Pedagogical Research
2024, 9(2), em0191
e-ISSN: 2468-4929
https://www.pedagogicalresearch.com

MODESTUM

Research Article **OPEN ACCESS**

Mathematics teachers' beliefs about mathematics, its teaching, and learning: The case of five teachers

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Citation: Kasa, Y., Areaya, S., & Woldemichael, M. (2024). Mathematics teachers' beliefs about mathematics, its teaching, and learning: The case of five teachers. *Pedagogical Research, 9*(2), em0191. https://doi.org/10.29333/pr/14172

ARTICLE INFO

Received: 14 Nov. 2023

Accepted: 26 Jan. 2024

ABSTRACT

This study explores the views of mathematics teachers at Assosa University regarding effective mathematics teaching. The study used a case study method as a mode of inquiry through semi-structured interviews with five mathematics teachers at Assosa University. The data were analyzed thematically. The results showed that mathematics teachers view memorization, practice, and concrete experiences as ways to enhance their students' understanding of mathematics. Teachers expressed different views on the nature of mathematics. Some teachers have an instrumental view, while others have a problem-solving view, and others have a Platonist view of the nature of mathematics. The study showed important arguments about the belief systems of mathematics teachers, showing that teachers' beliefs about the nature of mathematics provide the basis for their mental models of mathematics teaching and learning. The study revealed those mathematics teachers' beliefs about effective mathematics teaching influence their educational practices. The study suggests that developing the expertise of teachers in higher education institutions by changing their entrenched beliefs about mathematics needs particular attention, which ultimately affects their practice.

Keywords: nature of mathematics, teacher's belief, teaching and learning mathematics, understanding

INTRODUCTION

There is no universal agreement among researchers and the public on what constitutes "*good or effective mathematics teaching*." One's conception of mathematics influences desirable ways of teaching and learning it. How do we perceive that effective mathematics teaching has an impact on classroom practices? Thom (1973) noted this when he stated that all mathematical pedagogy, even if scarcely coherent, rests on the philosophy of mathematics. Understanding the philosophy of mathematics and addressing significant issues about its nature can resolve disagreements both in public and in researchers about what constitutes effective mathematics teaching. Effective mathematics teaching is a complex topic that requires deep understanding of the philosophy of mathematics. Classroom practices are heavily influenced by how we perceive mathematics teaching, and it is essential to address significant issues regarding the nature of mathematics. Thom's (1973) observations from 1973 still hold true today (Anthony & Walshaw, 2009; Ernest et al., 2016; Hamami & Morris, 2020), as all mathematical pedagogy rests on the philosophy of mathematics, even if it is not always coherent. By understanding this philosophy, we can develop new ways of teaching and learning that are more effective and engaging for the students.

Although informal learning plays an essential role in student learning, students acquire much of their knowledge and develop thinking skills during classroom instruction (Cai, 2007). Educational researchers have long attempted to identify the critical features of classroom instruction that effectively foster student learning. Among these critical features, teachers' beliefs constitute an essential construct in mathematics teaching and learning (Cai, 2007; Cai & Wang, 2010; Leatham, 2007). Teachers' beliefs are shaped by their experience, training, and personal philosophies, which influence their instructional decisions and practices. Research has shown that teachers with strong beliefs about the importance of student engagement, active participation, and conceptual understanding create more effective learning environments (Hiebert & Grouws, 2007). Additionally, teachers who believe in the potential of all students to succeed and provide opportunities for them to take ownership of their learning can positively affect student achievement (Cai & Wang, 2010). However, teachers' beliefs can also be limited if they have misconceptions or biases regarding certain groups of students or instructional strategies.

Although there is no consensus among researchers and the public on what effective mathematics teaching should look like and should be, no one questions that the instructional practices of teachers are influenced by both their cultural beliefs and conceptions of effective teaching (Bryan et al., 2007; Yang et al., 2020). Previous research has identified how beliefs, particularly beliefs about teaching and learning, influence teaching practices (Leatham, 2007; Muhtarom et al., 2019; Ren & Smith, 2018; Speer,

2008; Yang et al., 2020). Teachers' perceptions of the nature of mathematics and the teaching and learning of mathematics influence how students learn mathematics through their understanding (Xie & Cai, 2018). A teacher's manner of presenting mathematics indicates what he/she believes is essential, thereby influencing how students understand and learn mathematics (Cai, 2004; Carrillo-Yañez et al., 2018; Thompson, 1992). The studies cited above have shown those teachers' perceptions of the nature of mathematics and their teaching practices can have a significant impact on students' mathematical development.

