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# Imaginary dialogues and teacher role: Examining the cognitive and linguistic processes of preservice mathematics teachers

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**ABSTRACT**

In this study, the cognitive-linguistic features of teacher roles and word patterns were explored through imaginary mathematical dialogues constructed by preservice mathematics teachers. First, preservice mathematics teachers were given mathematical communication training. This training is a dialogic training on how to conduct mathematical dialogues from beginning to end by introducing mathematical dialogues to pre-service teachers. After 14-week communication lessons, preservice mathematics teachers were asked to write an imaginary mathematical dialogue. The preservice mathematics teachers wrote imaginary dialogues according to four discourse types (Teacher, Teacher-Students, Teacher-Student, Student-Student). The data of the study were analysed by content analysis. Content analysis was carried out in qualitative analysis software. It was concluded that pre-service teachers were able to create quality and different types of mathematical dialogues by using discourse types. Mathematical dialogues were found to be affected by the linguistic features used by the pre-service teachers. For this reason, it was revealed that the word patterns used by the teacher are important. Strong relationships were found between the teacher's roles and the word patterns used in imaginary mathematical dialogues. It was also found that some of the word patterns were specific to the discourse type.

**Keywords:** imaginary dialogues, dialogic training, communication lesson, cognitive-linguistic features

## INTRODUCTION

Mathematical dialogues have been a subject of interest throughout history. The tradition of dialogues between Socrates and his students in ancient times continued in the Renaissance period, where mathematical dialogues between teachers and their students were documented. During this period, there were mathematical dialogues including students' questions to teachers and the responses given to them. In the 19th century mathematical dialogues, students developed their own methods for calculation, and the explanations of these methods were acknowledged. These examples have influenced the scientists of the 21st century and shaped mathematical dialogues to international standards. It is indisputable that mathematical dialogues are an essential component of mathematics, as noted by Mendez (2001). Mathematical dialogues help the teacher scaffolds the space that the pupil needs to construct her/his knowledge (Ramis-Conde & Hope, 2020). Because the dialogues are used as a thinking tool (Mercer & Littleton, 2007). Imaginary mathematical dialogues are a useful tool for observing a student's thought process, and they facilitate an understanding of students' perceptions and challenges encountered during the learning process (Wille, 2008). Moreover, students' self-writing of dialogues involving mathematical argumentation is considered the first step in developing pre-formal arguments and proofs, as well as conceptualizing proof (Askevold & Lekaus, 2018). Mathematical dialogues in the classroom or the imaginary mathematical dialogues written by imagining the classroom involve mathematical discourse between teachers and students. Mathematical dialogues, which is indicative of a high-quality classroom experience as it involves the explanation and discussion of thoughts, as well as the defence of mathematical ideas (Walshaw & Anthony, 2008). International standards also highlight the importance of allowing students to express their views through mathematical discourse within mathematical dialogues (National Council of Teachers of Mathematics [NCTM], 2000). Imaginary mathematical dialogues are also utilized to understand students' mathematical thoughts (Wille, 2017b). Thus, mathematical discourse within mathematical dialogues/imaginary mathematical discourse serve as a bridge for students to express their opinions.The discourses of teachers have been shown to be influential in initiating mathematical dialogues and accessing mathematical ideas, as well as in closing those dialogues (Demirbağ, 2017; Demirci Celik & Baki, 2023). Therefore, the type and quality of teachers' discourse play an important role in conceptual comprehension (Chapin et al., 2003). Therefore, it is important for teachers to be aware of the types of discourses and to guide classroom dialogues accordingly. Future mathematics teachers who possess this awareness and knowledge will be able to steer teacher-student dialogues effectively during their teaching careers, and in doing so, enhance the

quality of mathematical communication. Wille (2017a) suggests that preservice teachers can positively influence students' mathematical learning by gaining awareness of discourse when writing their imaginary mathematical dialogues. The teacher has an important role in encouraging to students' learning by participating in mathematical dialogues (Akçakoca et al., 2024). Therefore, in this study, the role of the teacher in classroom dialogues and the word patterns used according to the role will be determined by focusing on the process of writing imaginary mathematical discourse by pre-service teachers.

## Conceptual Framework

According to sociocultural theory, learning is conceptualized as a process shaped through discursive practices in which individuals participate via classroom interactions (Vygotsky, 1978). From this perspective, cognitive development becomes visible not through individual mental operations alone, but through the roles and forms of participation that teachers and students assume within discourse (Rogoff, 1995). Classroom discourse is regarded not merely as a means of transmitting mathematical content, but as an interactional context in which cognitive processes are jointly constructed (Mercer & Howe, 2012). In this regard, teachers' discursive choices are closely related to how students engage in mathematical thinking and the types of cognitive actions they perform (Sfard, 2008). Within this conceptualization, attention shifts to the nature of classroom dialogue as an organizing component of mathematical discourse. In particular, differences in the quality of classroom dialogue are considered consequential for how mathematical ideas are explored, negotiated, and sustained over time.

Quality classroom dialogues on mathematical discourse between teachers and students are defined as interactive, rich, and meaningful, as well as continuous (Piccolo et al., 2008). There are several studies that classify the types of classroom-based mathematical discourse, speech, and discussion, considering them as tools for mathematical dialogues (Chapin et al., 2003; Hufferd-Ackles et al., 2004; Kazemi & Hintz, 2014; Mercer, 1995; Richards, 1991). Mathematical dialogues emerge as a result of explanations related to in-class questions and issues. Because the teacher orchestrates mathematical dialogues (Herbel-Eisenmann et al., 2013; Kooloos et al., 2020) the teacher has an important role in the emergence of mathematical dialogues (Wachira et al., 2013). Because mathematical dialogues are influenced by the linguistic features used by the teacher (Ng et al., 2021). Lemke (1990), who conducts studies about in-class dialogues, states that the most commonly used dialogue type by teachers during lessons is the triadic pattern. He argues that the triadic pattern is established through the teacher's question, the student's response, and the teacher's evaluation. Following on Lemke's (1990) dialogue types, Cazden (2001) suggests that this dialogue consists of three stages: teacher's initiation of discourse (Initiation), the students' responses (Response), and the teacher's evaluation (Evaluation). Another dialogue model is the classroom dialogue model, which is argued by Mortimer and Scott (2003) and presents different structures. Therefore, Mortimer and Scott's (2003) dialogue model can be used to consider the interaction levels between teachers and students. Celik (2019) proposes dialogue model which consists of four discourse types based on grounded theory and an analysis of approximately 500 hours' video recordings. The four discourse types suggested by Celik (2019) have both similarities and differences with Mortimer and Scott's (2003) dialogue model. For example, the non-interactive/authoritarian dialogue type in Mortimer and Scott's model is referred to as Teacher discourse type, as teachers' discourses are more common in this type of dialogue. Interactive/Dialogue-based dialogue type, on the other hand, is associated with *Student-Student* discourse type since classroom interaction is multidimensional. Therefore, there are similarities between Mortimer and Scott's (2003) dialogue model and Celik's (2019) discourse types model. However, Celik (2019) also identifies the *Teacher-Student* discourse type as a result of video analysis, which is different from the dialogue model because more than one student participates in the discourse simultaneously. Similarly, discourse types are named based on whether or not the interaction occurs between teachers and students and by placing classroom interaction at the core. The Teacher-Student discourse type has been identified as another discourse type, which examines students' interactive participation in mathematical discourse under the guidance of teachers. Thus, four in-class discourse types emerged: Teacher, Teacher-Students, Teacher-Student, and Student-Student.

