results

Variables		MTS	CTS	Academic Achievement
MTS	r	1		
	p			
	n	346		
CTS	r	.496**	1	
	p	.000		
	n	346	346	
Academic achievement	r	.031	.134*	1
	p	.568	.012	
	n	346	346	346

Table 8. Results of simple regression analysis of whether students' CTS scores predicted their MTS scores

Dependent variable	Independent variable	B	Standard error	B	t	p	R	R^2	F	p
	Constant	25.787	2.888		8.929	.000				
MTS	CTS	.417	.039	.496	10.584	.000	.496	.246	112.020	.000

Table 9. Results of simple regression analysis of whether students' academic achievement predicted MTS scores

Dependent variable	Independent variable	B	Standard error	B	t	p	R	R^2	F	p
	Constant	53.861	3.678		14.645	.000				
MTS	Academic achievement	.449	.786	.031	.571	.568	.031	.001	.326	.568

analysis results, the performance disparity between 5th, 7th, and 8th grade students favored 6th grade students. Similarly, for students from CTS, it was observed that the mean scores exhibited a significant difference based on grade level (F[3.346]=2.013, p>.05). The total score averages for CTS students, from lowest to highest, were, as follows: 6th grade (M=70.75), 5th grade (M=72.13), 8th grade (M=72.54), and 7th grade (M=74.87).

Findings Related to the Third Sub-Problem

In line with the third research problem, Pearson's correlation analysis was performed to determine a significant relationship between mathematical thinking of students and their critical thinking and academic achievements. Findings are given in **Table 7**.

According to **Table 7**, a significant positive and moderate relationship was observed between MTS scores and CTS scores of the students (r=0.496; p<0.05). In addition, a positive and low level correlation was observed between MTS and CTS scores of students and academic achievement. These findings of Pearson's correlation analysis are thought to indicate that for regression analysis, the assumption of multiple co-connectedness is met.

Findings Related to the Fourth Sub-Problem

In line with the fourth research problem, the data obtained on whether CTS scores of the students predicted MTS scores are given in **Table 8**. Examining **Table 8**, it can be seen that CTS scores of the students significantly predicted their MTS scores (R^2=.246, F=112.020; p<.005). Students' predictive skill self-efficacy explained 24.6% of the total variance in predictive ability. In other words, as the critical thinking skills of the students increase, their mathematical thinking skills also increase.

Findings on the Fifth Sub-Problem

In line with the fifth research problem, the data obtained on whether the academic achievement of the students predicted MTS scores are given in **Table 9**. Examining **Table 9** is examined, it is seen that the academic achievement of the students did not predict their MTS scores (R^2=.001, F=.326; p>.05). Students' academic achievement explains about 0.1% of the total variance in predictive ability.

Findings on the Sixth Sub-Problem

CTS scores and academic achievements of the students in line with the research problem, together with the data obtained for MTS scores are given in **Table 10**. Examining **Table 10**, it can be seen that CTS scores and academic achievements of the students together significantly predicted MTS scores (R^2=.247; p<.005). Students' CTS scores and academic achievement together explain

Table 10. Results of multiple regression analysis of students' academic achievement with CTS to predict MTS scores

Dependent variable	Independent variable	B	Standard error	B	t	p	R	R^2	F	p
	Constant	27.954	4.027		6.942	.000				
MTS	CTS	.421	.040	.501	10.586	.000	.497	.247	56.243	.000
	Academic achievement	-.533	.690	-.037	-.773	.440				

about 25.0% of the total variance of MTS scores. In other words, as students' CTS scores and academic achievements increase, MTS scores also increase.

DISCUSSION & CONCLUSIONS

This research, which examines the relationship between students' mathematical and critical thinking skills and academic achievements, is based on data obtained from 346 students. The critical and mathematical thinking levels of the students are at a moderate level, mathematical and critical thinking skills do not differ according to gender. In addition, MTS score averages of male students are higher than those of female students, and CTS score averages of female students are higher than those of male students. The study, "development of mathematical thinking in teacher candidates," conducted by Alkan and Bukova-Guzel (2005), explored the mathematical thinking levels of prospective teachers and found no significant gender-based differences. In his research titled "an evaluation of mathematical thinking skills," Duran (2005) investigated the impact of various variables related to mathematical thinking on 15-year-old students as part of PISA assessment, predicting their proficiency in mathematical thinking skills. The findings indicated that male students outperformed their female counterparts, and individuals who had undergone preschool education exhibited higher success rates compared to those without preschool education. In the studies on the critical thinking skills of individuals and gender in the field literature (Ay & Akgol, 2008; Cikrikci, 1996; Gelen, 1999; Kaya, 1997; Kurum, 2002; Walsh & Hardy, 1999), it has been shown that female students have higher critical thinking power than male students. As a result, while the findings are in favor of female students in some studies that have examined mathematical thinking skills according to gender (Pajares & Miller, 1994; Tella, 2011), these results are in favor of male students in other studies (Junge & Beverly, 1995). Accordingly, it can be said that this research finding is partially similar to findings in the field literature.