Understanding beliefs is essential for understanding teachers' thoughts and feelings about educational reform. Teachers' beliefs may serve as feedback to understand how they form views on educational reform. Teachers' beliefs about education reform can help authorities to implement reforms in the education system (Bas, 2021). Therefore, it is apparent that teachers' beliefs are fundamental to transforming the education system through education reform (Fullan, 2007). To comprehend the role of teachers' beliefs in educational reform, it is important to explore the factors shaping these beliefs. Research has shown that various factors can influence teachers' beliefs, including their personal experiences, cultural backgrounds, and professional training (Pajares, 1992). Additionally, the broader social and political context in which teachers operate can affect their beliefs about education reform. For example, economic pressure or changes in government policies may influence how teachers view the need for reform in the education system. Given these complex and multifaceted influences on teachers' beliefs, any efforts to promote educational reform must consider the diversity of perspectives and experiences among educators.

In recent years, there have been many reform efforts in Ethiopia in both public and private higher education institutions (HEIs), intending to improve higher education and create student-centered teaching and learning processes (Bishaw & Melesse, 2017; Dejene, 2019). These reforms mainly occur at the institutional level, such as program changes (for instance, from three to four years of college in the 2019/2020 academic year). Additionally, efforts have been made to improve the quality of teaching and learning by incorporating technological and modern pedagogical approaches. The government has also invested in infrastructure development, such as building new campuses and dormitories, to accommodate the growing number of students. There have been initiatives to increase access to higher education for marginalized groups, including women and people with disabilities. Despite these efforts, challenges remain in the Ethiopian higher education system, including inadequate funding and limited research capacity. However, with continued commitment from both the government and HEIs, it is hoped these challenges can be addressed, and Ethiopia's higher education system will continue to improve and contribute to the country's development.

The reforms depended essentially on the change in the entire professional field of teachers from the educational field (in the college of natural and computational sciences) to the applied field in the 2009/2010 academic year. HEIs have undergone a wave of reforms to prepare teachers. However, less attention has been paid to developing the teaching skills required for good practice. Programs such as the higher diploma program are in place in HEIs to fill gaps in teachers' pedagogical skills and give them licenses to teach in universities. However, a shift to student-centered teaching requires more remarkable changes. To this end, Ernest (1989) had the following to communicate on mathematics teaching reform:

> It depends fundamentally on the teacher's beliefs, particularly on the teacher's conception of the nature of mathematics and mental models of teaching and learning mathematics. Teaching reforms cannot occur unless teachers' deeply held beliefs about mathematics and teaching and learning change. Furthermore, these changes in beliefs are associated with increased reflection and autonomy on the part of the mathematics teacher (p. 249).

Most of the reform efforts, such as changing the mode of delivery from parallel to a modular system and increasing the number of years from three to four to the Bachelor of Science program, are happening at the institutional level. However, reform efforts that ignore individual teachers' preparation to prepare them for effective teaching and thus professional development are unlikely to achieve the required objective. There should be an effort to improve teachers' professional development and their beliefs/conceptions to implement a program as planned. If we are to change teaching practices in HEIs, it is vital to study teachers' views on effective mathematics teaching. It is important to provide opportunities for ongoing professional development, such as workshops and conferences that focus on effective teaching strategies and techniques. By investing in professional development of teachers, we can create a culture of effective teaching practices that will ultimately benefit students at an institutional level.

Understanding the beliefs of mathematics teachers in the context of Assosa University is crucial for several reasons. Firstly, it allows for a comprehensive analysis of the factors influencing their teaching practices and pedagogical approaches. Secondly, it provides valuable insights into the effectiveness of current teaching methods and curriculum design within the specific context of Assosa University. By exploring these beliefs, this study aims to contribute to the existing body of knowledge and facilitate improvements in mathematics education at Assosa University. Moreover, Assosa University operates within a unique cultural and educational context, potentially influencing how teachers interpret and implement effective teaching strategies. Exploring the specific beliefs of these teachers and their application in classroom is crucial to understanding their impact on student learning.

THEORETICAL CONSIDERATIONS

According to Xie and Cai (2018), the way teachers perceive mathematics is shaped by their beliefs about it. A mathematics teacher's understanding of subject includes their knowledge of it and their beliefs about learning and teaching it, as explained by Ernest (1989). While knowledge is important, it does not explain why different teachers approach mathematics in different ways. For instance, two teachers may have similar knowledge of mathematics, but one may adopt a problem-solving approach, whereas the other may prefer a more didactic approach, as noted by Pajares (1992) and Philipp (2007). Also, a teacher's beliefs about learning and teaching mathematics can be influenced by their own experiences as learners and teachers, as well as cultural and societal factors. Beliefs can shape way they design and deliver instruction, interact with students, and assess learning outcomes.