## Relevant Studies

Research on dialogues in mathematics classrooms highlight the need for teachers to be aware of how students participate and talk in classroom discourses. To develop this awareness, mathematics teacher educators require professional development (Razfar, 2012). Numerous studies have been conducted to investigate the development of mathematical discourse in both experienced and preservice teachers. For instance, Bennett (2010) investigated how two preservice mathematics teachers developed the mathematical discourse of their entire class with the guidance of a mentor. By the end of the term, these teachers engaged more students in discussions, asked more questions that aimed to assess comprehension, and reduced the time spent on delivering instruction. Kabael et al. (2017) conducted a study comparing the mathematical discourse and discourse analysis practices of a middle mathematics teacher trained in mathematical communication and an experienced teacher who was not trained in this area. The study revealed that the mathematical discourse and discourse analysis practices differed significantly between the preservice and experienced teacher**.**

There exists a body of literature that investigates how preservice mathematics teachers develop mathematical discourse in their classrooms. For instance, Mosvold (2015) examined how preservice teachers developed mathematical discourse by creating lesson plans with mentor teachers to improve their classroom discourse. The study found based on observational data that with structured and frequent support from their mentors, preservice teachers demonstrated visible improvements in their ability to involve a higher number of students in classroom debates and make mathematical operations more easily. In addition, preservice teachers used imaginary dialogues as a means of improving their mathematical discourse and knowledge. Spangler and Hallman-Thrasher (2014) asked preservice teachers to write imaginary mathematical dialogues, based on student responses that were correct, partially correct, or incorrect, to enhance their understanding of teaching mathematics. Their study revealed that imaginary dialogues that incorporated mathematical tasks assisted preservice teachers in improving their knowledge of

mathematical content and in turn, benefited their students. In a study about preservice teachers' experiences of using imaginary mathematical dialogues, Brodahl and Wathne (2018) provided preservice teachers with a problem situation and an initial mathematical dialogue, and asked them to write imaginary dialogues by completing the initial dialogue. Preservice teachers used this imaginary dialogue in the classroom where they conducted a practice lesson. After this experience, it was determined as to how preservice teachers perceived the role of imaginary mathematical dialogues in how students explained their mathematical thinking. A descriptive analysis was conducted under four categories of teacher perceptions, such as positive, negative, mixed, and natural. The study found that the majority of preservice teachers had positive perceptions. In contrast to such studies, the present study focuses not on preservice teachers' perceptions, but on the discursive features and teacher roles reflected in the imaginary mathematical dialogues they construct across different discourse types.

### Rationale and Significance of the Research

The literature on mathematical dialogues highlights the importance of teachers and teacher training in effectively explaining students' mathematical thinking. Thus, preservice teachers need to be cognizant of the different types of discourse. For example, Demirbağ's (2017) study on discourse types found that preservice science teachers who employed science-related knowledge in small and large group discussions were able to foster the development of arguments. Similarly, Kersaint (2015) noted that teachers can direct mathematical dialogues to encourage student participation. However, Peressini and Knuth (1998) observed that teachers' dialogue tended to be monologic, highlighting the need for teachers to develop more dialogic discourse. In this context, the present study discusses the four discourse types developed by Celik (2019), namely Teacher, Teacher-Students, Teacher-Student, and Student-Student, which encompass both monologic and dialogic discourse. Additionally, Hiebert et al. (2003) emphasize that many teachers lack experience with mathematical discourse during their teacher education, leading to difficulty creating an appropriate learning environment for students. Overall, it is evident that teachers require specific training in the development of mathematical discourse to effectively utilize it in the classroom. Bennett's (2010) study, for instance, observed classrooms and found that teachers struggled to promote mathematical discourse among students. To address this issue, it is important to provide professional support and training to teachers, such as a "Mathematical Communication" course. In this course, preservice teachers can learn about different types of mathematical discourse and how to guide dialogues, which would help them to increase the quality of mathematical communication in the classroom. Several studies have explored the effectiveness of such courses in improving mathematical discourse in the classroom. For instance, Ramis-Conde and Hope (2020) provided training to preservice teachers on the techniques of conducting mathematical dialogues and found that it gave children an opportunity to think about constructing mathematical concepts on their own. Furthermore, Wille (2017a) asked preservice teachers to write imaginary dialogues in learning algebra and found that it raised their awareness of potential errors and misconceptions in their use of mathematical discourse. Therefore, it is crucial for preservice teachers to receive proper training on how to conduct mathematical dialogues, from start to finish, in order to promote quality mathematical communication in the classroom. This can be achieved through pedagogical courses during pre-service teacher education.

The proposed study on how preservice teachers develop mathematical dialogues after receiving training on discourse types will make a significant contribution to the literature. Ramis-Conde and Hope's (2020) study, which also focused on preservice teachers' mathematical dialogues after providing training, primarily addressed student learning and motivation. However, this current study aims to investigate how preservice teachers create mathematical dialogues, making it distinct from Ramis-Conde and Hope's study. By exploring whether preservice teachers provide opportunities for students to participate, how they perceive their role as a teacher, and what language patterns they use while creating mathematical dialogues, this research will make valuable contributions to mathematics education literature. To gain insight into how preservice teachers create mathematical dialogues, this study employs imaginary mathematical dialogues, which can provide a better understanding of preservice teachers' thinking processes (Wille, 2017a). After participating in a Mathematical Communication course designed to introduce and improve the creation of mathematical dialogues, preservice teachers were asked to produce imaginary dialogues. This approach will help to illuminate how preservice teachers develop mathematical dialogues in practice, providing valuable insights that can inform teacher education programs and ultimately lead to better-quality mathematical communication in the classroom.

The present study aimed to address the following research questions, focusing on the experiences of preservice mathematics teachers:

1. What is the role of the teacher in creating discourse types related to terminology, as perceived by preservice mathematics teachers?
2. Which word patterns can teachers use when creating discourse types related to terminology, according to preservice mathematics teachers?
3. What is the relationship between the role of the teacher and the word patterns used when creating discourse types related to terminology, as perceived by preservice mathematics teachers?

## METHODOLOGY

The research process, including the pilot study, lasted approximately two years. The study was designed as action research in order to improve the Mathematical Communication course in a way that supports preservice mathematics teachers' ability to construct imaginary mathematical dialogues and to refine the instructional process based on the data obtained during the course practice. The main reason for structuring the study as action research was that it did not merely aim to describe an existing situation, but rather to develop an instructional practice through a cycle of planning, practice, evaluation, and revision.

The research was conducted within the framework of the four main steps of action research identified by Mills (2000): determining the focus area, collecting data, analyzing and interpreting data, and developing an action plan. These four steps were addressed in an interconnected way throughout the two-year research process. In the first year, a pilot practice of the Mathematical Communication course was carried out. The data obtained during this process provided a basis for determining the focus area, interpreting the initial findings, and developing an action plan that guided the second-year practice. In the second year, the Mathematical Communication course was restructured in line with this action plan, and the main practice was conducted.

In the first-year practice, the Mathematical Communication course was carried out with a group of preservice mathematics teachers. During this process, preservice teachers received training on the structure of imaginary mathematical dialogues, classroom mathematical communication, and different discourse types. At the end of the course practice, feedback was collected from the participants regarding the content and delivery of the course. This feedback showed that preservice teachers particularly emphasized the teacher's role in imaginary mathematical dialogues. Accordingly, the focus area of the study was determined as teacher roles and word patterns used in imaginary mathematical dialogues. Based on these results, an action plan was developed, and it was decided that the second-year practice would be structured more systematically around teacher roles, word patterns, and discourse types.

In the second-year practice, the Mathematical Communication course was conducted with a new group of preservice mathematics teachers in accordance with this action plan. In this practice, teacher roles, word patterns, and the ways in which different discourse types affect the quality of imaginary mathematical dialogues were addressed more explicitly and systematically. The participants examined examples of imaginary mathematical dialogues, discussed different discourse types, and focused on how teacher roles and linguistic choices shape classroom mathematical communication. At the end of the course, the participants were asked to write imaginary mathematical dialogues based on the discourse types addressed in the course. In addition, interviews were conducted with the participants regarding the imaginary mathematical dialogues written in the second-year practice.

Thus, the research process was completed by determining the focus area based on the findings obtained from the pilot practice conducted in the first year, developing an action plan based on these findings, restructuring the second-year practice within the framework of this plan, having the participants write imaginary mathematical dialogues, and evaluating the instructional process in light of the data obtained through the interviews conducted regarding these dialogues. In this respect, the study was conducted based on the action research cycle of planning, practice, evaluation, and revision.

### Research Participants and Research Process

The participants of this study comprised 39 middle school preservice mathematics teachers who were enrolled in the Mathematical Communication Course. The course plan for the Mathematical Communication course for preservice teachers was developed, and in the first weeks of the training, general information was provided to the preservice teachers about the importance of mathematical communication and its role as a tool for students to explain mathematical thoughts (**Table 1**). Examples were given on how mathematical dialogues were utilised in the mathematical communication process to raise awareness of preservice teachers about mathematical dialogues. In the following weeks of the training, the types of discourse that underpin mathematical dialogues (i.e., Teacher, Teacher-Students, Teacher-Student, Student-Student) were explained by giving examples. Indicators for each of the discourse types were introduced to preservice teachers. Preservice teachers were shown that discourse types in a mathematical dialogue form the general structure of the dialogue, and that the dialogue itself has indicators (e.g., motivation, explaining mathematical thoughts, and reaching mathematical ideas). Sample dialogues from real classroom environments were presented to the preservice teachers through video recordings and written transcripts. Preservice teachers were asked to focus on the indicators in the formation of discourse types while watching these videos. Examples from short sample mathematical dialogues were provided to show how indicators for types of discourse were formed. Later, sample written dialogues were provided to preservice teachers, showing how discourse types were formed from start to finish. In these dialogues, preservice teachers were expected to fill in the blanks within the mathematical discourse that were left blank. Finally, preservice teachers were asked to identify the indicators of the four discourse types in the written mathematical dialogues. During the training, the unclear aspects of mathematical dialogues were clarified for preservice teachers through question and answer sessions. For example, it was noticed that the "following from the blackboard/smartboard" indicator, which is one of the indicators in a Teacher discourse-type dialogue, was understood as the moments when students only looked at the board. Later, it was understood that the teacher was speaking to engage students in the lesson so that they would listen carefully to the mathematical terms explained. Additionally, it was observed that some preservice teachers tried to determine a different indicator in each line. With the individual feedback given to students, it was shown that some lines would combine to form an indicator.

**Table 1.** Summary of the course plan for the Mathematical Communication Course

| Week | Theme | Course Process |
|---|---|---|
| Week 1 | The importance and presentation of mathematical communication, mathematical dialogue, and mathematical discourse | Presentation of general and mathematics-specific concepts in the literature |
| Week 2 | The relationship between mathematical communication, mathematical dialogue, and mathematical discourse | Examination of dialogues created with mathematical terms |
| Week 3 | Presentation of discourse types through sample dialogues from video recordings | Discussion over discourse types |
| Week 4 | Presentation of discourse types through sample dialogues from video recordings | Discussion over discourse types |
| Week 5 | Presentation of discourse types through sample dialogues from video recordings | Discussion over discourse types |
| Week 6 | Analysis of discourse types through sample dialogues and transcripts from video recordings | Recognition of indicators about discourse types |
| Week 7 | Analysis of discourse types through sample dialogues and transcripts from video recordings | Recognition of indicators about discourse types |
| Week 8 | Analysis of discourse types through sample written video recordings | Analysis of discourse types and indicators |
| Week 9 | Determination of differences and indicators between discourse types | Experience of writing imaginary discourses |
| Week 10 | Writing imaginary dialogues appropriate for *Teacher* discourse type | Writing mathematical dialogues in four discourse types by selecting any of the outcomes of middle school mathematics curriculum |
| Week 11 | Writing imaginary dialogues appropriate for *Teacher-Students* discourse type | |
| Week 12 | Writing imaginary dialogues appropriate for *Teacher-Student* discourse type | |
| Week 13 | Writing imaginary dialogues appropriate for *Student-Student* discourse type | |
| Week 14 | Writing imaginary dialogues involving all discourse types | |

## Data Collection

Preservice teachers were first introduced to how mathematical dialogues are constructed. This introduction included discussions on the structure of mathematical dialogues, the role of teacher questions and responses, and the ways in which different discourse types can shape classroom mathematical communication. Then, the preservice teachers wrote imaginary dialogues according to the discourse types. The discourse types used in the study were Teacher, Teacher–Students, Teacher–Student, and Student–Student discourse. These discourse types were introduced to the participants before they began writing their imaginary mathematical dialogues.

After the 10th week, semi-structured interviews were conducted with the preservice teachers about the imaginary dialogues they had written. The purpose of these interviews was to clarify the participants' intentions while writing the dialogues and to obtain more detailed information about how they constructed teacher roles, selected word patterns, and connected the dialogues to possible classroom practices. The preservice teachers were interviewed about the dialogue they had written for each discourse type. Since there were four different discourse types, four interviews were conducted with each preservice teacher. Each interview focused on one discourse type and the participant's dialogue written for that discourse type. In this way, the interviews allowed the researcher to examine the participants' reasoning separately for each discourse type.

The imaginary dialogues written by the preservice teachers according to the discourse types were collected every week. Thus, the data collection process was carried out gradually throughout the implementation rather than at a single point in time. This weekly collection process enabled the researcher to follow how the participants constructed mathematical dialogues across different discourse types. How the preservice teachers wrote the imaginary dialogues was determined by asking questions about the dialogues they had written. During the interviews, follow-up questions were also asked when participants' explanations needed clarification or when their use of particular word patterns required further explanation.

The questions included:

- ✓ • "What is the role of the teacher in the …… discourse type when writing mathematical dialogues related to the terminology?"
- ✓ • "Did you use specific word patterns while writing mathematical dialogues related to the terminology in the ……. discourse type? If yes, please provide examples."
- ✓ • "In the ……... discourse type, how did you develop the dialogues related to the terminology in a real classroom environment, or did you not?"

These questions were designed to reveal both the cognitive and linguistic dimensions of the imaginary dialogues. The first question focused on the teacher role constructed by the participant, the second question focused on the word patterns used in the dialogue, and the third question focused on the relationship between the written dialogue and a possible real classroom context.

## Data Analysis

The qualitative data analysis software MAXQDA 2020 was used to analyze the preservice teachers' views and imagined mathematical dialogues. The data were first examined holistically and then analyzed in greater depth through content analysis.

In this process, themes and subthemes related to the teacher roles and word patterns used in the construction of mathematical dialogues were identified.During the coding process, each imagined mathematical dialogue was analyzed separately within its own discourse type. In this regard, the dialogues written according to the Teacher, Teacher–Student, Teacher–Students, and Student–Student discourse types were examined individually. In each dialogue, the teacher role represented according to the discourse type was first identified, and then the word patterns reflecting or supporting this role were coded.For example, the dialogues in the Teacher discourse type were examined in terms of the teacher role represented in the text. In this discourse type, the teacher's role as someone who establishes authority, conveys knowledge, stands out through expertise, and guides the lesson was more dominant. Expressions such as "Now I will explain this concept," "First, I will explain this rule to you.","You should use this term in this way," and "Pay attention, the important point here is this" were evaluated as statements reflecting the teacher's authority and control over the discourse. These expressions were coded under word patterns such as "establishing authority," "conveying knowledge," "explaining the term/formula," and "drawing attention." Similarly, expressions such as "This concept is defined as ..." and "You should remember this term because we used it in the previous lesson" were associated with the teacher's expert role, command of terminology, and function of helping students recall prior knowledge.

The reliability of the data was ensured through temporal consistency. Accordingly, the coding process was carried out twice within a six-month period. The coding conducted at different times was compared to identify similarities and differences. In the second round of coding, expert opinion was obtained, and codes that were semantically close to one another were combined under more comprehensive headings. Thus, the codes were finalized in a way that better reflected the word patterns and teacher roles expressed in the preservice teachers' imagined mathematical dialogues.

In determining the code names, not only the literal meanings of the expressions but also their functions within the dialogue were taken into account. For example, in the Teacher–Students discourse type, expressions such as "Let's think about this together now," "How did we reach this conclusion together?" and "Who in the class would like to say the name of this term?" were coded in the first round of coding as "guiding students to think together," "reaching a conclusion together," and "asking a question to the class," respectively. In the second round of coding, it was observed that these expressions served the function of involving the whole class in a shared process of thinking and speaking; therefore, in line with expert opinion, they were combined under the code "collectively involving students in the discourse."

