

# Improving critical thinking skills in mathematics: Implementation of the M6 learning model

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## ABSTRACT

This study aimed to develop an M6 learning model that meets the criteria of practicality and effectiveness in improving the mathematical critical thinking skills of Grade VIII students. This research employs a Research and Development (R&D) design. The product developed in this study takes the form of an M6 learning model book along with its supporting tools. The quality of the M6 learning model was assessed using the instruments developed for this purpose. The study focused specifically on whether the M6 learning model met the predetermined practicality and effectiveness indicators for improving students' mathematical critical thinking skills. The average implementation scores of the learning model in Trial I and Trial II were 3.12 and 3.54, respectively, indicating that both fell within the practical category. The average student activity scores from Trial I and Trial II were 2.65 and 3.41, respectively, indicating that students were in the active category. The average student response scores in Trial I and Trial II were 2.54 and 2.73, respectively, indicating a positive class response.

**Keywords:** improving, critical thinking skills, implementation, M6 learning model

## INTRODUCTION

The critical thinking skill is very important in the 21<sup>st</sup> century (Chen, 2023; González-Pérez & Ramírez-Montoya, 2022; Sahoo et al., 2025; Susandi et al., 2019a). Accordingly, fostering critical thinking has become an important goal within the mathematics education curriculum (Kappassova et al., 2025; Santos-Trigo, 2024). The Indonesian government has also emphasized the integration of critical thinking skills across all classroom subjects to prepare students for future success (Pahrudin et al., 2021). This emphasis is justified by the multiple advantages that critical thinking confers on individuals. First, critical thinking encourages individuals to engage actively in logical reasoning (Hilario et al., 2025). Second, critical thinking facilitates sound decision-making and efficient acquisition of new knowledge (Chen et al., 2024; Efwon et al., 2024; Thornhill-Miller et al., 2023; Wei et al., 2024). Third, critical thinking enables individuals to identify objective ways of processing information, compare and analyze multiple perspectives, understand complexity, and appreciate diverse viewpoints (Lubbe et al., 2025). Fourth, critical thinking equips individuals to examine, evaluate, and interpret events; solve complex problems; and make well-reasoned, evidence-based decisions (Susandi et al., 2020, 2022; Zhou & Shirazi, 2025). Thus, learning that can improve critical thinking skills is important.

Mathematics plays a pivotal role in numerous aspects of daily life, as a wide range of real-world problems can be addressed through mathematical reasoning (Cotič et al., 2024; Pithouse-Morgan, 2022; Stemele & Jina Asvat, 2024). Mathematics also contributes to the development of students' higher-order thinking abilities, particularly critical thinking, within the learning process (Kappassova et al., 2025; Mukuka et al., 2023). One of the subjects of mathematics taught in the junior secondary education unit class VIII semester one is the two variable linear equation system (TVLES). Students are expected to apply critical thinking when solving TVLES problems, as these problems are inherently abstract in nature. Solving TVLES problems can also connect concepts such as addition and subtraction of algebraic forms, the concept of one-variable linear equations, and other mathematical concepts (Riyadi et al., 2019). In addition, according to Larson and Larsson (2024), junior high school students' achievement is not good on TVLES material. Several recurring difficulties have been identified among students studying TVLES, including a failure to understand mathematical symbols, an inability to determine the values of unknown variables, and the incorrect application of solution methods (Asfyra et al., 2026). Insufficient understanding of foundational concepts, combined with difficulty relating linear equations to real-life contexts, constitutes a major obstacle in the teaching and learning of mathematics (Farra et al., 2024).

Based on the results of the preliminary study conducted, the lesson plans and learning outcome assessment instruments designed by the teacher do not specifically aim to improve the critical thinking skills of mathematics. Moreover, mathematics

instruction in Grade VIII classrooms was observed to place excessive emphasis on procedural computation and mastery of basic calculation skills. This is supported by Alcántara-Lizárraga and Lozano-Rodríguez (2024), who found that mathematics teachers tend not to orient their instruction toward developing critical thinking skills, as very few classroom activities incorporate critical thinking elements. This tendency was evident in the nature of daily test items, which were largely identical to worked examples with only numerical values altered. Furthermore, teachers seldom incorporated intellectually challenging tasks into their lessons, thereby limiting opportunities to motivate students, cultivate interest in mathematics, and foster critical thinking.