Table 1. Philosophies about nature of mathematics

Philosophy	Mathematics is viewed, as follows:
Instrumentalist	Mathematics is a bag of tools.
Platonist	Mathematics is a static, unified body of knowledge.
Problem-solving	Mathematics is a dynamic, problem-driven discipline.

Table 2. Teacher's role & intended outcome of instruction

Teacher's role	Intended outcome
Instructor	Skills mastery with correct performance
Explainer	Conceptual understanding with unified knowledge
Facilitator	Confident problem posing and solving

Table 3. Models of learning mathematics

View of learning	Model of learning mathematics
Active construction of knowledge	Active construction of an understanding model
Passive reception of knowledge	Reception of knowledge model
Development of autonomy and interest in the learner	Exploration and autonomous pursuit of own interests model
Submissive and compliant learner	Compliant behavior and mastery of skills model

Ernest (1989) identified three interconnected components that comprise teachers' mathematical belief systems. These components include their perception of the nature of mathematics, their model of mathematics teaching, and their view of the process of learning mathematics. According to Ernest (1989), a teacher's view of the nature of mathematics is crucial because it has a significant impact on their beliefs about teaching and learning mathematics. Therefore, when studying effective mathematics teaching, it is essential to examine teachers' perspectives on mathematics and its teaching and learning (Cai & Wang, 2010). Ernest's (1989) ideas have been widely adopted and utilized by researchers in education (Beswick, 2012; Bryan et al., 2007; Muhtarom et al., 2019; Yang et al., 2020). Understanding a teacher's perspective on mathematics is essential to effective teaching and learning. It influences their beliefs about how best to teach the subject and what methods to use.

Beliefs About Nature of Mathematics

The way a teacher perceives mathematics can be seen as conscious or unconscious thoughts, ideas, meanings, regulations, mental images, and preferences regarding the subject (Cai, 2007; Cai & Wang, 2010). These views form the foundation of their philosophy of mathematics, although some teachers may not have clear philosophies (Ernest, 1989). Three philosophies of mathematics related to teachers' conceptions or views of the nature of mathematics are distinguished by their observed occurrence in mathematics teaching (Beswick, 2012; Ernest, 1989). The first is the instrumentalist view, which considers mathematics a collection of facts, rules, and skills used for superficial purposes. The second is the Platonist view, which views mathematics as a fixed and unified body of knowledge discovered rather than created. The third is the problem-solving view, which views mathematics as a constantly evolving field of human creation and innovation, rather than a completed product, with its outcomes subject to modification. **Table 1** summarizes these perspectives.

Models of Mathematics Teaching

The way mathematics is taught depends on the teacher's understanding of the teaching roles, actions, and classroom activities associated with it (Ernest, 1989). Thompson (1992) identified several traits that reflect a teacher's conception of mathematics teaching, including the goals of the mathematics program, teacher's role, student's role, appropriate classroom activities, instructional approaches, mathematical procedures, and acceptable outcomes. Differences in teachers' views on mathematics are linked to variations in their perspectives on teaching it. Multiple models or perspectives have emerged from research in mathematics education to describe teachers' teaching beliefs (Xie & Cai, 2018). Ernest (1989) suggested three teaching models based on teachers' roles and the intended outcomes of instruction. The models used are listed in **Table 2**. The instructor teacher model emphasizes student performance and the mastery of mathematical rules and procedures in mathematics instruction. The explainer teacher model focuses on a conceptual understanding of the content itself. The facilitator teacher model centers on learners' personal construction of mathematical knowledge in instruction.

Models of Mathematics Learning

Teachers' perceptions of mathematics teaching are closely linked to their beliefs about the mathematics learning process. This includes their understanding of how mathematics is learned, the mental activities and behaviors involved in the learning process, and what activities are considered suitable for effective learning, as stated by Ernest (1989) and Thompson (1992). Ernest (1989) identified two crucial concepts that determine the models of learning mathematics: first, whether learning is viewed as an active process of constructing knowledge that is interconnected and meaningful or as a passive reception of knowledge; and second, whether learning is viewed as developing autonomy and interest in the learner or as creating a submissive and compliant learner. Based on these constructs, four simplified models of learning mathematics can be outlined: a model that emphasizes compliance and skill mastery, a model that focuses on the reception of knowledge, a model that promotes active construction of understanding, and a model that encourages the exploration and autonomous pursuit of one's interests. **Table 3** summarizes the different views on mathematics learning and their corresponding models.

Table 4. Teachers' beliefs systems (adapted from Beswick, 2012 & Ernest, 1989)

Beliefs about nature of mathematics	Teacher's role	Beliefs about mathematics teaching	Beliefs about mathematics learning
Instrumentalist	Instructor	Content-focused with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Explainer	Content-focused with an emphasis on understanding	Active construction of understanding
Problem-solving	Facilitator	Learner focused	Autonomous exploration of own interest

Belief systems refer to how an individual's beliefs are organized around a particular idea or object. These systems consist of primary or derivative beliefs that can be central or peripheral. It is important to note that teachers' beliefs do not exist in isolation, and are often clustered together (Philipp, 2007). Belief systems are dynamic and can change as individuals evaluate their beliefs about their experiences (Thompson, 1992). **Table 4** illustrates how these beliefs are connected, with the beliefs in the same row being theoretically consistent and those in the same column forming a continuum. However, according to some researchers, individual teachers' beliefs may not fit into a single category.

THE PROBLEM

The educational literature contains diverse interpretations and definitions of the beliefs/conceptions held by mathematics teachers (Beswick, 2012; Philipp, 2007; Thompson, 1992). Beliefs refer to the psychological understandings, premises, or propositions that individuals consider true, while conceptions encompass general ideas or mental frameworks that include beliefs, meanings, concepts, rules, mental images, and preferences (Philipp, 2007). Teachers' knowledge systems heavily rely on their beliefs.

Reform efforts have been undertaken by the Ministry of Science and Higher Education (MoSHE) to expand HEIs to achieve quality and access through the reform of undergraduate programs. Most of the reform efforts, such as changing the mode of delivery from parallel to a modular system and increasing the number of years from three to four of the Bachelor of Science Program, are happening at the institutional level. These changes aim to improve the quality of education and provide students with more opportunities for practical experiences. MoSHE has also been working on improving access to higher education by increasing the number of institutions and expanding their capacity. Overall, these reform efforts are crucial to ensure that students receive high-quality education that prepares them for success in their future careers.

However, reform efforts that ignore individual teachers' preparation and professional development are unlikely to achieve the required objectives. There should be an effort to work on teachers' professional development and their beliefs/conceptions to implement a program as planned. Key stakeholders paid little attention to the professional development of individual teachers, especially when institutional reforms focused only on curriculum changes. This lack of attention to teacher development can lead to resistance and ineffectively implementing new programs. Teachers need ongoing support and training to integrate new curricula and teaching methods into their classrooms successfully. Professional development should be tailored to individual teachers' needs and include opportunities for collaboration and reflection. Additionally, it is important to address teachers' beliefs and conceptions about teaching and learning, as these can affect their willingness to embrace change. By prioritizing teachers' professional development, education reform efforts can ensure successful implementation and improve student outcomes.

Researchers have developed theories on teachers' beliefs and how they influence classroom teaching. However, they developed these theories mainly by studying pre-service, in-service, and practicing teachers in elementary and secondary schools (Speer, 2008). Usually, the researchers were primarily university teachers but had not investigated their own beliefs about the nature of mathematics, teaching, and learning. In addition, they did not examine how these beliefs affected teaching and student learning in HEIs. In Ethiopia, little is known about university mathematics teachers' conceptions of practical mathematics teaching concerning the nature of mathematics, mathematics teaching, and mathematics learning. This lack of research leaves a gap in our understanding of how mathematics is being taught at the university level in Ethiopia. Without this knowledge, it is difficult to make informed decisions on how to improve the quality of mathematics education. It is important for researchers to investigate the beliefs and practices of university mathematics teachers to gain insight into how they approach teaching and learning.

Effective teaching is a crucial aspect of higher education, particularly in mathematics. This study examines the beliefs about the effective teaching of university teachers in mathematics from a higher education perspective. The results will significantly contribute to our understanding of mathematics teaching and the belief systems of university teachers. This study addressed the following two research questions:

1. What do university mathematics teachers consider effective mathematics teaching?
2. What common beliefs do university mathematics teachers have about teaching mathematics?

METHOD

Research Design

The study used a descriptive case study to understand university mathematics teachers' beliefs about the effective teaching of mathematics instead of judging and evaluating their beliefs. Educational literature uses several approaches to investigate

teachers' belief systems (Mosvold & Fauskanger, 2013; Philipp, 2007). The present study involved qualitative research because it enables researchers to elicit participants' inner experiences to help them extract and explore meanings across and within cultures instead of manipulating experimental variables (Corbin & Strauss, 2014). The utilization of qualitative research methods enabled us to delve deeper into understanding the belief systems of teachers, encapsulating their individual perspectives and interpretations. Through the exploration of these internal experiences, we were able to unveil the multifaceted elements that mold teachers' convictions about effective mathematics teaching. These elements encompass their educational backgrounds, personal experiences, and cultural influences.

This methodology offers invaluable insights, shedding light on the diverse array of beliefs harbored by teachers. It contributes significantly to fostering a more nuanced and comprehensive understanding of effective mathematics instruction. The meticulous examination of individual and collective belief systems illuminates the underlying principles and practices perceived as effective in the realm of mathematics teaching. It draws attention to the inherent variability in pedagogical beliefs, reflecting the richness of the instructional tapestry woven by teachers from varied backgrounds and experiences.

By capturing the subtle intricacies of teachers' perceptions and beliefs, qualitative research serves as a powerful tool to inform and refine educational strategies and policies, ensuring they are resonant and equitable, catering to the evolving needs and expectations of both teachers and students in the domain of mathematics education.

Instead of judging or evaluating teachers' beliefs, the study aimed to understand their perspectives on effective mathematics teaching. A descriptive case study allows for in-depth exploration of a particular context, Assosa University in this case, without imposing external standards or predetermined criteria. This aligns with the desire to discover and appreciate the unique tapestry of beliefs woven by these specific teachers. The study focused on eliciting and unpacking participants' inner experiences, which quantitative methods would not capture effectively. Qualitative research, specifically in-depth interviews in this case, enables the researchers to dive deep into individual worldviews, understanding the reasons and influences behind beliefs. This fosters a nuanced comprehension of their educational philosophy and teaching practices.

Study Group

Five mathematics teacher, teaching the course "applied mathematics I" for engineering students at Assosa University, voluntarily contributed to the study in the 2020/2021 academic year. These participants were selectively chosen for the research, rooted in criteria such as their experience, expertise, and willingness to participat. Each had preceding experience instructing "mpplied mathematics I" to pre-engineering students and delivering "applied mathematics" lessons to professional engineers.

Out of seven teachers responsible for teaching the course, two were not included in the selection; one, a recent BSc degree recipient, was excluded due to insufficient teaching experience. Another teacher, who was female, was currently on maternity leave. Consequently, the research was confined to the participation of five teachers. This careful selection ensured that the study was enriched with profound insights and reflections from teachers with substantial exposure to and understanding of applied mathematics, thereby facilitating a more nuanced exploration of teaching methodologies and belief systems in applied mathematics instruction within the engineering domain.

The identities of the participating teachers were anonymized and coded as T_{01}, T_{02}, T_{03}, T_{04}, and T_{05} to maintain confidentiality. All participants in this study were male and held academic positions as lecturers. Of the participants, T_{02} and T_{04} possessed bachelor's degrees in mathematics education, while T_{01}, T_{03}, and T_{05} earned their undergraduate degrees in applied mathematics. Moreover, each of the five teachers held a master's degree in applied mathematics, each with differing specializations.

The teachers brought varied levels of teaching experience to the study, with some having imparted knowledge for over a decade and others relatively new to the educational field. This diverse range of experience added a rich, multifaceted perspective to the study, allowing for a more comprehensive exploration and understanding of the diverse teaching methodologies and belief systems in applied mathematics instruction. This diversity in academic background, specialization, and experience enabled a thorough and nuanced exploration into the intricacies of teaching applied mathematics, providing diverse insights and reflections pivotal to the research's objective of understanding effective mathematics instruction methodologies within the context of engineering education.

Data Collection

Data about the beliefs of mathematics teachers regarding effective teaching of mathematics was collected through semi-structured interviews. Ernest's (1989) concepts of belief systems of mathematics teachers were utilized as a framework for designing and analyzing the data in this study. The researchers adapted interview questions from Cai (2007) and grouped them into three categories:

(1) questions about the nature of mathematics, including its substances, acceptance of truth, and abstractness,

(2) questions about teaching, such as the qualities of an effective teacher and lesson, and

(3) questions about learning, including the definition of understanding and the role of memorization and practice in developing students' understanding.

Each of the five teachers was interviewed using semi-structured questions to understand their views on the nature of mathematics, mathematics teaching, and mathematics learning processes. Semi-structured interviews allow us to understand not only what teachers believe but also why they hold their beliefs. The first author conducted interviews, which lasted between nine and 23 minutes each, and the transcripts were three-six pages long. The interviews were audio-recorded and transcribed verbatim.

Table 5. Teachers' views about mathematics

Mathematics is	T_{01}	T_{02}	T_{03}	T_{04}	T_{05}
Knowledge from real life	*	*	*	*	*
Practical in daily life and in sciences	*	*	*		
Relevant to solve real life problems	*	*	*		
Science of logic		*		*	
Way of thinking		*			
Study of numbers			*	*	
Described by its contents [e.g., geometry, algebra, etc.]			*	*	*
Abstract	*	*			

Note. *Indicates that teacher has a corresponding statement

Data Analysis

In conducting this study, we employed a meticulous three-phase approach to scrutinize the transcribed data derived from our interactions with university mathematics teachers. This strategy, entrenched in validated research methodologies, facilitated the exploration and categorization of teachers' philosophical convictions concerning mathematics and its associated instructional and learning processes. Our systematic method enabled the identification of recurrent themes and patterns within the teachers' feedback, yielding insightful perspectives on efficacious mathematics teaching strategies.

The rigorous analysis executed in this study ensures the reliability and replicability of our findings, significantly bolstering the overall validity of our research outcomes. The structured analytical framework adopted in this research, not only unraveled the intricate tapestry of teachers' beliefs and understandings but also provided a robust foundation for discerning the pedagogical implications of these beliefs in the realm of mathematics education.

In the succeeding paragraph, the distinct procedures executed during each phase of our meticulous analysis are delineated in detail, providing a clear and comprehensive overview of our systematic approach to data interpretation and synthesis.

We employed three phases for the coding and analysis of transcribed data. First, we began with an open coding approach, a process of developing categories of concepts and themes emerging from all transcribed data, making no prior assumptions (Corbin & Strauss, 2014; Miles & Huberman, 1994; Saldaña, 2016) (open coding). Second, we re-examined all the data using a start list of codes developed to specifically address research on university mathematics teachers' beliefs about mathematics and the closely related beliefs on the teaching and learning of mathematics (axial coding). This phase involves comparing and contrasting codes to identify similarities and differences, leading to a more nuanced understanding of the data. Finally, we examined common themes as teachers expressed in their responses during the interviews (selective coding). This final phase involves selecting the most significant and relevant themes that emerged from the previous phases.

To ensure the trustworthiness of the qualitative data on mathematics teachers' beliefs, the researchers employed several meticulous measures. Firstly, they grounded their study in Ernest's (1989) established framework for understanding teacher belief systems, providing a robust foundation for data interpretation. Additionally, by adapting established interview questions from Cai (2007) and categorizing them into specific domains (nature of mathematics, teaching, and learning), the researchers ensured consistency and focus while allowing for flexibility to probe deeper into individual teachers' perspectives. Furthermore, limiting the interview sample to five participants enabled in-depth exploration of each teacher's views, minimizing the risk of superficial conclusions. Utilizing semi-structured interviews facilitated delving beyond what teachers simply believe to understand the "why" behind their viewpoints, enriching the data with context and rationale. Lastly, the verbatim transcription and audio recording of interviews created a reliable and transparent record for subsequent analysis, enhancing auditability and confirmability of the findings.

We considered ethical issues when collecting and analyzing data and disseminating reports. The participating teachers signed an informed consent form. We collected the data by developing rapport with the study participants and maintaining trust by explaining that the study would not harm them. Regarding the confidentiality and involvement of any deception activity, we shared our notes and all information with the participants to avoid misinterpretation of the data. We used pseudonyms to protect the identity of the participants and assured them that the information would be used exclusively for research purposes.

RESULTS

The presentation of the results was divided into three categories: teachers' beliefs regarding the nature of mathematics, their convictions about teaching mathematics, and their beliefs about how mathematics is learned. For mathematics teaching to be effective, teachers should possess all three belief systems.

Teachers' Beliefs About Nature of Mathematics

The participating teachers shared their opinions on the definition of mathematics, stating that it is a logical and necessary component of daily life that involves a specific way of thinking. However, there is a range of responses from teachers when asked to answer the same question. Some define mathematics in terms of practical application, as seen in **Table 5**, which outlines the perspectives of the five university mathematics teachers. Typically, when discussing the nature of mathematics, teachers highlight their relationship with real-world problems and how they can be used as a useful tool for efficient problem solving. They believed that mathematical knowledge is acquired through real-world problems involving numbers.

The five teachers reached a consensus that mathematics is derived from real-life situations. Three participants emphasized the practicality and relevance of mathematics in everyday life. T_{02} expressed that mathematics is vital for comprehending real-life problems and provides a means of solving them. T_{01} viewed mathematics as a science that measures everyday activities, making them both quantifiable and practical. T_{03} described mathematics as a tool for analyzing reality in other subjects. Although T_{01} acknowledged mathematics as an abstract subject, it was also a science of logic that promoted critical and logical thinking. T_{04} concurred with this view by stating that mathematics is a science of logic that enhances logical reasoning skills. Mathematics is closely linked to calculations and involves the study of numbers. One interviewee stated that mathematics primarily involves the study of numbers (T_{03}), whereas another called it a scientific field that focuses on numbers and their calculations (T_{04}). Some participants defined mathematics based on its content, such as geometry, algebra, and calculus, as mentioned by the two interviewees (T_{03} and T_{05}).

About the substance of mathematics, it is described by teachers in two ways: either by its content, such as algebra and geometry, or as a discipline that includes various methods and applications. Some teachers view algebra, geometry, and calculus as the substance of mathematics with practical uses in real life (T_{03}, T_{04}, and T_{05}). However, others see the substance of mathematics as a problem-solving process, way of thinking, and the use of formulas that require scientific skills and rationality (T_{01} and T_{02}).

All teachers disagreed with the statement that "*some people believe that many things in mathematics should simply be accepted as true and remembered without explanation*" when it was presented to the participants. They asserted that not all mathematical concepts are accepted without scrutiny, especially at the university level, where students must remember learned concepts. While memorization is crucial for saving time, the verification of these concepts is necessary.

Knowledge acquisition is crucial for mathematics learning. Teachers share this belief and consider mathematics to be a science of reasoning in which facts are proven to be true. It is not sufficient to simply accept concepts as true; they must be proven through axioms and postulates (T_{02}). The importance of proof and verification in mathematics was emphasized by some teachers who stressed the need to justify and think logically (T_{01} and T_{02}). The validity of the theorems must also be checked by proving or disproving statements (T_{01} and T_{05}). Overall, the interviewed teachers highlighted the significance of the learning process, proof, and verification of mathematics.

The acceptance of truth in mathematics is closely tied to the philosophy of mathematics and nature of mathematical concepts. Mathematics is viewed as a constantly evolving field of human creativity and invention–a product of culture. It is a process of inquiry and discovery, not a finished product, and its findings are subject to revision (Ernest, 1989). One teacher interviewed (T_{03}) believed that:

> … not everything in mathematics can be accepted as true and memorized, as humans create mathematics and are fallible. Instead of blindly accepting mathematical formulae, identify errors and make improvements. If an error is found in an accepted formula, it must be changed.

The participating teachers had varying views on whether mathematics was an abstract subject or not. Some believe that mathematics is not abstract and that there are simple concepts in the subject (T_{01}, T_{03}, and T_{05}). Others have acknowledged that mathematics may appear abstract, but it is actually a reality that can be observed (T_{02} and T_{04}). One teacher also mentioned that the abstract nature of mathematics may come from the methods or techniques used to teach it (T_{02}).

Teachers' Beliefs About Mathematics Teaching

During the discussions, the attendees shared their opinions on teaching mathematics, focusing on both the teaching methods and teachers themselves. They agreed there was no universally accepted standard for effective mathematics teaching. The interviewed teachers talked about the qualities of a successful teacher, the features of an ideal lesson, and the appropriate use of manipulatives and concrete models.

The participants held strong opinions about the qualities that make mathematics teachers more effective. They based their ideas on their own experiences, those of their colleagues, and beliefs about what constitutes good teaching in school or university settings. They unanimously agreed that a thorough understanding of mathematics was crucial for effective teaching. One teacher summed up the characteristics of an effective teacher as being well-prepared, knowledgeable about the topic, able to explain concepts, and able to focus on the needs of individual students (T_{01})

During conversations with the teachers who participated in this study, certain qualities were identified as important for an effective teacher. These included having a strong grasp of both what to teach and how to teach it, possessing good character, and adhering to a code of ethics. Some also noted that effective teachers should be punctual, prepared, and treat all students equally without discrimination. Additionally, they should serve as role models by exhibiting positive personal qualities and using a student-centered approach in the classroom. Finally, some suggested that teachers wear teaching gowns.

In addition, teachers have identified certain qualities that make mathematics lessons more successful. These include the use of clear and comprehensible teaching methods, incorporation of straightforward visual aids, and focus on student engagement, gradual introduction of concepts from basic to advanced, and demonstration of how these concepts connect to real-world situations. Another important aspect of successful mathematics lessons is teachers' ability to create a positive and supportive learning environment. This can be achieved through effective classroom management, encouraging student participation and collaboration and providing opportunities for students to ask questions and receive feedback. In addition, teachers should strive to make mathematics lessons relevant and meaningful to their students by incorporating real-life examples and applications.

Teachers' Beliefs About Mathematics Learning

In discussions about mathematics learning, the participants emphasized the importance of understanding and how teachers could facilitate this. They also considered the roles of memorization, practice, and concrete experiences. **Table 6** presents a summary of the teachers' perspectives on mathematics learning, all of whom agreed that understanding is a crucial aspect. However, there are varying views among teachers regarding what constitutes understanding and promoting it among students. One teacher defined understanding as gaining an authentic or common sense perspective of what needs to be known or understood (T_{01}).

Gaining a comprehensive understanding of mathematics is crucial for both students and for achieving educational objectives. All informants believed that learning mathematics with understanding was essential. One informant emphasized the importance of understanding in learning mathematics, stating that it's enjoyable to delve into the topic, but without paying close attention and comprehending it, one can grow to dislike it (T_{03}). Another participant, T_{04}, added that:

> … Understanding is fundamental, and that memorization or cramming is unnecessary. For instance, if one grasps the concept of integrating or derivating two functions, they can apply it even if the function changes owing to an exponent or coefficient modification. Failure to understand this concept can lead to mathematical failure.

Teachers have different approaches to helping students understand mathematics. Some believe that it is crucial for students to grasp definitions and examples to comprehend mathematical concepts. Teacher T_{04} emphasized the importance of understanding the definition, as it serves as a foundation for everything else. Once the definition is understood, its application becomes easier. Teacher T_{05} suggests that teachers should explain the definition and theorem briefly and then provide proofs and additional examples to elaborate on them. To ensure effective learning, teachers should focus on teaching fundamental concepts relevant to students' prior knowledge. Avoid repetition and irrelevant materials. Instead, teachers should emphasize building a strong foundation for basic knowledge through frequent and thorough instruction (T_{01}).

To facilitate students' understanding and learning of mathematics, it is important for teachers to ensure that the concepts are understood. This can be achieved by selecting teaching methods tailored to the students' level of understanding. T_{02} suggested that teachers should be knowledgeable about their students' understanding levels and use techniques to simplify the lesson. By doing so, the lesson becomes more manageable for learners. One of the participants (T_{03}) suggested that, to help students learn mathematics with understanding, they should be encouraged to pay more attention to the lesson and relate the concepts to real-life problems.

When asked about concrete experiences in teaching mathematics, all participants reacted positively, particularly in primary schools. However, at the university level, there has been a shift towards abstract mathematical thinking. Despite this shift, teachers still should incorporate concrete experiences into their lessons to help students understand complex mathematical concepts. Teachers must be skilled in selecting materials and activities to engage their students and facilitate learning. In doing so, they can help students develop a deeper understanding of mathematics and build a strong foundation for future learning. Ultimately, concrete experiences are an essential tool for effective mathematics instruction at all levels of education.

The literature has presented two perspectives on memorization: the ability to recall relevant information and the connection to repetitive learning and practice. However, in this study, the teachers did not make a clear distinction between these two perspectives, and instead emphasized the importance of memorization in learning mathematics and achieving student understanding. The participants acknowledged that memorization is crucial for higher education students, who have learned a significant amount of mathematics and cannot constantly verify everything. According to T_{01}, memorization is an important aspect of learning mathematics because not all mathematical formulas can be proven or demonstrated. Therefore, memorizing them is necessary instead of constantly testing and proving theorems and formulas (T_{01}). Another teacher (T_{04}) added that memorization is useful for students to remember formulas and definitions. One teacher (T_{02}) emphasized the importance of memorization by capitalizing on the term when he said:

> Acquiring mathematical knowledge relies heavily on memorization abilities. Memorization involves retaining and collecting information. Learners must memorize the fundamental principles that serve as the foundation for each lesson. By establishing a connection between these principles and the current topic, students can make the learning process less complex.

Mathematics involves numerous formulas that require a certain level of memorization for effective learning. According to one participant, knowing the formulas is crucial for problem-solving; thus, students need to memorize them along with basic facts (T_{03}). Another participant noted that while memorization is important for learning mathematics, understanding concepts is more beneficial for meaningful learning, particularly at higher levels (T_{05}). