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Integrating cooperative learning with Polya's problem-solving model in grade 9 mathematics

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ABSTRACT

This study examined whether an integrated cooperative-learning and Polya-based intervention was associated with greater gains in ninth-grade students' mathematical problem-solving competence than individually implemented Polya-based instruction. A quasi-experimental pretest-posttest design was conducted with 436 students from 10 classes in Ho Chi Minh City, Vietnam, over four months. The primary analysis used difference-in-differences estimation, while propensity-score plots were used as supplementary descriptive evidence of between-group patterns. Students' competence was assessed across four dimensions: identifying, planning, performing, and re-evaluating. The intervention group showed larger gains than the comparison group across all four dimensions, with the largest estimated effects in identifying and re-evaluating. These findings suggest that structuring peer interaction around explicit problem-solving stages may strengthen students' mathematical reasoning and reflective review in lower-secondary classrooms.

Keywords: cooperative learning, mathematical problem-solving competence, Polya's problem-solving model, Student Teams-Achievement Divisions, quasi-experimental design

INTRODUCTION

Developing mathematical problem-solving competence is a central aim of contemporary mathematics education because students are expected not only to reproduce learned procedures but also to interpret unfamiliar situations, select appropriate strategies, implement solutions, and evaluate the reasonableness of their results (Lee & Chen, 2015; OECD, 2017). Yet many students continue to struggle when required to transfer previously learned knowledge to non-routine mathematical tasks, a difficulty often described as inert knowledge (Renkl et al., 1996). This challenge suggests that possessing mathematical knowledge alone is insufficient; students also need support in activating, regulating, and applying that knowledge during problem solving (Park, 2023).

Polya's four-step model of understanding the problem, devising a plan, carrying out the plan, and looking back remains one of the most influential frameworks for teaching mathematical problem solving (Lee & Chen, 2015; Polya, 1945). Its strength lies in making the problem-solving process explicit and teachable. However, individually implemented Polya-based instruction does not always lead to consistent improvements in students' performance. Novice problem solvers often move too quickly to computation, underuse planning and reflection, and struggle with the metacognitive demands of the full problem-solving cycle (Kaitera & Harmoinen, 2022; Schoenfeld, 1992; Silver et al., 2005; Sweller, 1988). In such cases, explicit heuristics may be necessary but not sufficient, especially when students work without timely feedback or external regulation.

One possible response to this limitation is to integrate explicit problem-solving heuristics with structured cooperative learning. Recent work on collaborative problem solving in mathematics classrooms further suggests that the quality of peer interaction and teacher intervention is central to how students construct mathematical understanding together (Cao, 2024). Social scaffolding and collaborative regulation can create opportunities for students to explain their reasoning, compare strategies, justify their choices, and monitor one another's thinking during mathematical work (Slavin, 1995; Tang et al., 2023; Vygotsky, 1978; Wismath & Orr, 2015). Among cooperative learning models, Student Teams-Achievement Divisions (STAD) is particularly relevant because it combines group interdependence with individual accountability (Slavin, 1995, 2014). Prior research has reported positive effects of cooperative learning on mathematics learning and has begun to explore the integration of cooperative structures with Polya's problem-solving process (Capar & Tarim, 2015; Černilec et al., 2023; Dookurong et al., 2025; Yapatang & Polyiem, 2022). However, existing studies have primarily evaluated intervention effectiveness using overall achievement scores or global measures of problem-solving performance. Consequently, relatively little is known about how different dimensions of mathematical problem-solving competence respond to integrated cooperative learning and problem-solving instruction. Understanding such differential responsiveness is important because competence development may vary across dimensions rather than occurring uniformly across all aspects of mathematical problem solving.

This question is particularly relevant in lower-secondary mathematics classrooms, where improving students' problem-solving competence remains an important instructional priority. A 2025 bibliometric analysis highlights that research on mathematical problem solving in secondary education has undergone a significant expansion in the last five years (Cuong et al., 2025). Despite the rapid growth of research on mathematical problem solving, relatively few intervention studies have examined outcomes at the level of individual competence dimensions. Most studies continue to report overall performance gains, making it difficult to determine which dimensions of competence benefit most from instructional support. A multidimensional perspective may therefore provide a more detailed understanding of competence development in mathematics classrooms. To address this gap, the present study examines the effects of an integrated STAD-Polya intervention on ninth-grade students' mathematical problem-solving competence in Vietnam. A quasi-experimental pretest-posttest design was implemented with 436 students from 10 classes in Ho Chi Minh City over a four-month intervention period. Experimental classes received the integrated STAD-Polya intervention, whereas comparison classes followed Polya's problem-solving steps individually. To estimate differential change over time, the study employed Difference-in-Differences (DID) analysis (Angrist & Pischke, 2009; Wooldridge, 2010) and used propensity-score-based diagnostics as supplementary descriptive support for between-group comparability and post-intervention distributional patterns (Fan & Nowell, 2011; Zhan et al., 2019). Students' competence was examined across four dimensions: identifying, planning, performing, and re-evaluating. Accordingly, the study addresses the following research questions:

RQ1. To what extent is an integrated STAD-Polya intervention associated with greater gains in students' mathematical problem-solving competence than individually implemented Polya-based instruction?

RQ2. How do the estimated effects of the intervention vary across the four dimensions of competence: identifying, planning, performing, and re-evaluating?

LITERATURE REVIEW

Development of Mathematical Problem-Solving Competence in Mathematics Education

Mathematical problem-solving competence is widely regarded as a core goal of mathematics education because it requires students not only to recall procedures but also to interpret unfamiliar situations, organize relevant information, select appropriate strategies, implement solutions, and evaluate the reasonableness of their results (Lee & Chen, 2015; OECD, 2017). In this sense, problem-solving competence is broader than routine task performance; it reflects students' capacity to mobilize mathematical knowledge flexibly in response to non-routine problems.

In the present study, this competence is operationalized through four related dimensions derived from Polya's problem-solving framework: identifying the mathematical situation, planning a solution, performing the selected strategy, and re-evaluating the result (Lee & Chen, 2015; OECD, 2017; Polya, 1945). This multidimensional view is important because students may perform unevenly across stages of problem solving. Some learners can begin a task successfully but struggle to formulate a plan, while others may complete calculations without adequately reviewing the logic or validity of their answers. This observation suggests that mathematical problem-solving competence should not be viewed solely as a single outcome variable. Rather, examining competence through multiple dimensions may provide a more nuanced understanding of how students respond to instructional intervention. A multidimensional perspective can reveal patterns of growth that remain hidden when analysis is limited to overall performance measures. Recent mathematics education research has similarly emphasized the value of examining learning progressions and developmental trajectories in specific mathematical domains, highlighting the importance of understanding how different aspects of competence develop over time (Seah & Horne, 2026; van Dijke-Droogers et al., 2025).

Polya's four-step model remains one of the most influential approaches to teaching mathematical problem solving because it makes the problem-solving process explicit and teachable (Polya, 1945). Empirical work has continued to show the instructional value of Polya-based approaches for improving students' reasoning and supporting more systematic engagement with mathematical tasks (Kaitera & Harmoinen, 2022; Lee & Chen, 2015; Pradana, 2024). At the same time, the effectiveness of Polya's model depends not only on presenting the four steps, but also on creating learning conditions in which students can explain, test, and refine their thinking while moving through those steps. Recent work therefore points to the importance of interaction, discussion, and collaborative regulation in strengthening students' problem-solving processes (Tang et al., 2023; Wismath & Orr, 2015).

Integrating STAD Cooperative Learning with Polya's Problem-Solving Steps

Cooperative learning provides one possible way to strengthen the instructional use of problem-solving heuristics. Grounded in the work of Slavin, cooperative learning emphasizes structured peer interaction, shared responsibility, and active participation rather than passive reception of instruction (Slavin, 1983, 1987, 1995). In such settings, students are expected to discuss ideas, support one another's learning, and take responsibility both for group work and for individual understanding. This structure is relevant to mathematical problem solving because it can create opportunities for students to verbalize reasoning, compare alternative strategies, and monitor their progress during difficult tasks.

Among cooperative learning models, Student Teams-Achievement Divisions (STAD) is especially suitable for classroom-based mathematics instruction because it combines heterogeneous grouping, team support, individual assessment, and achievement recognition within a clear and manageable instructional sequence (Slavin, 1995, 2014; Yapatang & Polyiem, 2022). These features make STAD particularly compatible with Polya's model. Heterogeneous grouping can broaden the range of ideas available during problem solving, while individual accountability helps ensure that collaborative work does not replace personal understanding. Furthermore, cooperative learning is reinforced by the foundational theory of social interdependence (Lewin, 1935), which explains that student motivation is characterized by positive and negative interdependence among group members. Positive interdependence creates a dynamic, interactive learning environment that encourages mutual progress and supports group members in achieving common goals. Negative interdependence, on the other hand, creates competition among group members, fostering self-esteem and the drive to surpass rivals, thereby motivating each individual within the group to achieve higher performance than the rest of the group ultimately. In this way, STAD may provide a practical structure for guiding students through identifying, planning, performing, and re-evaluating in a more explicit and socially supported manner.

Previous research has reported positive effects of cooperative learning on mathematics-related outcomes and has increasingly explored how cooperative structures can be combined with Polya's problem-solving process. Meta-analytic evidence suggests that cooperative learning is generally beneficial for mathematics achievement and attitudes (Capar & Tarim, 2015). Classroom-based studies have also shown that heterogeneous collaboration can support mathematical learning and problem solving more effectively than homogeneous or individually oriented arrangements in some settings (Černilec et al., 2023; Klang et al., 2021). More directly, recent studies have begun to examine instructional designs that combine cooperative learning with Polya's problem-solving stages (Dookurong et al., 2025; Mawardati et al., 2022; Yapatang & Polyiem, 2022). However, the available evidence remains limited in several important respects. First, much of the existing literature evaluates intervention outcomes using overall achievement measures or global indicators of problem-solving performance, providing limited insight into how different dimensions of competence respond to instructional support. Second, relatively few studies have employed rigorous quasi-experimental designs to examine multidimensional competence outcomes. Third, evidence from lower-secondary classrooms in contexts such as Vietnam remains scarce.

This gap is important because stronger claims about program effectiveness in education depend on methodologically robust evaluation designs rather than isolated classroom impressions (Slavin, 2008). Accordingly, the present study addresses these gaps by examining whether an integrated STAD-Polya intervention is associated with greater gains in mathematical problem-solving competence than individually implemented Polya-based instruction. Importantly, the study adopts a multidimensional competence framework that examines intervention effects across four dimensions of competence: identifying, planning, performing, and re-evaluating. This approach enables the investigation of differential competence growth across dimensions rather than relying solely on overall performance measures. By doing so, the study contributes a more detailed understanding of how instructional interventions influence different aspects of mathematical problem-solving competence.

MATERIALS AND METHODS

Research Design

This study employed a quasi-experimental pretest-posttest design to examine whether an integrated STAD-Polya intervention was associated with greater gains in students' mathematical problem-solving competence than individually implemented Polya-based instruction. The intervention was conducted over four months, from January to May 2025, and involved 436 ninth-grade students from 10 intact classes across five secondary schools in Ho Chi Minh City, Vietnam. Five classes were assigned to the experimental condition ($n = 216$), and five classes were assigned to the control condition ($n = 220$).

Because intact classes were used, students were not randomly assigned to conditions. To improve baseline comparability, class allocation was based on students' previous semester mathematics achievement, and all participants completed a pre-test before the intervention. This design enabled the study to compare differential change in competence over time between the two instructional conditions. Because assignment occurred at the class level, the findings are interpreted as quasi-experimental estimates of differential change rather than as definitive causal effects.

Instructional Content and Intervention Procedures

The instructional content was equivalent across conditions and covered functions, quadratic equations in one variable, systems of equations, and solid geometry, in alignment with the Vietnamese mathematics curriculum framework (Ministry of Education and Training, 2018). The difference between the two conditions lay in the instructional organization rather than in the mathematical content.

In the experimental classes, instruction was delivered through an integrated STAD-Polya format that combined the cooperative-learning structure of Student Teams-Achievement Divisions with Polya's four-step problem-solving process (Polya, 1945; Slavin, 1995, 2014). Students were organized into heterogeneous teams of four to six members on the basis of their prior problem-solving competence. Each lesson followed a common sequence. First, the teacher introduced the problem situation and clarified the targeted stage of the problem-solving process. Second, students discussed the task within their teams to identify and understand the problem, propose possible solution plans, and agree on the most appropriate approach, corresponding to understanding the problem and devising a plan. Third, students worked individually within their teams to carry out the agreed solution, corresponding to carrying out the plan. Fourth, they reviewed their own work and engaged in peer checking to compare alternative solutions and evaluate the reasonableness of their answers, corresponding to looking back. Finally, the teacher conducted whole-class consolidation to synthesize key mathematical ideas and common errors. Individual accountability was maintained through quizzes, worksheets, or oral responses, whereas group recognition was provided through classroom-based

Table 1. Framework for assessing mathematical problem-solving competence (Adapted from Ministry of Education and Training, 2018)

No	Content	Score				
		1	2	3	4	5
1	**Identifying competence**					
1.1	Reasoning and expressing problem information					
1.2	Analyzing and synthesizing relevant information and data					
1.3	Defining the problem and viewing it from different perspectives					
1.4	Orienting how to solve the problem					
2	**Planning competence**					
2.1	Identifying the relationship between the hypothesis and the conclusion					
2.2	Establishing a mathematical model to describe the problem					
2.3	Using mathematical language to present the solution idea					
2.4	Finding multiple solutions					
2.5	Evaluate the advantages and disadvantages of each solution					
3	**Performing competence**					
3.1	Applying knowledge to solve problems					
3.2	Using tools and supporting means to solve problems					
3.3	Presenting the solution logically					
3.4	Explaining the correctness of the solution					
3.5	Find a simpler solution or apply it in practice					
4	**Re-evaluating competence**					
4.1	Reviewing the results after implementation					
4.2	Identify the causes of the results and find alternative solutions					
4.3	Evaluating peers' solutions					
4.4	Model similar problems in learning and life					

rewards or performance feedback. This instructional structure was intended to preserve the heuristic sequence of Polya's model while embedding it in a socially regulated learning environment that encouraged explanation, comparison, and reflective review (Wismath & Orr, 2015; Yapatang & Polyiem, 2022).

In the control classes, the same mathematical content was taught through Polya-based problem solving without cooperative learning. Students were guided through the same four problem-solving stages; however, no formal team structure, peer-support routine, or group-recognition mechanism was implemented. As a result, problem solving in the control condition remained individually organized, although teacher guidance on the problem-solving sequence was retained.

To support consistency across classrooms, teachers received standardized guidance and used common instructional materials. Implementation fidelity was monitored through three random, unannounced classroom observations during the intervention period. All pre-test and post-test scripts were scored by an experienced independent teacher who was blind to group assignment.

Measurement of Mathematical Problem-Solving Competence

Students' mathematical problem-solving competence was assessed using two researcher-developed tests administered before and after the intervention. The pre-test and post-test were designed to mirror the structure and scope of the students' first-semester and second-semester mathematics examinations, respectively. Although the two tests were not identical forms, they were constructed to represent the same competence framework, comparable content scope, and a common scoring rubric. Accordingly, change over time was interpreted at the level of shared competence dimensions rather than item-by-item equivalence. To strengthen content validity, both instruments were reviewed by experienced ninth-grade mathematics teachers for curriculum suitability and alignment with students' cognitive levels. The assessment framework was developed based on the mathematical problem-solving competence structure specified in the Vietnamese General Education Curriculum (Ministry of Education and Training, 2018). Prior to implementation, the framework, assessment indicators, and instructional model were reviewed by a panel of five experts in educational science and mathematics education. Experts evaluate three aspects of the proposed framework: (a) content relevance, (b) clarity and appropriateness of language and presentation, and (c) methodological considerations. Collectively, these results provide evidence of satisfactory content validity for the assessment framework and instructional model. Revisions were subsequently made based on expert feedback before the instruments were finalized for use in the study.

The assessment framework followed the problem-solving competence structure prescribed by the Ministry of Education and Training (2018) (**Table 1**). In the present study, mathematical problem-solving competence was operationalized through 18 sub-indicators grouped into four dimensions: identifying, planning, performing, and re-evaluating. The identifying dimension comprised four indicators, the planning dimension comprised five indicators, the performing dimension comprised five indicators, and the re-evaluating dimension comprised four indicators.