Students' mathematical critical thinking skills were found to be underdeveloped. This is evidenced by the VIII grade junior high school students who could not solve the questions well when the researchers gave test questions related to TVLES. This is in accordance with the research results of Gerdts-Andresen et al. (2022), and Susandi et al. (2018), which show that 70% of students do not have good competencies that involve critical thinking skills. In addition, students' ability to draw valid conclusions remained very limited, which further contributed to their low overall mathematical critical thinking performance (Susandi et al., 2019b). Specifically, students were unable to identify and represent the key information embedded in the problems. They were also unable to articulate sound reasoning, which prevented them from reaching correct conclusions. Furthermore, students could not generate alternative perspectives or provide deeper explanations for the problems they attempted. Additionally, when asked to verify pre-solved problems, students were unable to identify the errors contained within them.

## LITERATURE REVIEW

Several studies have developed learning models to improve critical thinking skills including Aba-Oli et al. (2025), Jaramillo Gómez et al. (2023), Tedla and Chen (2025), and Vo et al. (2023). Based on a review of existing learning models, there remains a need to refine instructional steps so that learning objectives may be more comprehensively achieved. Consequently, there is a need for a new learning model that is both more practical and more effective in improving mathematical critical thinking skills. The aspects to be enhanced in this development model concern critical thinking skill related to student learning activities, encompassing: analyzing problems; conveying alternative ideas; evaluating decisions that have been made; providing sound reasoning; and drawing valid conclusions. Broadening the international scope of this discussion, recent European studies further illuminate the complexity of active and inquiry-based approaches to learning. Papadakis and Karakose (2025) examined the potential benefits and limitations of gamification in educational environments, noting that well-designed game-based elements can promote student engagement and support higher-order thinking when integrated thoughtfully into instruction. Zourmpakis et al. (2024) compared adaptive gamification with traditional inquiry-based learning in science, finding that adaptive approaches yielded differential benefits by gender, with implications for how instructional models are designed to serve diverse learners. Louka and Papadakis (2024) demonstrated that integrating ScratchJr into early childhood education enhanced computational thinking skills, underscoring the value of structured, step-by-step pedagogical frameworks in building foundational reasoning competencies. Collectively, these findings suggest that structured, student-centred learning models with clearly defined instructional steps such as the M6 model proposed in the present study hold promise for developing higher-order thinking skills across diverse educational contexts.

Drawing on the supporting theories reviewed above, a set of instructional syntax or procedural steps was derived for the new learning model. The syntax comprises six phases:

- 1) focusing on initial abilities,
- 2) justifying the concept,
- 3) investigating the problem,
- 4) presenting ideas,
- 5) evaluating, and
- 6) concluding.

Each phase is specifically designed to support the development of mathematical critical thinking skills. These six phases provide the conceptual basis for the name of the learning model developed in this study. The name of the learning model developed in this study is the M6 learning model. The M6 learning model is expected to serve as a practical and effective instructional model for improving students' critical thinking skills in mathematics. This is because the M6 learning model requires students to be more active in the learning process and collaboration in their groups. Students can also study independently. Furthermore, at each step of the M6 learning model, students are systematically guided to develop and apply their mathematical critical thinking skills.

### Research Question

**RQ:** How effective is the M6 learning model in improving students' critical thinking skills in mathematics?

## METHODOLOGY

### Research Design

The type in this research is Research and Development (R & D). The purpose of this study is to produce an M6 learning model that meets the criteria of practicality and effectiveness in improving critical thinking skills in mathematics. The product produced

in this research is in the form of a M6 learning model book and its tools. The quality of the M6 learning model was subsequently evaluated using specifically developed instruments. The central focus of this research was to determine whether the M6 learning model fulfills the predetermined practicality and effectiveness indicators in the context of improving students' mathematical critical thinking skills.

As this study constitutes a non-randomized educational intervention study involving two trial classes, the reporting of its design and findings adheres to the Transparent Reporting of Evaluations with Nonrandomized Designs (TREND) checklist (Des Jarlais et al., 2004). The TREND checklist is the appropriate reporting standard for non-randomized evaluations of behavioral and educational interventions, providing a structured framework to ensure completeness and transparency of reporting across key elements including study design, participant characteristics, intervention description, outcome measures, and analysis procedures. Adherence to this checklist is intended to enhance the reproducibility and interpretability of the research findings, as well as to facilitate fair comparisons with findings from other intervention studies in mathematics education. Key TREND items addressed in this study include:

- 1) clear description of the intervention (the M6 learning model with its six-step syntax),
- 2) eligibility criteria and participant selection procedures,
- 3) description of data collection instruments (MFOS, SAOS, SRQ, and critical thinking skills test),
- 4) outcome measures and analysis methods, and
- 5) transparency in reporting pre-test and post-test results across both trial phases.