Strong relationships were identified between teacher roles and the word patterns used in the dialogues, and these relationships were displayed in MAXQDA maps. MAXQDA maps were used to visualize the relationships among teacher roles, word patterns, and discourse types. Through these maps, it was shown more clearly which teacher roles were more intensely associated with which discourse types and word patterns.The preservice teachers were coded as T1, T2, ... T39, and in the findings section, the analysis process was supported with selected examples from the dialogues written by the preservice teachers.

## FINDINGS

After the training of preservice mathematics teachers, the role of the teacher in writing imaginary mathematical dialogues differs according to the discourse type. It can also be said that word patterns used in writing imaginary dialogue differ depending on the discourse type. Therefore, the findings of the research are presented according to the types of discourse used in the process of creating imaginary mathematical dialogue: Teacher, Teacher-Students, Teacher-Student, Student-Student discourse types.

### Teacher Discourse Type

In this type of discourse in which teachers' discourses are active, teachers have roles such as *being an expert in mathematical terminology, having authority and being active and energetic, and being competent with the terminology* whilst talking about the terms. When preservice teachers write imaginary mathematical dialogues, they need to be the expert in giving basic terminological information and summarising the subject. For example, P20 said:

> Teachers' role is to convey the terms and symbols that give direct information, to deliver to students what is written in the book, in the same way that is described in the book or in a way that they know. Their role does not allow students to interpret and comprehend the information in their own terms. Sometimes it's a word we actually know, but we can't even express the simple term we know and ramble on it because we don't remember its definition in the book and we are not allowed to provide a definition by ourselves. Again, students, for whom teachers summarise and revise the subject, are not allowed to think about the subject in a different way. For example, the fact that teachers do not even perform an activity mentioned in the book but rather explain it to students in a brief way in the dialogue I wrote is an example of this situation....

Thus, teachers must be experts in terminology when transferring terminological knowledge. Teachers' authority was also more visible in their discourse. That is, statements, such as the inability to question teachers' discourse or the fact that they give commands to students about what to do, show that teachers have authority. Preservice teachers emphasised that teachers ought to know the terminology and have a command of the terms by using correct and clear terms. Considering that teachers will provide the information directly to students, they needed to have a good command of the terms for enabling rote-learning in students. P22 supported this:

> While the teacher writes the mathematical dialogues within the scope of the terminology in the discourse type, the teacher's role is to make students memorise the terms, unfortunately. The teacher tries to load more information on to

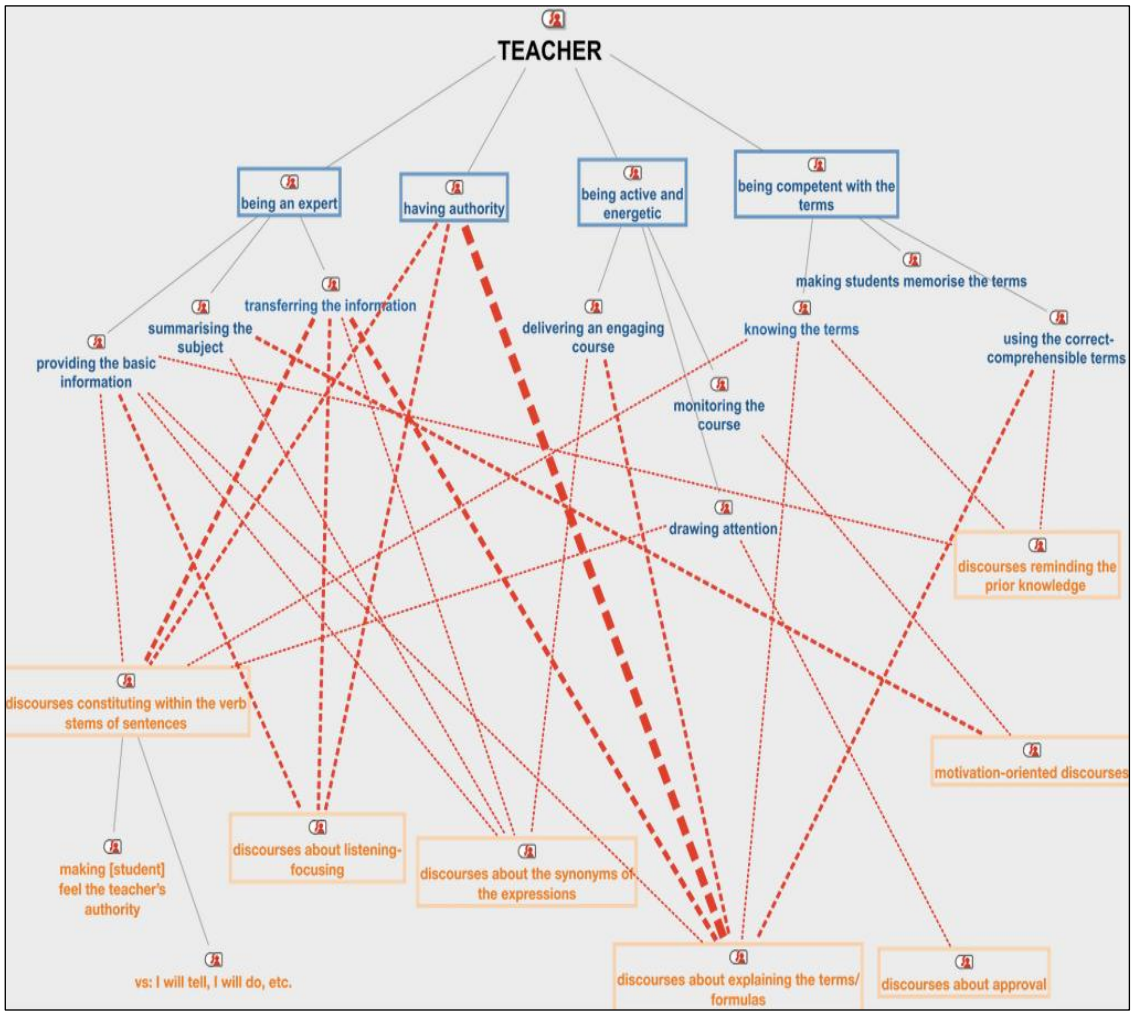


**Figure 1.** The relationship between teachers' roles and the word pattern used in the Teacher discourse type (Source: Authors' own elaboration)

Note. Blue codes represent the teacher role; orange codes represent word patterns. This representation is the same on all maps. The thickness of the lines represents the frequency.

> students through rote and expects that all of this information will remain in the student's memory. They don't care much about permanent learning; they give them the knowledge to be memorised and make them memorise the terms and wait for them to solve the question when students face it, in the way that the teacher has already explained.

After the training was received, preservice teachers stressed that they needed to be active and energetic as part of their teacher role during the class. In this context, teachers ought to teach mathematical terminology through an engaging lesson delivery. In addition, students would be expected to keep high attention in the mathematics lesson and follow the course accordingly. P11's statement supports this claim:

> Teachers' role is to track students' engagement with the lesson and to encourage students to participate as much as possible.

It is explained above how preservice teachers see the role of the teacher while writing the imaginary discourses in the Teacher discourse type at the end of the training. When the relationship between word patterns and teachers' role is examined, word patterns seemed to be different according to teachers' role. The relationship between teachers' role and word patterns is shown in **Figure 1**.