Each sub-indicator was scored using a five-level rubric reflecting the quality of students' responses: Level 1 indicated an absent or incorrect response, Level 2 a partially correct response, Level 3 a basically correct response, Level 4 a largely correct response, and Level 5 an accurate, complete, and well-developed response. Scores for each competence dimension were calculated as mean scores on a 1-to-5 scale, with higher scores indicating higher competence. To support scoring consistency, detailed scoring descriptors were developed for each performance level. Level 1 represented an absent or incorrect response,

Table 2. Problem-solving competence of students in two groups before the experiment

Competence	Control (1) *n* = 220	Experiment (2) *n* = 216	D (1-2)	*t*	*Pr*
	Mean (SD)	*Mean (SD)*			
Identifying	3.04 (0.46)	3.03 (0.46)	0.01	0.17	0.87
Planning	2.96 (0.58)	3.04 (0.51)	-0.08	1.60	0.11
Performing	3.07 (0.45)	3.10 (0.52)	-0.02	0.49	0.63
Re-evaluating	3.03 (0.67)	2.99 (0.76)	0.04	0.58	0.56

Table 3. The propensity score of the two groups before the experiment

Variable	Mean			t-test
	Experiment	*Control*	*%bias*	t (p.value)
pscore	0.49828	0.49667	2.7	0.29 (0.7740)

whereas Level 5 represented a complete, accurate, and well-justified response. Intermediate levels reflected increasing degrees of correctness, completeness, and quality of reasoning. All pre-test and post-test scripts were scored by an experienced mathematics teacher who was not involved in the instructional intervention and was blind to students' group assignment. To further reduce potential scoring bias, neither the classroom teachers nor the researcher participated in the primary scoring process. After scoring was completed for each participating school, the researcher independently re-scored a random sample of approximately 10–15% of the scripts as a quality-control procedure. Any discrepancies were reviewed against the scoring rubric and resolved through discussion. This process was intended to strengthen scoring consistency and reduce potential assessor bias.

The internal consistency of the tool was tested using Cronbach's alpha coefficient based on the entire study sample (n = 436). All four subscales exceeded the usual threshold of 0.70, indicating reliability ranging from acceptable to strong. The alpha coefficients were 0.858 for identification, 0.878 for planning, 0.764 for implementation, and 0.934 for reassessment, respectively. Simultaneously, the correlation coefficient between each item and the adjusted total score for the 18 indicators ranged from 0.381 to 0.870, indicating that each indicator made a significant contribution to its respective aspect. Furthermore, the validity assessment results through principal factor analysis (see Appendix A) confirmed that the 18 observed variables (representing 18 criteria) formed four principal factors: Identifying (4 observed variables); Planning (5 observed variables); Performing (5 observed variables); Re-evaluating (4 observed variables) and factor loading coefficients >0.5. The above results mean that the set of observed variables correctly measures the component competencies within the problem-solving competency structure. Taken together, these results support the use of the Ministry of Education and Training (2018) framework for assessing mathematical problem-solving competence in the present quasi-experimental context.

Data Analysis

The primary analysis used a Difference-in-Differences (DID) framework to estimate whether the intervention group showed larger pretest-posttest gains than the comparison group (Wooldridge, 2010). For each competence dimension, the model included indicators for group, time, and their interaction. The group indicator captured baseline between-group differences, the time indicator captured overall pretest-posttest change in the comparison group, and the interaction term captured the additional change associated with the integrated STAD-Polya intervention. The interaction coefficient was interpreted as the estimated intervention effect for each competence dimension. Because the intervention was assigned to intact classes rather than randomly assigned individuals, the estimates are interpreted as quasi-experimental comparisons of differential change rather than as definitive causal effects.

The propensity score analysis method was used only as a supplementary descriptive diagnostic method to the Difference-in-Differences analysis results. This analysis aimed to verify whether the increase in competency scores of the experimental group compared to the control group was due to the effect of the intervention or due to sample differences. If the analysis results showed a p-value of ATT (Average Treatment Effect on the Treated) <0.05, it would confirm that the change in competency in the experimental group was the effect of the intervention. All statistical analyses were performed using STATA 17 and SPSS 25.5.

RESULTS

Problem-Solving Competence of Students in Two Groups Before the Experiment

Students' problem-solving competence was measured across four dimensions: identifying, planning, performing, and re-evaluating. As shown in **Table 2**, the two groups were highly comparable at baseline across all four dimensions. Independent-samples t-tests revealed no statistically significant differences in mean scores (all $Pr > .05$), and the observed between-group differences were negligible in magnitude (all $D < 0.10$). These results suggest that the experimental and control groups were broadly comparable in their problem-solving competence at pre-test. Taken together, these findings provide support for the assumption that the two groups began the intervention with broadly comparable levels of observed problem-solving competence.

Furthermore, the results of the pre-experiment balance test of the propensity score (pscore) of the two groups (see **Table 3**) showed that the average value of the experimental group was 0.498 and the control group was 0.497, with a very low difference (%bias) of 2.7% (much smaller than the commonly recommended threshold of < 10%). Along with this, the t-test for the pscore had a p-value of 0.774 (> 0.05). This result confirms that there is no statistically significant difference between the average bias scores of the two groups.