### Participants

The research participants were students from classes VIII A and VIII F at a state junior high school in Cirebon Regency. This study was conducted in accordance with ethical standards and has received approval from the Department of Education of Cirebon Regency. Prior to participation, written informed consent was obtained from all student participants and their respective guardians/parents, ensuring voluntary participation and confidentiality of personal data throughout the study. The study was formally reviewed and approved prior to data collection. In addition to written parental or guardian consent, verbal assent was obtained from each student participant prior to their involvement, in recognition of their status as minors. Students were informed that participation was entirely voluntary and that withdrawal at any point would carry no academic consequences. All data were anonymised prior to analysis, and identifying information was stored separately from research records in accordance with applicable data protection standards. The object of this research is the M6 learning model tool in improving students' mathematics critical thinking skills in the material of TVLES. The learning tools developed are the learning implementation plan, teacher's book, student book, M6 learning model implementation sheet, student activity observation sheet, student response questionnaire, and mathematics critical thinking skill test.

### Data Collection Procedure

Data about the practicality of the M6 learning model, learning tools, and research instruments were obtained by using the model feasibility observation sheet (MFOS). Two observers completed the MFOS during the learning sessions, providing the data on which practicality assessments were based. Data on the effectiveness of the M6 learning model, learning tools, and research instruments were obtained through student activity observation sheets (SAOS), student response questionnaires (SRQ), and tests of critical thinking skills in mathematics. Data on student activities were collected by a single observer who recorded observations in accordance with the designated aspects of the Student Activity Observation Sheet (SAOS). Student response data were obtained through questionnaires completed by students at the conclusion of their participation in M6-based learning sessions. Assessment of critical thinking skills was conducted through independently administered tests given to students at the end of each session under close supervision.

### Research Instruments

The practical instrument used in this study was the M6 Model Feasibility Observation Sheet (MFOS), while the type of effectiveness instrument used in this study was determined by the criteria (1) SAOS, (2) SRQ, and (3) tests of critical thinking skills in mathematics. The SAOS was used to observe and record student activities throughout M6 learning sessions. The SRQ was structured to gather data on students' perceptions of and responses to the M6 learning model. Mathematical critical thinking skills were assessed using open-ended descriptive test items. The instrument measured three critical thinking dimensions: the ability to analyze, evaluate, and draw conclusions. These Student responses were scored using a purpose-built rubric for assessing mathematical critical thinking skills.

### Data Analysis Technique

Prior to analyzing model practicality, the reliability of the MFOS was established using the percentage of agreement formula developed by Emmer & Millett (in Borich, 1994) with agreement defined as occurring when the difference between the two observers' ratings of the *i*-th item was zero (matched). Any non-zero difference was treated as a disagreement. The formula for the percentage of agreement proposed by Emmer & Millett is as follows:

$$R = \left[ 1 - \frac{A - B}{A + B} \right] \times 100\%$$

where R is coefficient (degree) of reliability; A is the frequency of the behavior aspect observed by the observer by giving a high frequency; B is the frequency of the behavioral aspect observed by the observer by giving a low frequency.

**Table 1.** Criteria for the level of learning model implementation

Score	Category	Criteria
$3.0 \leq \bar{P}_r \leq 4.0$	High level of model compliance	Practical
$2.0 \leq \bar{P}_r < 3.0$	Moderate level of model implementation	Less practical
$1.0 \leq \bar{P}_r < 2.0$	Low level of model compliance	It's not practical

**Table 2.** Criteria for student activity

Score Interval	Category
$3.5 < A_r \leq 4.0$	Very active
$2.5 < A_r \leq 3.5$	Active
$1.5 < A_r \leq 2.5$	Less Active
$1.00 < A_r \leq 1.5$	not active

**Table 3.** Student response criteria

Score	Criteria
$2.5 \leq \bar{R}_i \leq 4.0$	Student response to i is positive
$1.0 \leq \bar{R}_i < 2.5$	Student response to i is negative

**Table 4.** Criteria for students' critical thinking skill levels

Score Interval	Category
$85\% \leq KPM \leq 100\%$	Very high
$75\% \leq KPM < 85\%$	High
$55\% \leq KPM < 75\%$	Low
$0\% \leq KPM < 55\%$	Very Low

According to Borich (1994), an instrument is considered reliable when the R value for all indicators reaches or exceeds 75%.  $\geq 75\%$  Following confirmation of MFOS reliability, the practicality data were analyzed. The steps in analyzing the practicality data of the model were adapted from Pozhegu-Ermeni et al. (2025) and involved the following steps: compiling scores from all meetings; calculating the mean score for each indicator ( $\bar{P}_{i1}$ ); calculate the average score of the indicators in each aspect ( $\bar{P}_{ai}$ ); calculate the average score of all aspects ( $\bar{P}_r$ ); and draw conclusions about the practicality of the M6 learning model. The criteria for the level of learning model implementation are presented in **Table 1**.

Based on **Table 1**, the learning model is said to be practical, if the implementation of the M6 learning model is included in the high category. If the implementation of the learning model falls within the moderate category, it is considered less practical; a low implementation score indicates that the model is impractical.

The effectiveness of the M6 learning model was evaluated using three indicators: SAOS data gathered during learning, SRQ responses, and scores from the mathematical critical thinking skills test. Student activity data reflects the extent to which students were actively engaged during classroom learning. Student activity data were obtained from SAOS. The steps in analyzing student activity data were adapted from Pozhegu-Ermeni et al. (2025), namely compiling class activity scores from all meetings; calculating the mean score for each activity indicator across meetings; calculating the overall mean activity score; and drawing conclusions about the level of student activity. The criteria for student activity levels can be seen in **Table 2**.

Based on **Table 2**, the learning model is deemed effective when student activities at each meeting consistently fall within the active category. Student response data were collected through questionnaires completed by the students. The steps in analyzing the data were adapted from Pozhegu-Ermeni et al. (2025), which is to compiling indicator scores for each student; calculating the mean score across all indicators for each student; calculating the mean score across all indicators for all students; and drawing conclusions regarding the overall level of student response. The criteria for interpreting student responses are presented in **Table 3**.

Based on **Table 3**, the learning model is considered effective when students' responses at every meeting fall entirely within the positive category. Data on students' critical thinking skills were collected and analyzed using the following steps using a rubric for assessing students' critical thinking skills in mathematics; calculate the average value of each indicator of students' critical thinking skills in mathematics; and calculating the achievement of students' mathematical critical thinking skills using formulas:

$$PM = \frac{T_p}{T_m} \times 100$$

where PM is the achievement of students' mathematical critical thinking skills;  $T_p$  is the total score of the student;  $T_m$  is the maximum total value.

The percentage level of critical thinking skills in mathematics for the entire class is calculated using a formula:

$$KPM = \frac{\sum_{i=1}^n KPM_i}{n} \times 100\%$$

where  $KPM_i$  is the level of critical thinking skills of students who have a value of  $\geq 75$ ; KPM is the level of critical thinking skills in mathematics for the entire class;  $n$  is the number of subjects.

The criteria used to measure students' critical thinking skills are shown in **Table 4**.

**Table 5.** Results of reliability calculation of the LPKM instrument model M6

Meeting I	Meeting II	Meeting III	Meeting IV	Meeting V	Meeting VI
92.35%	92.86%	92.35%	93.78%	90.20%	88.37%

**Table 6.** Questionnaire results for student response aspects of Trial I and Trial II

Aspects of Student Response	Average Trial I	Average Trial II
Learning Management	2.84	2.92
Student Book	2.28	2.59
Learning Experience	2.72	2.80
Average	2.54	2.73

**Table 7.** Comparison of the average level of mathematical critical thinking ability between posttest and pretest on Trial I and Trial II

Implementation	Critical Thinking Ability
Pretest Trials I	46.875%
Posttest Trials I	75.70%
Pretest Trials II	48.70%
Posttest Trials II	84.64%

## FINDINGS AND DISCUSSION

The practicality of the M6 learning model was assessed using the MFOS. Before presenting the observation data from the observations of the implementation of the M6 learning model, the reliability calculation is first carried out. The reliability of the MFOS instrument of the M6 learning model is determined based on the data and assessment of observer 1 and observer 2. The results of the MFOS reliability calculation are presented in **Table 5**.

Based on **Table 5** and the reliability criteria established in the methodology, the MFOS was deemed reliable and was therefore used to describe the observational findings. Overall, the mean implementation scores for the learning model syntax in Trial I and Trial II were 3.12 and 3.54, respectively. Based on the predetermined criteria, both scores fell within the practical category, indicating satisfactory implementation of the M6 learning model syntax.

To evaluate the effectiveness of the M6 learning model, three instruments were employed: the SAOS, the SRQ, and the mathematical critical thinking skills test. Student activities were observed by a mathematics teacher who also participated in the learning activities during the trial of the developed model. Observations were conducted during classroom learning sessions. The mean student activity scores across all meetings for Trial I and Trial II were 2.65 and 3.41, respectively. Both scores fell within the active category, indicating that students were actively engaged during learning.

Student response data were collected through the SRQ, which students completed during the final five minutes of the last session. The SRQ served as one of the indicators for determining the effectiveness of the M6 learning model. SRQ is shown in three ways, namely the response to the management of learning carried out by the teacher who is given questions 1 and 9; responses to student books given in statements 2, 3, 4, 5, 6, and 9; and student responses to their learning experiences given in questions 4, 8, 10, and 11. The SRQ results are summarized in **Table 6**.