**Figure 1** shows that there are stronger links between teachers' role and some word patterns in the process of writing imaginary mathematical dialogues. There is a close connection between the teacher's display of their authority in mathematical dialogues and the pattern of explaining the terms/formulas themselves, compared to other relational ties. Similarly, teachers seem to have many roles between their explanation of the terms/formulas themselves and the word pattern. For example, there are links between the phrase used by the teacher to explain the terms/formulas themselves and their competence with the terms, being an expert in terminology, and being active and energetic throughout the lesson delivery. P18 expressed the teachers' role as being the expert on the subject and, hence, responsible to convey the information:

**Table 2.** The word pattern that the preservice teacher used and the imaginary mathematical dialogue about this word pattern

| Word pattern | Imaginary mathematical dialogue |
|---|---|
| In Teacher discourse type, dialogues that had more intense teacher discourses than student discourses were created. Having examined the dialogue written under the heading of terminology and following the meaning of terminology, it appeared to be the content about terms, symbols, and mathematics. The teachers made the in-depth explanations about the mathematical terminology themselves. As seen in the dialogue, student discourses are too short, clear, and limited. Additionally, the teacher read the definition himself without letting the student ask if they could read it. There was nearly no exchange among students regarding the subject. In the third line of the dialogue, the teacher initially gave the definition of that subject once they moved to that subject, instead of explaining it over an example. | Teacher: Now my friends, let's look at what ratio means. Stop looking around and start focusing on your book. Okay now, read with me. Elmas, open your book.<br>Beril: Should I read, teacher?<br>Teacher: I am reading, you just listen. The comparison of the measures of two quantities with each other in the form of division is called ratio. The ratio of a and b is shown as a/b.We find ratio by dividing two things into proportions. We will try to find this ratio [now]. For example, Cevriye's height is 140, Ada's height is 150. The ratio of Cevriye's height to Ada's height…<br>Akay: 140/150<br>Teacher: Yes, what did he say, first he said Cevriye's height and then Ada's height. You directly said 140/150, you are correct. But if we simplify it, it would be better and becomes 14/15. Thus, by dividing them into each other, we find the simplest form of the ratio. Now let's continue with the examples related to this. Follow me from the book, they are all there. |

Note. The above dialogue consists of 20 lines. The table only shows the relevant part of the word pattern used.

> Students are not given the opportunity to talk about mathematical terminology. In this type of discourse, it is understood that the teacher informs about the new subject, but the teacher himself will explain the terminology related to the subject. It can be said that this is more motivating to make students listen to mathematical discourses related to terminology. Formulas and rules are transferred directly to students.

This preservice teacher used the word pattern to explain the terms:

> Whilst writing the mathematical dialogues within the scope of the terminology in the Teacher discourse type, I first explained the subject in detail to the student through specific word patterns based on a routine as taught to us and gave the rules, then I reinforced the subject by solving some problems. Something like getting feedback from the student is not given much importance in this type of discourse….

Considering word patterns used to create imaginary mathematical dialogue by preservice teachers, the discourse reminding of prior knowledge, not giving approval, and motivation-oriented discourses are less connected with other word patterns and teacher roles. There is a relationship, although weak, between the discourses reminding the prior knowledge and the role of mastering the terms. The discourses created within the verb stem of the sentences, which are one of the other word patterns, also vary among teacher roles. There are discourses such as *"....I will tell, I will do..."* that make the teacher's authority felt in the discourse, in which the verb stems of the sentences are constructed within the word patterns used in the Teacher discourse type. P7 supported this:

> "Since it is the Teacher discourse type, I paid particular attention to use patterns such as, I will explain, I will do, I will show, in my dialogue in general as the teacher explains the terms and mathematical expressions in Teacher discourse type.

Preservice teachers also stated that they tended to use word patterns to make students listen and pay attention to the terminology, since the teacher's discourses were more active in this discourse type. Another word pattern was the discourse about the synonyms of mathematical expressions. When looking at teachers' roles in this word pattern, it is seen that the teacher is an expert and has authority. The imaginary mathematical dialogue that can exemplify this situation and the word pattern used are given in **Table 2**.

### Teacher-Students Discourse Type

In writing imaginary mathematical discourses for this type of discourse, preservice teachers emphasised the importance of the teacher being central in the classroom. Teachers believed that they would simultaneously be able to include students in mathematical discourse. When there was a teacher at the core of teaching, they should use the terms correctly/clearly.

Another role of teachers was determined to encourage students to participate in the discourse collectively by taking a student-centred approach in teaching. P26 supported this:

> The teacher should be a guide, should always put the class at the core, and should always exemplify their thoughts with real-life examples so that class participation is actively ensured.

According to preservice teachers, in relation to writing imaginary mathematical dialogue, teachers could collectively engage students with the mathematical discourse by asking short-answer questions or by asking terms that the whole class could

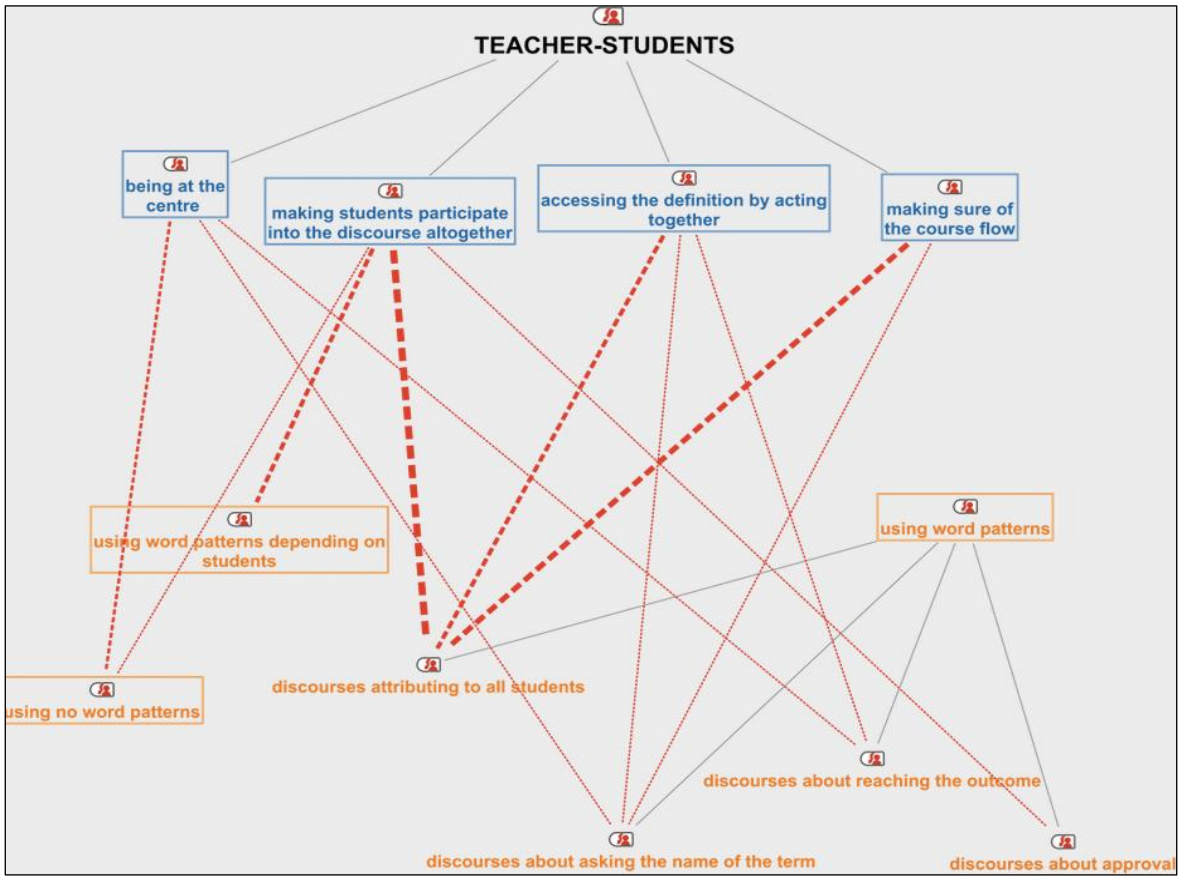


**Figure 2.** The relationship between teacher roles and the word patterns used in the Teacher-Students Discourse Type (Source: Authors' own elaboration)

participate in mathematical discourse at the same time. In addition, it is among teachers' roles that students in the class must reach the definition together with the teacher, as they collectively participate in the mathematical discourse. The following statement illustrates this:

> The teacher reminds students in the class of the prior knowledge before the class, and informs them that they will participate in the mathematical discourse at the same time. Students simultaneously express their ideas about the terminological part, confirming statements are made by the teacher, and students give answers such as yes/no and a collective discourse is provided. Thus, the steps in mathematical discourses are applied. For example, when the teacher asks about taking the square root of X number and asks if it would be, say, 4, more than one student should respond with 'yes'. In other words, teachers' role is to communicate with more than one student in the classroom.