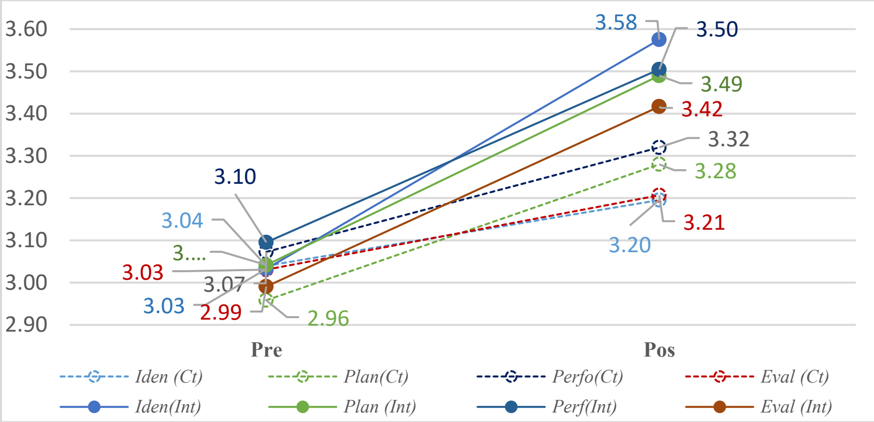

Figure 1. Differential growth across dimensions of mathematical problem-solving competence (Source: Authors' own elaboration)

Table 4. Measurement results using the DID model

No	Competence	β_1	β_2	β_3	Pr(β_3)
1	Identifying	0.01	0.16	0.39	0.000
2	Planning	-0.08	0.32	0.13	0.016
3	Performing	-0.02	0.25	0.16	0.000
4	Re-evaluating	0.04	0.18	0.25	0.000

Note: β_1, the difference between the two groups when selecting samples; β_2 is the change (after-before) of the control group; β_3, the difference of the experimental group – the difference of the control group

Table 5. PSM analysis results

Variable	Sample	Treated	Controls	Difference	S.E.	t
Identifying	Unmatched	3.58	3.20	0.38	0.035	10.77
	ATT	3.57	3.20	0.37	0.037	9.97
Planning	Unmatched	3.61	3.18	0.43	0.040	10.65
	ATT	3.61	3.18	0.43	0.043	9.99
Performing	Unmatched	3.53	3.28	0.25	0.044	5.81
	ATT	3.53	3.28	0.25	0.046	5.53
Re-evaluating	Unmatched	3.42	3.21	0.21	0.052	4.07
	ATT	3.43	3.18	0.25	0.054	4.71

Figure 1 illustrates changes in competence scores across the four dimensions from pre-test to post-test. Although both groups demonstrated improvement over time, the experimental group showed consistently larger gains than the comparison group across all competence dimensions.

Differential Effects of the STAD-Polya Intervention Across Competence Dimensions

Table 4 reports the DID estimates for the four competence dimensions. Regarding RQ1, the integrated STAD-Polya intervention was associated with greater gains in mathematical problem-solving competence than individually implemented Polya-based instruction. The interaction coefficients were positive and statistically significant for *identifying* (β_3 = 0.39, *Pr* < *.001*), *planning* (β_3 = *0.13, Pr* = *.016*), *performing* (β_3 = *0.16, Pr* < *.001*), and *re-evaluating* (β_3 = *0.25, Pr* <*.001*). Regarding RQ2, these differences should be interpreted as differences in coefficient magnitude rather than as formally tested differences between dimensions.

Furthermore, the results of the pedagogical experiment's impact assessment on students' problem-solving abilities using the PSM method (see **Table 5**) show that the average impact on the experimental group (ATT) confirmed that the intervention program yielded positive, strong, and statistically significant results (t>1.96, corresponding to a significance level of P<0.05). Specifically, participation in the program improved abilities by 0.25 to 0.43 score, compared to the unrealistic scenario where they did not participate. In addition, the slight decrease in the coefficient of difference (Difference) and t values from the Unmatched row to the ATT row in some variables reflects that part of the initial effectiveness was actually caused by the phenomenon of sample bias. However, after balancing the sample using the Kernel algorithm in PSM, the pure practical effectiveness (ATT index) of the program remained very high and statistically significant. This result demonstrated the effectiveness of the pedagogical experiment method and confirmed the reliability of the DID evaluation results above.

DISCUSSIONS

The results of this semi-experimental study indicate that the integrated STAD-Polya intervention was associated with greater progress in the problem-solving abilities of 9th-grade students compared to teaching based on the Polya method implemented individually. The DID estimates were positive and statistically significant for all four dimensions assessed—identification, planning, implementation, and reassessment. Furthermore, the impact of the pedagogical intervention on students' problem-solving abilities was also confirmed through PSM analysis. In summary, these findings suggest that combining structured collaborative learning with clear problem-solving methods can support student development across multiple stages of the problem-solving process, particularly in aspects related to problem interpretation and critical thinking.

The pattern of results is notable because the estimated advantage of the intervention was not limited to a single dimension. The largest differential gain was observed for identifying, followed by re-evaluating, with smaller but still statistically significant gains for performing and planning. This pattern suggests that the intervention may have helped students engage more systematically with problem interpretation and reflective review, while also supporting strategic planning and solution execution. Such an interpretation is broadly consistent with prior work indicating that novice learners often underuse planning and review processes when solving problems individually (Silver et al., 2005) and that discussion-based learning environments can make strategic thinking more visible during problem solving (Russo & Minas, 2020; Tang et al., 2023; Yapatang & Polyiem, 2022). Theoretically, George Polya's (1945) problem-solving process, with its four closely linked steps following an ascending logic from Understanding the Problem => Planning => Implementing the Plan => Reviewing, provides a systematic cognitive framework for structuring mathematical thinking. Over time, it will form habits for students, transforming them from passive learners into active learners, confident in solving difficulties. Furthermore, integrating cooperative learning into each problem-solving step helps break down a complex problem into smaller stages, and the solution is carried out in groups, with students working together, supporting each other, practicing together, and finding solutions more easily—something not possible when individuals solve problems independently. Furthermore, the findings in this study can be explained through the theory of social interdependence (the foundational theory of cooperative learning), according to which positive interdependence leads to interaction that promotes and stimulates the development of thinking and self-reliance, while negative interdependence creates competition and increased critical thinking among students in the group. From another perspective, is that the STAD structure creates repeated opportunities for students to articulate information, justify strategies, and compare alternatives while moving through Polya's stages. In this sense, the cooperative format may have functioned as a form of social scaffolding that expands students' zones of teaching and learning, allowing mathematical ideas to be developed with timely peer and teacher support rather than through isolated individual effort (Norton & D'Ambrosio, 2008; Vygotsky, 1978). The present findings therefore support the instructional value of integrating cooperative structures with Polya's model, rather than treating heuristic instruction and collaboration as separate pedagogical choices.

Informal classroom observations during implementation generally align with quantitative results. Experimental classes appear to have more frequent peer interpretation and broader participation across achievement levels compared to control classes, where responses were typically dominated by a small number of students. However, these observations should be interpreted cautiously as they are primarily used to monitor implementation rather than being analyzed through a formal qualitative protocol. Nevertheless, our observations are consistent with previous studies showing that cooperative learning can promote engagement and commitment in math classrooms (Drakeford, 2012; Mendo-Lázaro et al., 2022).

Although the study yielded positive results, several limitations remain: The intervention was conducted in intact classes rather than through random assignment, which could lead to an imbalance in the number of academically strong/weak students in each experimental and control group; the pedagogical experiment was not conducted on a large scale across many different regions, resulting in results that are not representative of the overall population; and another issue is that to effectively organize cooperative learning, teachers need to be provided with essential knowledge and become familiar with the teaching style of facilitator-to-facilitator. Furthermore, teachers also spend considerable time dividing students into cooperative learning groups to ensure equal abilities among the small groups. Given the above limitations, we recommend that for more accurate results, similar studies should be conducted on a broader scale, with cross-comparisons of effectiveness across different regions, and that teachers should also equip themselves with knowledge and skills in organizing cooperative learning. Despite some limitations, this study has provided important evidence that the integrated STAD-Polya method can be a promising approach to enhancing students' problem-solving abilities, giving teachers more suitable options in teaching problem-solving skills – a crucial skill of the 21st century.

CONCLUSIONS

This study examined whether an integrated STAD-Polya intervention was associated with stronger gains in ninth-grade students' mathematical problem-solving competence than individually implemented Polya-based instruction. Conducted with 436 students from 10 classes in Ho Chi Minh City, Vietnam, the study found positive and statistically significant DID estimates across four dimensions of competence: identifying, planning, performing, and re-evaluating. These findings suggest that combining a structured cooperative-learning format with explicit problem-solving heuristics may support gains across multiple dimensions of mathematical problem-solving competence more effectively than individually implemented Polya-based instruction alone.

The main contribution of the study lies not only in evaluating the effectiveness of an integrated STAD-Polya intervention, but also in examining intervention effects through a multidimensional competence framework. By analyzing intervention effects across four competence dimensions—identifying, planning, performing, and re-evaluating—the study demonstrates that competence growth was not uniform across dimensions. This multidimensional perspective provides a more nuanced understanding of instructional impact than would be possible through overall competence scores alone. By organizing peer interaction around Polya's stages, the STAD structure may help students make their reasoning more explicit, sustain attention to planning, and engage more consistently in reflective review. Despite its limitations, the study provides strong scientific evidence for teachers to consider regarding an active teaching method that promotes students' problem-solving abilities.

Implications for Practice

For classroom practice, the findings suggest that teaching mathematical problem solving may benefit from more than presenting a sequence of solution steps. Teachers may also need to structure how students discuss, justify, and review mathematical ideas while working through those steps. In this respect, the integrated STAD-Polya approach offers a practical way to connect heuristic instruction with peer explanation and individual accountability.

For schools and curriculum designers, the results suggest that cooperative structures can be incorporated into regular mathematics lessons without changing the core content of the curriculum. Rather than treating collaboration as an additional activity, teachers can organize it around identifying the problem, planning a strategy, performing the solution, and re-evaluating the result. This model may support a shift from a result-oriented to a process-oriented mathematics classroom. It may be especially useful in contexts where students tend to focus on obtaining answers quickly without adequately articulating or reviewing their reasoning.

First, teachers should form heterogeneous groups while maintaining individual accountability so that students can benefit from peer explanation without losing responsibility for their own learning (Capar & Tarim, 2015).

Second, teachers should allocate explicit discussion time for planning and for the re-evaluating phase, since these stages are often underused in individual work. Prompting students to compare strategies, justify decisions, and review the reasonableness of answers may strengthen the quality of mathematical reasoning (Hidayat et al., 2025; Thi-Nga et al., 2024).

Third, teachers should align classroom tasks and group procedures with Polya's four stages so that collaboration supports the problem-solving process rather than becoming general discussion. In practice, this means using structured prompts, team roles, and brief whole-class consolidation to keep students focused on the mathematical purpose of each stage.

Limitations

First, the study used intact classes instead of randomly assigning students to new classes, although the groups were assessed as similar in problem-solving abilities before the experiment, using intact classes would not completely eliminate the issue related to relationships between students, which could lead to the effectiveness of cooperative learning groups (for example, if the group has students with good relationships beforehand, the group learning effectiveness is high; conversely, if the group has students with poor relationships beforehand, the group activity effectiveness may be low), which could increase or decrease the effectiveness of cooperative learning. Second, the study was conducted in an urban area over a period of four months, which limits the ability to generalize the findings. Third, PSM analyses were used as an additional descriptive support method rather than the primary basis for estimating effectiveness.