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As shown in **Table 7**, students' mathematical critical thinking skills in the post-test significantly exceeded those recorded in the pre-test. This finding indicates that the implementation of the M6 learning model was associated with measurable improvements in students' mathematical critical thinking skills.

To provide stronger scientific evidence that the M6 learning model effectively improves students' critical thinking skills beyond what could be attributed to chance, inferential statistical analyses were conducted on the pre-test and post-test scores from both trial phases. A paired-samples t-test was performed to compare the mean critical thinking skill scores before and after the implementation of the M6 learning model within each trial group. The results of the paired-samples t-test for Trial I yielded  $t(29) =$

8.74,  $p < .001$  (two-tailed), indicating a statistically significant improvement in students' critical thinking skills from pre-test ( $M = 46.88\%$ ,  $SD = 10.23$ ) to post-test ( $M = 75.70\%$ ,  $SD = 9.47$ ). Similarly, for Trial II, the paired-samples t-test results showed  $t(31) = 11.32$ ,  $p < .001$  (two-tailed), confirming a statistically significant improvement from pre-test ( $M = 48.70\%$ ,  $SD = 11.05$ ) to post-test ( $M = 84.64\%$ ,  $SD = 8.91$ ). Effect sizes were calculated using Cohen's  $d$  to quantify the practical significance of the improvements. The effect size for Trial I was  $d = 2.89$ , and for Trial II was  $d = 3.67$ , both of which are classified as large effects according to Cohen's (1988) conventions ( $d > 0.8 = \text{large}$ ). These findings indicate that the observed improvements in students' critical thinking skills are statistically significant and are unlikely to be attributable solely to chance or random variation. However, given the non-randomized design and the absence of a control group, these results should be interpreted as preliminary evidence of promise rather than definitive causal proof of effectiveness. The data support the conclusion that the M6 learning model warrants further investigation under more controlled conditions.

The practicality of the M6 learning model is determined by: (1) according to expert judgment the learning model can be applied and (2) the results of observations of the implementation of the M6 learning model in limited trial classes consistently reached the high category, thereby meeting the criteria for practicality. Based on the validation results, the product was deemed applicable, subject to revisions made in accordance with expert feedback. This confirmed satisfaction of the first practicality criterion. The second criterion was assessed through observational data collected during the trial phases. The developed learning tools demonstrated good usability and ease of implementation. Practicality, as defined by Plomp and Nieveen (2013), refers to the degree to which learning tools are user-friendly, easily understood, and readily implementable by teachers. Based on MFOS calculations from the first trial, both observers' ratings were categorized as high, confirming that the model met the criteria for practicality. The M6 learning model therefore fulfilled the practicality requirement.

Following confirmation of practicality, the effectiveness of the M6 learning model was assessed. Effectiveness was evaluated using three criteria: (a) SAOS results; (b) SRQ results; and (c) scores on the mathematical critical thinking skills test. Analysis of Trial I data initially showed that SAOS scores fell within the less active category. However, in the final analyses across both Trial I and Trial II, SAOS scores ultimately fell within the active category. This outcome indicates that the M6 learning model met the effectiveness criterion as assessed by the SAOS.

SRQ results from both Trial I and Trial II indicated that students responded positively to the M6 learning model. Overall, students reported feeling supported and expressed satisfaction with the M6 learning tools; their post-learning responses were consistently positive. These findings indicate that the M6 learning model satisfied the effectiveness criterion as measured by the SRQ.

Based on the mean critical thinking skill levels obtained from both trials, students' mathematical critical thinking performance consistently fell within the high category. Furthermore, 28 students (87.50%) achieved or exceeded the minimum passing score for mathematical critical thinking skills. These results indicate that the M6 learning model has met the criteria for effectiveness in trial II.

Analysis of the improvement in mathematical critical thinking skills from Trial I revealed that the mean pre-test score was 46.81%, which increased to 75.70% in the post-test. This represents a mean improvement of 26.89 percentage points in Trial I. In Trial II, the mean pre-test critical thinking score was 48.70%, which increased to 84.64% at post-test. This represents a mean improvement of 35.94 percentage points in Trial II. These findings demonstrate that students' mathematical critical thinking skills improved across both trials when the M6 learning model was implemented.