According to the grade level, mathematical thinking skills of the students differ significantly. This does not apply to critical thinking skills. It can be seen that both mathematical and critical thinking skills develop as the grade level increases, but this value is at the highest level at the 5th grade and decreases during the transition to the 6th grade. In some studies, significant differences have been found in terms of critical thinking skills according to the grade levels of the students. In many of these studies, Kokinda (1989) found that there was an increase in the same direction between critical thinking power and deductive, inference, and evaluation of arguments sub-scales; Adams (1999), Brooks and Stephard (1992), Cikrikci (1992), Evcen (2002), Miller (1992), and Scott et al. (1998) found that there was an increase in the same direction between critical thinking power and the grade level. In this study, although there is no significant relationship between the grade level and academic achievement of the students, critical thinking skills increase as the grade level increases and the 5-8th grade level increases. It has been revealed that the difference between grade levels was in favor of the 6th graders. Koksal and Cogmen (2018) concluded that there is a significant difference in critical thinking skills of secondary school students in favor of female students according to gender, but there is no significant difference according to grade levels. Kurum (2002) found that there was a difference in the interpretation subtest in favor of university 2nd year students. The reason for this situation may be related to the cognitive development levels of 6th grade students, their transition from abstract thinking to concrete thinking process.

It is seen that there is a significant and moderate relationship between students' mathematical thinking skills and critical thinking skills. This does not apply to the academic achievement variable. In this study, it was seen that students' CTS scores and academic success together significantly predicted MTS scores, but CTS scores had more effect. Accordingly, it can be said that as critical thinking skills of the students increase, their mathematical thinking skills can increase. In the study, where Tüzün and Cihangir (2020) determined the relationship between mathematical thinking stages and the mathematical self-efficacy of secondary school students, it was determined that there was a correlation between the students' mathematics course end of semester scores and mathematics self-efficacy and mathematical thinking stages. In Karakoca's (2011) study, it was seen that there was a significant differentiation in the variable of mathematical achievement in mathematical thinking situations in problem-solving, and in the study of Alkan and Bukova-Guzel (2005), there was a linear relationship between the scores of analysis courses of mathematics teacher candidates and mathematical thinking. Mubark (2005) found a significant relationship between mathematical thinking skills and mathematical achievement in his study. In a study conducted by Kocaman (2017), the researcher stated that there was a positive significant relationship between mathematical thinking skills and achievement, and in a study by Nepal (2016), it was reported that there was a strong correlation between mathematical thinking skills and Mathematics Achievements. Accordingly, it can be said that this research finding differs from the findings of the field literature, but it is similar to the conclusion in a study by Alkan and Bukova-Guzel (2005) that there is no relationship between university entrance exam scores and 1st grade semester averages of prospective teachers and their mathematical thinking skills. In addition, studies on the relationship between critical thinking and academic success (Elci et al., 2020; Mete, 2021; Ozcan, 2017) found that there is a significant relationship between secondary school students' critical thinking tendencies and academic success. This research has partially revealed different results in the literature findings.

Consequently, it was observed that there was a relationship between students' critical and mathematical thinking skills, both critical and mathematical thinking skills of students did not differ significantly according to gender, and mathematical thinking skills differed significantly according to grade level. It was concluded that both mathematical and critical thinking skills of the students increased according to their grade levels.

Recommendations

It is important to raise students' mathematical and critical thinking levels to the highest level. In this context, it is recommended that teachers be included activities aimed at developing these skills in students in classroom environments and increase the number of activities aimed at developing these skills in the curriculum. Conducting research on more limited groups of students through clinical or semi-structured interviews and addressing how critical and mathematical thinking are cognitively can be useful in terms of reaching deeper information on the subject. Experimental studies can be carried out with students, teachers and teacher candidates for all sub-dimensions of mathematical and critical thinking. This research can be applied in different types of schools, in different regions, and at different grade levels, thus expanding the scope of the research.

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Declaration of interest: No conflict of interest is declared by the author.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

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