Also, teachers had a role in ensuring the flow of the lesson. While the teacher provides the flow of the lesson, it is emphasized that the students should concurrently participate in the mathematical discourse regarding the example from daily life given by the teacher. Preservice teachers stated that, while students were concurrently participating in the mathematical discourse, teachers should manage the class by maintaining their authority. For example, P4 said:

> The teacher protects his authority. They manage the class and maintain their dominance. They appeal to the majority. Their role is to provide the flow of the lesson. The students in the class also proceed by confirming the information with the teacher….

**Figure 2** presents the relationship between teacher roles and word patterns in the Teacher–Students discourse type. In this discourse type, the teacher role was mainly associated with involving the whole class in the discourse, guiding students toward a shared conclusion, and responding to students' participation.

**Figure 2** shows that there is a difference the teacher role and word patterns when writing imaginary mathematical dialogues, as laid out in the Teacher-Students discourse type. Word patterns related to discourse addressing all students have a very close relationship with many teacher roles. This word pattern mostly includes discourse for thinking together. Preservice teachers used such word patterns as: "*……let's think together, let's start, let's see, etc.*". The statement of P9 illustrates this:

> I used such patterns as 'Let's do it together, let's look at it together, did our average change?'.

Expressions aimed at engaging students within the class are also used in the patterns that address all students by evoking thinking together. Mostly second-person expressions such as "*Are we looking at the board? What are we saying? What are we*

*doing?*" are used. In addition, preservice teachers emphasised that they used expressions that addressed all students in their imaginary mathematical dialogues, and that teachers and students reached definitions by acting together.

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### Teacher-Student Discourse Type

Preservice teachers often stressed teachers' role in teaching the subject. They approached teachers' role in subject teaching in two different ways: student-centered and teacher-centered. In the latter, teachers should explain the terms adequately and accurately. They were regarded as having a central position in teaching since a dialogue between the teacher and a student developed while other students were left disengaged by the teacher. Contrarily, most preservice teachers stated that teaching was rather student-centred. They expressed teachers' role as making students think actively. Teachers needed to support students with the terms that they would have difficulty to comprehend. Preservice teacher coded P21 explained:

> Teacher helps students learn by communicating with a student or a small number of students in the Teacher-Student discourse type. They approach to the terms on the basis of a student's understanding and continues teaching the course accordingly, as in my dialogue, teacher provides the definition of a polygon after a student says 'it has many sides'.

Preservice teachers have identified another role of teachers as active communication established between the teacher and the student whilst writing imaginary mathematical dialogues in the Teacher-Student discourse type. In this context, it has been determined that the teacher should be in active communication with the student by establishing a one-to-one dialogue with the students. Preservice teachers stated that in this way students could be given the opportunity to express their thoughts. The preservice teachers emphasized that students could reach mathematical definitions more easily as they give the students a lot of opportunities to explain their mathematical thinking. Many preservice teachers stated that the role of teachers is not to ask questions to students or to directly give them the rule, but to encourage them to find the rule. For example, P9 illustrates this:

> The teacher involves them in the discourse by giving them an opportunity. He directs them. The teacher enables them to access information by asking leading questions. By reviewing the subject with previous information, the teacher enables students to explore it within a cause-effect relationship. For example, students know that unit squares are used when finding the area, and the teacher aims for students to better understand the subject by building new knowledge on this.

Therefore, another role of the teacher was determined: the teacher should also give student-specific answers in accordance with students' statements about mathematical terminology. P30's statement exemplifies this: "In the dialogues with the student, we should look more from student's window…"

Preservice teachers defined teachers' role as being more student-centric in explaining their mathematical thinking. Considering the relationship between word patterns and teachers' role in this discourse type, it is similar to the relationship in the Teacher-Students discourse type. While writing this type of discourse, word patterns should be used depending on students, as well as having those who use patterns and those who do not. Preservice teachers here differ based on their use of word patterns. The teacher role that preservice teachers determined and word patterns used and the relationship in-between are shown in **Figure 3**.

As seen in **Figure 3**, some of the preservice teachers used word patterns, some did not, and some others used them in accordance with students' mathematical discourses. There was a close link between the patterns that preservices used whilst writing imaginary mathematical dialogue in the Teacher-Student discourse type and the roles of the teacher. There is a very close connection between the teacher giving the student the opportunity to talk about mathematical terminology and them asking questions.

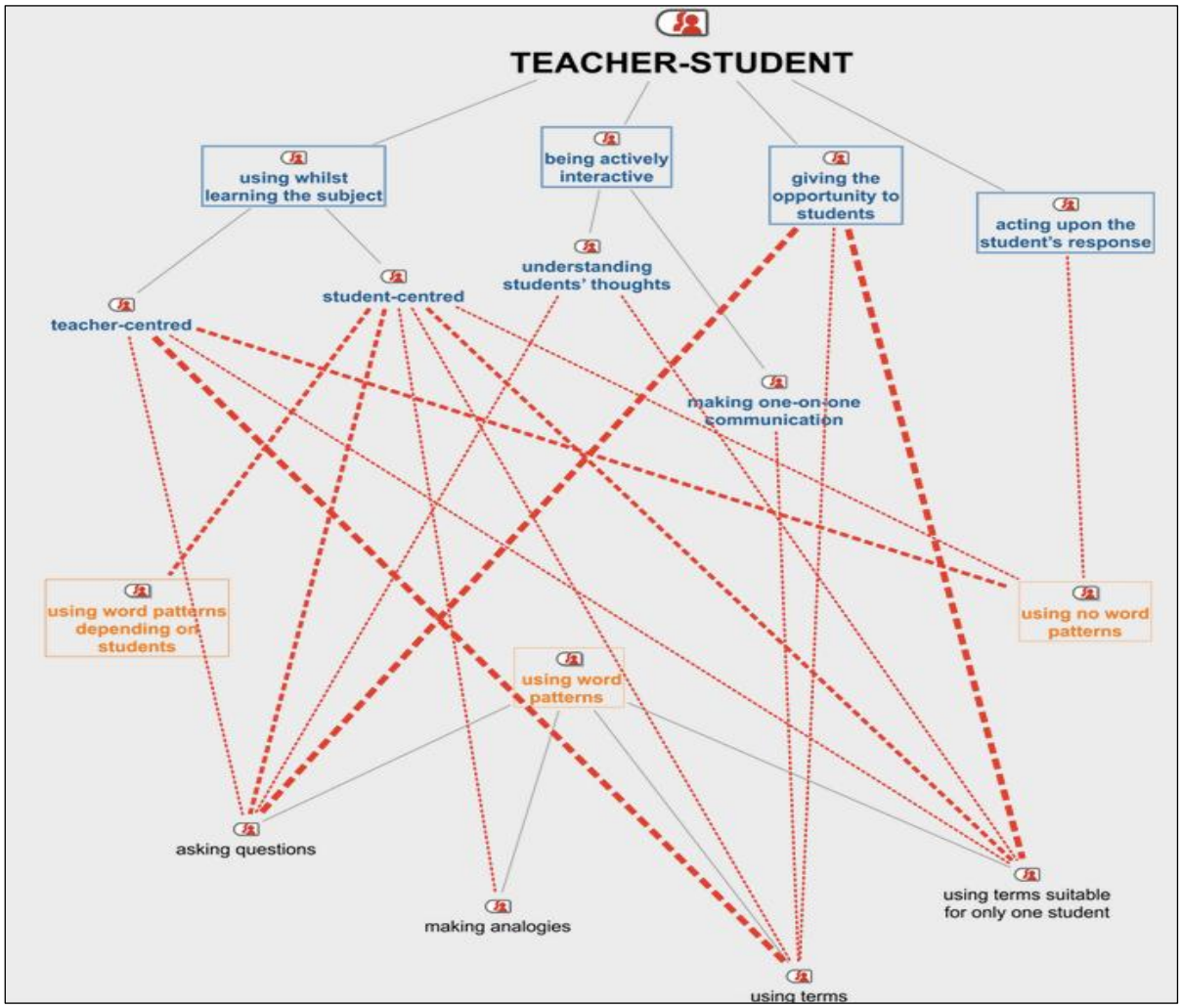


**Figure 3.** The relationship between teacher roles and the word patterns used in the Teacher-Student discourse type (Source: Authors' own elaboration)

When the word patterns for asking questions were examined within themselves, some preservice teachers used discourse about asking reason-why questions to students; some of them used discourse aimed at asking mutual questions. There are also preservice teachers who use both types of questioning. Preservice teacher coded P7 illustrates this with the following:

> As I wrote in the dialogue, I asked questions that would enable students to give answers about mathematical terms, and explain why their answer was like that, since in the Teacher-Student discourse type, the teacher and the student make sense of the terms by asking questions to each other.

For preservice teachers, it is important to give students an opportunity in the formation of Teacher-Student dialogues. **Figure 3** shows that there is a very close relation between the teacher's giving the student the opportunity to talk about mathematical terminology and the expressions addressing only one student. **Table 3** presents the responses of the preservice teacher coded P3, including the identified teacher's role, the word pattern, and representative expressions from the dialogue.

**Table 3.** The role of the teacher, the word pattern used, and the imaginary mathematical dialogue about this pattern

| Role | Word pattern | Imaginary mathematical dialogue |
|---|---|---|
| To create a suitable environment for students to ask questions about the question, to get their ideas about the subject, to reach the solution together, to establish generalizations and formulas together. Provide students with a simple level or elaboration of the subject. | For example: What do you think Burcu? Are we on the right track, do you have any other ideas, do you have any other suggestions that will lead us to a solution? | Teacher: .... Now, let's have fun and have a lesson following this successful match, let's include sports in the lesson. |
| | | Burcu: Which sports, Teacher? |
| | | Teacher: Which sports would you like to have, Burcu? We can give our first example based on that. |
| | | Burcu: I really like playing volleyball, I guess volleyball is my favourite sport. |
| | | Teacher: Then, let's take the volleyball court as an example for our subject of measuring area. Well, Burcu, can you tell us what the geometric shape of the volleyball court is? |
| | | Burcu:It is a rectangular court. |
| | | Teacher: Are the fields of the two teams equal then, my dear Burcu? Or can one of them be longer or shorter? |
| | | Burcu: It is equal to be fair for both teams, Teacher. |
| | | Teacher: Let's turn on our smart board and see a real volleyball court. Look, there is a real volleyball court here and it has real dimensions, as Burcu said, in a rectangular shape. |

Note. The dialogue above consists of 32 lines. The table presents the relevant part of the word pattern used.

### Student-Student Discourse Type

Teachers' role in the imaginary mathematical dialogues written for the mathematical conversations of students among themselves is essentially examined in three categories: being aware, guiding students, and controlling the information exchange. Teacher awareness to predicting students' answers can steer students' inner discussions. Most of the preservice teachers believed that teachers' guidance during students' discussions about terms would increase Student-Student conversations. Hence, the teacher ought to use appropriate learning-teaching methods in the mathematics lesson. Also, teachers' moderation in students' discussions could enable a healthy communication among them. Likewise, teachers' guidance while speaking about mathematical terms would allow students to discuss among themselves. Teachers could lead the mathematical discussions and allow students to discuss among themselves. It also was stated as another way of forming mathematical discourse among students is for their participation to be taken into consideration. Teachers could give real-life examples to foster this.

Having looked at the relationship between word patterns and teachers' roles, it seems to be similar to the relationship between Teacher-Students and Teacher-Student discourse types. While writing diaologue in this type of discourse, there are those who use, and do not use, word patterns as well as those who need to use them in accordance with students' needs. Preservice teachers differed based on their use of word patterns while writing imaginary mathematical dialogues. This differentiation is more apparent with the teacher being a guide to students. **Figure 4** shows teachers' role, word patterns, and the relationship in-between.

The relationships between the teacher role and the word patterns identified in the Student–Student discourse type are presented in **Figure 4**. Given the word patterns that teachers use while guiding students, there are those who use patterns and those who do not, as well as those who use patterns in relation to specific students. **Figure 4** shows that there are close relationships between the sub-components of teachers' guidance role, such as establishing healthy communication, being a guide, and ensuring the active participation of the class, and those who use patterns in relations to students. Other close relations between teachers' roles and the word patterns used are between teachers' role in controlling information exchange and their expressions of helping and making sense. P4 used patterns such as "*For example, you solve it, help each other*" or P9 said "*Help each other, did you measure?*". In addition, there are ties between the teacher's role of controlling the information exchange and the word patterns supporting/rejecting expressions and predictions. Also, there is a relationship, although weak, between all teacher roles in the formation of the Student-Student discourse type and those who do not use word patterns. Preservice teacher coded P27 said that they used general expressions:

> Actually, I didn't use it much, I tried to ask questions to both the individual student and the class in the form of 'we'. I don't think there is a particular pattern used. But I directed questions to the whole class and had everyone answer. I tried to add a friendly atmosphere to the discussion. I preferred to stay away from the teacher-oriented authoritarian structure…

Most of those who did not use a specific word pattern in the formation of the Student-Student discourse type stated that they addressed students in a conversational manner.

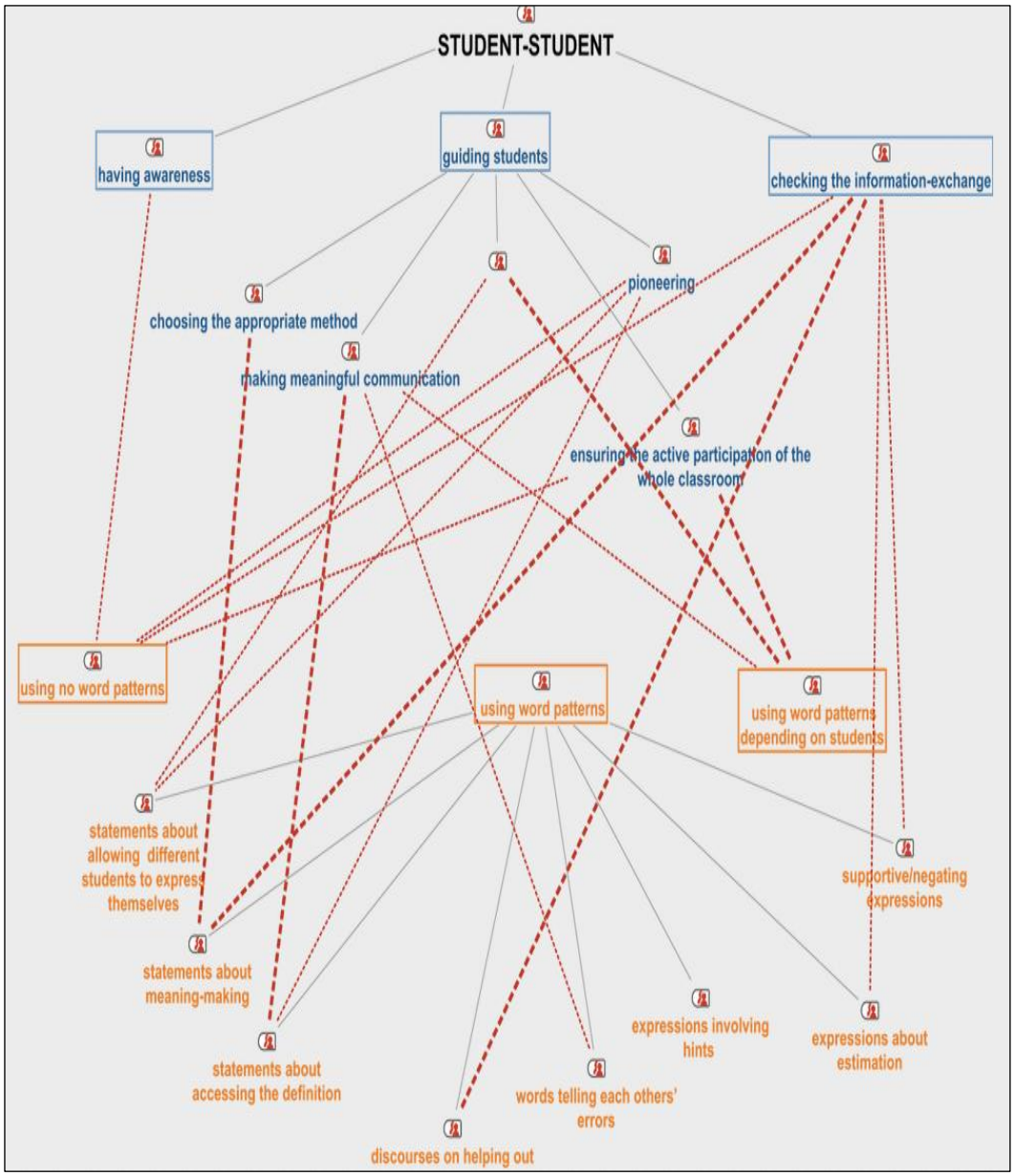


**Figure 4.** The relationship between teacher roles and the word patterns used in the Student-Student discourse type (Source: Authors' own elaboration)

## DISCUSSION, CONCLUSIONS, SUGGESTIONS

After completing the training on mathematical discourse, preservice mathematics teachers wrote imaginary mathematical dialogues in different discourse types. It was observed that the preservice teachers stated that the teacher could direct the discourse types while constructing mathematical dialogues and that the teacher's role changed according to the discourse type. This finding shows that the word patterns used in classroom mathematical communication have an important function in determining the teacher's role. In other words, the language used by the teacher is not merely a tool for conveying mathematical content; it also determines the teacher's position in the classroom, the nature of the interaction established with students, and the direction of the discourse. For example, in the Teacher discourse type, the teacher's position as the one who explains, guides, and transmits knowledge causes a more authoritative and teacher-centered role to become prominent. Such a discourse structure may limit students' active participation in the process of meaning-making and may therefore contradict the constructivist understanding of mathematics education, which prioritizes student participation, multiple ways of thinking, and the shared construction of meaning.

In terms of practice, this result shows that it is important for preservice teachers to recognize the pedagogical consequences of the word patterns they use. Preservice teachers should be able to distinguish which expressions position the teacher as someone who establishes authority, transmits knowledge, or directs the discussion, and which expressions encourage students to think, explain, justify, and construct shared meaning. Therefore, in teacher education processes, different discourse types should be addressed not only as forms of communication but also as pedagogical tools that shape the teacher's role and the

classroom mathematical learning environment. This finding is consistent with Demirbağ (2017), in his study about different discourse types, found that different discourse types contribute to the argument development of preservice teachers and that the role of the teacher is very important at every stage of the process from the negotiation of ideas, to ending the discussion. In addition, Lehesvuori et al. (2013) graphically showed the formation of different communication approaches with classroom discourse and determined that the dialogue practice was effective in learning science in a cumulative and meaningful way and the teacher had a role in the formation of this practice. In addition; Manouchehri and Enderson (1999), in their study on the promotion of mathematical discourse based on the in-class discourse, concluded that the teacher determines the classroom atmosphere although the discourse of the class is formed by students. As seen, it is not how the process starts as to what matters for the quality of the interaction, but how it develops and how the teacher uses the follow-up practice in role selection and dialogue conducting (Wells & Arauz, 2006). In this regard, the main role of the teacher seems to be the management of 'the communication traffic'. That is, it is to focus on topics and guide students to answer their own questions (Nystrand et al., 2003).

The fact that teachers' role is different according to different discourse types is one the major findings of this study. However, in studies examining teachers' role in mathematical discourse in the classroom, it has been determined that teachers' role is to guide students. For example, Rahal and Melvin (1998) found that preservice teachers who took courses in discourse teaching strategies in their undergraduate degrees determined that teachers' role was not limited to being the deliverer of information. Differently, from a sociocultural perspective, Sánchez and Garcia (2014) stated that teachers' role could be determined through mathematical discourse emerging between teachers and students. This perspective suggests that teachers can act as a guide who can overcome students' cognitive predicaments, as well as directly convey information to them or enable them to access information. In this context, preservice teachers who wrote imaginary mathematical dialogues for different discourse types stated that teachers' role might differ following their education. In their study, Ramis-Conde and Hope (2020) unearthed that teaching preservices how to conduct mathematical dialogue, using different styles in real classroom activities, and making them think about their use gives them a new perspective. In this context, it can be said that preservice teachers who have received such training before about writing mathematical dialogues have an idea about how to conduct the dialogues. Indeed, Blanton et al. (2001) emphasised in their study that we need to help preservice teachers develop practices that engages students with dialogues. In addition, Rahal and Melvin (1998) determined that the knowledge of preservice teachers who took courses on discourse teaching strategies in their undergraduate education gradually increased in understanding and applying teaching strategies after their education. In this context, it becomes more significant that preservice teachers will gain experience by taking the Mathematical Communication course. Hence, preservice teachers should gain experience during undergraduate education on how to conduct mathematical dialogues.

In line with another aim of this study, it was seen that preservice teachers expressed that the word patterns that they use while writing imaginary mathematical dialogues can be different in different discourse types. It is determined that they, after receiving training, recognise different discourse types and use patterns accordingly. Culican (2007), in his study in which teachers gained experience by examining detailed course transcripts and examined teachers' discourses over time, determined that there was a significant change and development in the word patterns in in-class discourse within the context of the Interaction Cycle, maximising student participation and learning. In this study, it was observed that while writing imaginary mathematical dialogues in different discourse types, preservice teachers used different word patterns depending on teacher's role. In fact, Ramis-Conde and Hope (2020) stressed the importance of associating linguistics and mathematics in mathematical dialogues, how preservice teachers relate and interact with students in the construction of knowledge. In this study, it was revealed how preservices interacted with students in different discourse types through imaginary mathematical dialogues. According to Clarke et al. (1993), how students' mathematical knowledge is formed and shaped can be explored by looking at their imaginary dialogues. Moreover, Wille and Boquet (2009), in their study on imaginary mathematical discourses, determined that students' mathematical thoughts could be estimated when they write a dialogue about their imaginary heroes.

Considering preservice teachers imaginary mathematical dialogues and how the word patterns they used while writing these dialogues were expressed, it became apparent that they used certain word patterns more frequently. Krussel et al. (2004) suggest that the word patterns in mathematical dialogues are related to purpose, metaphors, classroom organisation, etc. and identified by using certain keywords to present those relations. In-class discourse and normative interaction patterns knowingly guide and influence student learning for improvement (Pierson, 2008). It can be said that depending on the active or passive role of the teacher/student, these patterns become more prominent. However, Ng et al. (2021) show that certain features and word patterns embedded in mathematical dialogues can restrict or enable certain types of authority in the classroom. In this case, it comes to mind that certain word patterns may affect the role of the teacher. Also, teachers may have their own word patterns. In fact, Yang (2021) in his study examining the questions that teachers would use in the class in terms of students' interaction concluded that the word patterns peculiar to teachers embedded in the questions affected the interaction levels of students. In this study concerned with writing imaginary mathematical dialogues, it is found that reason-why type of questions were effective in the formation of the Student-Student discourse type. This finding is consistent on the analysis of classroom discourses in the mathematics learning and teaching in China. That chapter suggests that teachers' questions concerned with what, why, how are effective in student interaction (Li & Huang, 2013). Since the word patterns used by teachers guide the classroom interaction, preservice teachers/teachers need to raise their awareness about the word patterns they use.

## LIMITATIONS

This study is based on preservice teachers' views and the imagined mathematical dialogues they constructed, rather than on their actual classroom practices. Therefore, the findings reflect preservice teachers' conceptualizations of teacher roles and

mathematical discourse; they do not provide direct observations of how these roles and discourse patterns emerge in real classroom settings. Since the data obtained in the study are based on written dialogues, verbal and nonverbal communication elements that may be important in classroom interactions, such as gestures, facial expressions, tone of voice, emphasis, wait time, and immediate student responses, could not be included in the analysis. In addition, although imagined dialogues provide valuable data for revealing preservice teachers' pedagogical and discursive awareness, they may not fully reflect unexpected student responses, the interactional flow, and contextual variables in real classroom environments.

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