Future Research

Future research should replicate the intervention in more diverse school contexts, examine whether the observed gains are sustained over longer periods, and investigate more directly how peer interaction supports specific stages of students' problem-solving processes. If conditions permit, future studies should rearrange students in the experimental and control classes into new classes instead of keeping them in their original classes, which reduces the biases associated with the long-term relationships between students when divided into cooperative learning groups. Future research should also examine whether the effects of integrated STAD-Polya instruction are mediated by students' attitudes toward cooperation and learning motivation, as these mechanisms have recently been linked to mathematics achievement in other instructional settings (Zuo et al., 2025). Additional work may also compare this integrated approach with other cooperative-learning formats to determine which design features are most relevant for strengthening mathematical problem-solving competence. Future studies may also investigate whether different dimensions of mathematical problem-solving competence exhibit distinct developmental trajectories across longer instructional periods and across different educational contexts. In addition, future research on problem-solving skills development should consider the integration of metacognitive skills development. According to Panáčová and Veseláková (2026), metacognitive skills development and problem-solving skills development are mutually reinforcing.

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REFERENCES

Angrist, J. D., & Pischke, J.-S. (2009). *Mostly harmless econometrics: An empiricist's companion*. Princeton University Press. https://doi.org/10.1515/9781400829828

Cao, Y. (Ed.). (2024). *Students' collaborative problem solving in mathematics classrooms: An empirical study*. Springer. https://doi.org/10.1007/978-981-99-7386-6

Capar, G., & Tarim, K. (2015). Efficacy of the cooperative learning method on mathematics achievement and attitude: A meta-analysis research. *Educational Sciences: Theory & Practice, 15*(2), 553-559. https://doi.org/10.12738/estp.2015.2.2098

Černilec, B., Cotič, M., Felda, D., & Doz, D. (2023). Differences in students' mathematics knowledge in homogeneous and heterogeneous groups. *European Journal of Science and Mathematics Education, 11*(1), 15-32. https://doi.org/10.30935/scimath/12431

Cuong, L. M., Tien-Trung, N., Ngu, P. N. H., Vangchia, V., Thao, N. P., & Thao, T. T. P. (2025). Mathematics problem-solving research in high school education: Trends and insights from the Scopus database (1983–2023). *European Journal of Science and Mathematics Education*, 13(2), 77-89. https://doi.org/10.30935/scimath/16038

Dookurong, I.D., Yarhands, D.A., & Akwerty, E. (2025). A systematic review of the integration of collaborative learning and Polya's problem-solving strategy for enhancing preservice teachers' critical thinking. *International Journal of Studies in Education and Science*, 6(4), 395-428. https://doi.org/10.46328/ijses.5520

Drakeford, W. (2012). The effects of cooperative learning on the classroom participation of students placed at risk for societal failure. *Journal of Psychology Research, 2*(4), 239-246. https://doi.org/10.17265/2159-5542/2012.04.003

Fan, X., & Nowell, D. L. (2011). Using propensity score matching in educational research. *Gifted Child Quarterly, 55*(1), 74-79. https://doi.org/10.1177/0016986210390635

Hidayat, R., Mohd Saad, M. R., & Wewe, M. (2025). A meta-analysis of the effect of metacognitive instruction on mathematics achievement. *Cogent Education*, *12*(1), Article 2517510. https://doi.org/10.1080/2331186X.2025.2517510

Kaitera, S., & Harmoinen, S. (2022). Developing mathematical problem-solving skills in primary school by using visual representations on heuristics. *LUMAT: International Journal on Math, Science and Technology Education*, *10*(2), 111-146. https://doi.org/10.31129/LUMAT.10.2.1696

Klang, N., Karlsson, N., Kilborn, W., Eriksson, P., & Karlberg, M. (2021). Mathematical problem-solving through cooperative learning—The importance of peer acceptance and friendships. *Frontiers in Education, 6*, Article 710296. https://doi.org/10.3389/feduc.2021.710296

Lee, C. Y., & Chen, M. J. (2015). Effects of Polya questioning instruction for geometry reasoning in junior high school. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(6), 1547-1561. https://doi.org/10.12973/eurasia.2015.1419a

Lewin, K. (1935). Psycho-sociological problems of a minority group. *Character & Personality; A Quarterly for Psychodiagnostic & Allied Studies*, *3*, 175-187. https://doi.org/10.1111/j.1467-6494.1935.tb01996.x

Mawardati, R., Fahradina, N., Aini, R. A., & Pratiwi, N. (2022). Pembelajaran geometri berdasarkan teori Polya plus dengan model pembelajaran kooperatif tipe student team achievement division (Stad) di kelas viii smp it Daruzzahidin Aceh Besar [Geometry learning based on Polya plus theory with Student Team Achievement Division (STAD) cooperative learning model in class viii of Daruzzahidin Islamic Junior High School, Aceh Besar]. *Jurnal Pendidikan Matematika dan Sains, 1(2),* 45-56. https://ejournal.unida-aceh.ac.id/dikmas/article/view/402

Mendo-Lázaro, S., León-del-Barco, B., Polo-del-Río, M. I., & López-Ramos, V. M. (2022). The impact of cooperative learning on university students' academic goals. *Frontiers in Psychology, 12*, Article 787210. https://doi.org/10.3389/fpsyg.2021.787210