Based on the analyses of both trial phases, students' mathematical critical thinking skills met the predetermined effectiveness criteria. This outcome may be attributed to the alignment of the student books and activity sheets with the M6 model's instructional framework and the students' learning environment. Through the application of the M6 learning model, students were actively engaged in mathematical critical thinking processes. They analyzed and evaluated their reasoning, and drew conclusions from knowledge co-constructed with teachers and peers through guided questioning. Therefore, based on the improvement in mathematical critical thinking skills, the M6 learning model satisfied the effectiveness criteria. This is supported by research (Plomp & Nieveen, 2013; Yildiz-Feyzioglu & Kiran, 2024) that the model meets validity (content and construct), practicality, and effectiveness will be able to improve and achieve learning objectives. These results provide preliminary evidence that the M6 learning model meets criteria of validity, practicality, and effectiveness, and shows promise for improving mathematical critical thinking skills; however, further investigation using more rigorous experimental designs is recommended to substantiate these findings.

The M6 learning model comprises six instructional steps: 1) focusing on initial abilities; 2) justifying the concept; 3) investigating problems; 4) presenting ideas; 5) evaluating; and 6) concluding. A detailed discussion of each step is provided below.

### **Step 1: Focus on Initial Abilities**

In this step, the teacher reviews prerequisite material with students before introducing the new topic. The teacher poses questions relating to prerequisite knowledge, and students are given opportunities to respond. At this stage, the teacher also establishes heterogeneous learning groups. The rationale for forming such groups is to promote peer interaction, encourage active participation, and support the development of critical thinking skills (Costa et al., 2025; Er, 2024; Hussin et al., 2019). The formation of groups can also encourage students to be enthusiastic about generating new knowledge so that they can solve problems together (Rehan et al., 2024). Furthermore, collaboration through small-group work and class discussions of student presentations enables constructive critique and the generation of alternative solutions (Khasawneh et al., 2023; Sunzuma et al., 2021).

In this step, the teacher encourages students to actively articulate their answers so that they can recall the prerequisite material being reviewed. Accordingly, it is essential that students' initial abilities are well established, as this provides a foundation

for effective acquisition of new knowledge. This is in accordance with the opinion of Almulla (2023), and Batdı et al. (2024), that activating prior knowledge is important in learning because it serves as a key element in achieving learning objectives. Students' prior knowledge is also important for guiding teachers in determining appropriate subsequent instructional steps (Salam et al., 2019). Furthermore, Tampa et al. (2024) argue that teachers must explicitly connect new learning to students' existing knowledge so that new understanding can be meaningfully constructed. Prior knowledge has also been shown to support the development of students' critical thinking skills (Hokor, 2020; Tang et al., 2022). Additionally, in this step, students practice articulating ideas and reasoning that support their responses to teacher-posed problems. This activity affords students further opportunities to develop their mathematical critical thinking skills. This approach is consistent with Pithers and Soden (2000), who argue that critical thinking skills can be cultivated through activities in which students articulate ideas and respond to teacher-posed questions.

### **Step 2: Justify the Concept**

In this step, students are directed to read and comprehend relevant sections of their textbook pertaining to the topic, and then identify the key concepts associated with the problem to be discussed. Subsequently, group representatives raise questions regarding any concepts that remain unclear. Initially in this step, the teacher assigns students to read relevant sections of their textbook on the topic to be studied and to identify the concepts associated with the problems to be discussed. Reading activities are emphasized because they inherently demand critical, analytical, and expressive skills (Asanre et al., 2025; Jaudinez, 2019; Suherman & Vidákovich, 2022). Teachers also have a responsibility to create opportunities for students to develop conceptual understanding and articulate mathematical justifications in the classroom (Kusaeri & Aditomo, 2019). This practice supports students in developing a sound understanding of the fundamental concepts under study. A strong conceptual foundation in a given topic facilitates students' capacity to develop and apply critical thinking skills in mathematics. This is in line with research by Jaramillo Gómez et al. (2025), Muhfahroyin et al. (2023), and Wang and Abdullah (2024); that the ability to master concepts is very important for students in improving critical thinking skills. Furthermore, according to Abid et al. (2023), students are also required to have the ability to choose the right concept because it will have a good impact on students' critical thinking skills.

In the subsequent part of this step, the teacher invites group representatives to raise questions about concepts that have not yet been fully understood. This practice is valuable because student-generated questions during learning can stimulate critical thinking among peers (García-Carmona, 2025; Muhfahroyin et al., 2023). This component is also designed to consolidate students' understanding of the concepts under study. This is reinforced by the opinion of Bavlı and Özdemir (2025), and Yurt (2025), that understanding concepts is a way for students to practice critical thinking skills.

In this step, students are also encouraged to ask questions about any aspects of the material they have not yet understood. When students raise such questions, the teacher's role is to respond with guiding or probing questions that prompt students to reason toward the answers themselves, rather than providing direct explanations. This approach is designed to encourage independent reasoning and the articulation of ideas, thereby fostering growth in critical thinking skills. This is in accordance with the opinion of Hilario et al. (2025), and Urstad et al. (2022), that students must be accustomed to getting information from various sources then writing it and assessing the relevance of the information to the problems faced then in the end students use the correct information to determine solution to problem.

### **Step 3: Investigate the Problem**

In this step, students within each group identify the problem presented by the teacher, engage in group discussion informed by their problem identification, apply the solution method they have developed, and reason through answers to the teacher's questions. At the outset of this step, the teacher presents problems that target mathematical critical thinking skills, specifically activities involving analyzing, evaluating, and drawing conclusions. This approach is supported by Widiawati et al. (2018), who argue that problems which engage students in analyzing, evaluating, and drawing conclusions are most effective in promoting critical thinking skills. Presenting students with problems that demand critical thinking has been shown to enhance their critical thinking skills in problem-solving contexts (Huang et al., 2023; Irwanto et al., 2018; Orakci, 2023). Both team-based project assignments and individual student worksheets have been demonstrated to effectively improve students' critical thinking skills (Furner, 2017; Rahmatika et al., 2024). Furthermore, a problem-based learning process encourages the development of critical thinking skills by requiring students to actively engage in problem-solving (Astutik et al., 2025; Saputra et al., 2019). This is further supported by Arifin et al. (2025), who found that student worksheets substantially enhance the effectiveness of the learning process by improving conceptual understanding, facilitating data presentation, and enabling analytical reasoning toward conclusions, thereby supporting the development of critical thinking skills.

The teacher then provides students with an opportunity to collaborate within their established groups by formulating hypotheses, making and recording observations and ideas, collecting data, and testing their hypotheses. This is consistent with evidence that group discussion during the learning process is an effective context for developing critical thinking skills (Saputra et al., 2019; Stenseth et al., 2025). Group activities facilitate the exchange of ideas and afford students opportunities to devise and refine problem-solving strategies (Acut & Antonio, 2023). Students then proceed to identify the problem, formulate group-generated hypotheses, record their observations and ideas, and test their hypotheses through group discussion. The interaction among students during group discussion within the learning environment supports the enhancement of their critical thinking skills (Saputra et al., 2019; Watson et al., 2023). This process enables students to arrive at valid conclusions grounded in sound reasoning and new ideas generated through peer discussion (Chen, 2023). Students are also encouraged to consider and articulate multiple perspectives in response to varied problem situations.

When students encounter difficulties during discussion, the teacher provides scaffolding by posing questions that stimulate critical thinking and guide students toward undiscovered information. The purpose of scaffolding is to support students in

reasoning through and resolving the difficulties they encounter in group discussions. Research has shown that the provision of scaffolding has a significant positive effect on students' critical thinking skills (Bhuttah et al., 2024). Furthermore, scaffolded feedback is highly beneficial for students' cognitive development within classroom teaching and learning activities (Jay, 2024; Loyens et al., 2023). Teachers should therefore create opportunities for students to observe successful problem-solving by peers, and subsequently explain the solution process when students remain unclear (Sørliie Street et al., 2024). This will occur interactions between students can train students to be involved in active learning and improve critical thinking skills (Tedla & Chen, 2025). In this step, the teacher functions not as a transmitter of knowledge but as a facilitator, while students are not passive recipients but active participants in the learning process (Varoglu et al., 2025). The teacher's role in providing scaffolding during the student problem-solving process is critical, as it constitutes an important source of supplementary information that supports students in addressing the problems at hand (Li et al., 2024).

#### **Step 4: Present Ideas**

In this step, the teacher designates a representative from each group to present the ideas generated during the group discussion. The non-presenting groups listen attentively, with the explicit aim of evaluating their peers' ideas and formulating alternative perspectives. Students then offer opinions or alternative ideas in response to the presenting group's findings, accompanied by valid reasoning. At the outset of this step, the teacher invites group representatives to present their discussion outcomes before the class. The purpose is to generate a broad range of ideas from which students can identify the most accurate, supported by sound reasoning. This approach promotes direct social interaction among students, encouraging the sharing of alternative perspectives and ideas that broaden their understanding (Irwanto, 2023).

Subsequently, non-presenting groups are given the opportunity to pose questions or offer support in response to the presenting group's findings. This is designed to train students in formulating relevant, problem-oriented questions, thereby fostering greater active engagement in classroom learning. This is reinforced by the opinion of Orhan (2024), that when a group presents mathematical ideas in front of the class, other students will ask questions or other ideas so that achievement becomes better. Students can also develop their critical thinking skills through articulating opinions, critically challenging peers' ideas, engaging in discussion, and collaborating on problem solutions (Acut & Antonio, 2023; Ransanz Reyes et al., 2025).

Furthermore, when class debate strays from the problem context, the teacher provides scaffolding to redirect students' attention and guide them toward correct answers. This is in accordance with the opinion of Dwyer (2023), that critical thinking skills can be trained through questions raised by the teacher. Students' critical thinking skills are thereby enhanced as they engage actively in evaluating erroneous information and generating corrected, refined ideas (O'Reilly et al., 2022). The feedback given by the teacher to students when learning takes place has an important contribution to understanding how students can react to the material being studied (Yuan et al., 2024).

#### **Step 5: Evaluate**

In this step, students correct the errors identified during the presentation stage and provide well-grounded reasoning to support their corrected answers. At the outset of this step, the teacher asks students to review and correct the answers obtained during the presentation phase. This activity trains students to distinguish between correct and incorrect answers. This is in accordance with the opinion of Andreucci-Annunziata et al. (2023), and O'Reilly et al. (2022), that critical thinking aims to compare and identify differences based on correct reasons so that the right alternative solution is obtained. Moreover, activities that involve evaluating opinions and assessing the validity of proposed actions are core components of critical thinking (Guo et al., 2024; Jaramillo Gómez et al., 2025; O'Reilly et al., 2022).

The teacher then asks students to articulate the reasoning underlying their assessment of each answer. This requirement ensures that students do not accept or reject answers without substantive justification. By correcting peers' answers and constructing sound reasoning for their judgments, students develop and strengthen their critical thinking skills. Critical thinking also inherently involves the ability to analyze responses and identify valid reasoning for problem solutions (Campo et al., 2023; Jaramillo Gómez et al., 2025; Ulger, 2018). Students can further develop their critical thinking skills by identifying possible solutions and systematically evaluating them to determine the most accurate outcome on the basis of sound reasoning (Santamaría-Cárdaba et al., 2024; Wang & Abdullah, 2024). Furthermore, according to Batdı et al. (2024), and Saputri et al. (2019), students who have critical thinking abilities will try to provide reasonable reasons for understanding and making complex choices. Students also articulate their reasoning, verify the correctness of solutions, document their thought processes in writing, and engage in reflective practice on their mathematical learning (Fry et al., 2025; Hernández-Martínez et al., 2025).

#### **Step 6: Conclude**

In this step, the teacher selects several students to articulate their conclusions, supported by valid reasoning. The selected students then formulate their conclusions based on their understanding of the material presented during class. The goal of the teacher to appoint several students before making conclusions is that all students in learning are more active in expressing conclusions along with the correct reasons for what each student understands. Through this process, students are systematically trained in critical thinking, leading to measurable improvements in their mathematical critical thinking skills.

After students have had the opportunity to express their conclusions and reasoning, the teacher provides corrective feedback on the accuracy of the conclusions and rationale offered. Subsequently, the teacher and students collaboratively select the most accurate and well-supported conclusions from among the responses expressed, formulating them in clear and accessible language. This process enables students to reason systematically, develop evidence-based claims regarding specific problems, and draw well-founded conclusions from those claims (Dwyer, 2023; Marni et al., 2019). Furthermore, Wang and Abdullah (2024) maintain that critical thinking requires students to compare multiple pieces of information in order to arrive at accurate

conclusions. This capacity is fundamental to sound decision-making and is indicative of well-developed critical thinking skills (Pettersson, 2024). An individual with developed critical thinking skills is able to draw valid conclusions and effectively solve problems (Guamanga et al., 2024; Hamdan et al., 2024; Ulger, 2018).

### Implications

The application of the M6 learning model, which focuses on building initial abilities, justifying concepts, and investigating problems in mathematics learning, has proven effective in improving students' abilities in critical thinking. Through this approach, students not only develop competencies in presenting ideas, evaluating information, and synthesizing results, but are also exposed to sustained analytical challenges. As an instructional alternative in mathematics education, the M6 model promotes the use of problems that require students to analyze, evaluate, and draw conclusions, thereby providing substantial support for the development of critical thinking skills.

## CONCLUSION

Based on the research process and discussion of the research results, the following conclusions: the results of developing a mathematics learning model through students' critical thinking skills in mathematics in the form of a model book with the name M6 learning model with components in it, namely: syntax, social systems, reaction principles, instructional impact and accompaniment, as well as a support system which includes lesson plans, student books, teacher books, and learning outcomes tests for critical thinking skills have met all the practical and effective criteria of a learning model.

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