Ministry of Education and Training. (2018). *Chương trình giáo dục phổ thông môn Toán* [General education program for mathematics]. https://luatvietnam.vn/giao-duc/thong-tu-32-2018-tt-bgddt-bo-gddt-ban-hanh-chuong-trinh-giao-duc-pho-thong-163978-d1.html

Norton, A., & D'Ambrosio, B. S. (2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education, 39*(3), 220-246. https://www.jstor.org/stable/30034969

Organisation for Economic Co-operation and Development [OECD]. (2017). *PISA 2015 assessment and analytical framework: Science, reading, mathematics, financial literacy and collaborative problem solving* (Rev. ed.). OECD Publishing. https://doi.org/10.1787/9789264281820-en

Panáčová, J., & Veseláková, J. (2026). Pedagogical intervention with a mathematically gifted student–Expanding problem-solving strategies and developing metacognition: A case study. *European Journal of Science and Mathematics Education, 14*(3), 427-448. https://doi.org/10.30935/scimath/18785

Park, J. H. (2023). Revisiting creativity and critical thinking through content analysis. *Thinking Skills and Creativity*, 49, Article 101233. https://doi.org/10.1016/j.tsc.2023.101233

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton University Press. https://doi.org/10.1515/9781400828678

Pradana, L. N. (2024). Problem-solving strategy: Mathematical problem-solving model within the Polya framework. *KnE Social Sciences, 9*(6), 728-740. https://doi.org/10.18502/kss.v9i6.15327

Renkl, A., Mandl, H., & Gruber, H. (1996). Inert knowledge: Analyses and remedies. *Educational Psychologist*, *31*(2), 115-121. https://doi.org/10.1207/s15326985ep3102_3

Russo, J., & Minas, M. (2020). Student attitudes towards learning mathematics through challenging, problem solving tasks: "It's so hard–in a good way". *International Electronic Journal of Elementary Education*, *13*(2), 215-225. https://doi.org/10.26822/iejee.2021.185

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). Macmillan.

Seah, R., & Horne, M. (2026). A learning progression for understanding angle concepts. *Mathematical Education Research Journal, 38,* 511-535. https://doi.org/10.1007/s13394-025-00537-0

Silver, E. A., Ghousseini, H., Gosenpud, D., Cappuzello, L., & Strawhun, B. T. (2005). Moving from rhetoric to practice: Issues faced by teachers in having students explain their thinking and justify their solutions. *Journal of Mathematical Behavior*, *24*(3-4), 287-305. https://doi.org/10.1016/j.jmathb.2005.09.009

Slavin, R. E. (1983). When does cooperative learning increase student achievement? *Psychological Bulletin, 94*(3), 429-445. https://doi.org/10.1037/0033-2909.94.3.429

Slavin, R. E. (1987). Cooperative learning and the cooperative school. *Educational Leadership, 45*(3), 7-13. https://files.ascd.org/staticfiles/ascd/pdf/journals/ed_lead/el_198711_slavin.pdf

Slavin, R. E. (1995). *Cooperative learning: Theory, research, and practice* (2nd ed.). Allyn & Bacon.

Slavin, R. E. (2008). Perspectives on Evidence-Based Research in Education—What Works? Issues in Synthesizing Educational Program Evaluations. *Educational Researcher*, *37*(1), 5-14. https://doi.org/10.3102/0013189X08314117

Slavin, R. E. (2014). Cooperative learning and academic achievement: Why does groupwork work? *Anales de Psicología, 30*(3), 785-791. https://doi.org/10.6018/analesps.30.3.201201

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*(2), 257-285. https://doi.org/10.1207/s15516709cog1202_4

Tang, X., Liu, Y., & Milner-Bolotin, M. (2023). Investigating student collaborative problem-solving competency and science achievement with multilevel modeling: Findings from PISA 2015. *PLOS ONE, 18*(12), Article e0295611. https://doi.org/10.1371/journal.pone.0295611

Thi-Nga, H., Thi-Binh, V., & Nguyen, T.-T. (2024). Metacognition in mathematics education: From academic chronicle to future research scenario–A bibliometric analysis with the Scopus database. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(4), Article em2427. https://doi.org/10.29333/ejmste/14381

van Dijke-Droogers, M., Drijvers, P., & Bakker, A. (2025). Effects of a learning trajectory for statistical inference on 9th-grade students' statistical literacy. *Mathematical Education Research Journal, 37,* 187-210. https://doi.org/10.1007/s13394-024-00487-z

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press. https://www.jstor.org/stable/j.ctvjf9vz4

Wismath, S. L., & Orr, D. (2015). Collaborative learning in problem solving: A case study in metacognitive learning. *The Canadian Journal for the Scholarship of Teaching and Learning, 6*(3), Article 10. https://doi.org/10.5206/cjsotl-rcacea.2015.3.10

Wooldridge, J. M. (2010). Econometric analysis of cross section and panel data (2nd ed.). MIT Press.

Yapatang, L., & Polyiem, T. (2022). Development of the mathematical problem-solving ability using applied cooperative learning and Polya's problem-solving process for Grade 9 students. *Journal of Education and Learning, 11*(3), 40-46. https://doi.org/10.5539/jel.v11n3p40

Zhan, X., Sun, D., Qiang, C., Song, R., & Wan, Z. H. (2019). Propensity score analysis of the impacts of junior secondary students' participation in engineering practices on their attitudes toward engineering. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(11), Article em1765. https://doi.org/10.29333/ejmste/106761

Zuo, S., Qi, C., & Li, G. (2025). The relationship between project-based learning and mathematics achievement: mediating roles of attitudes towards cooperation and motivation. *Current Psychology, 44(2*), 15781-15794. https://doi.org/10.1007/s12144-025-08319-4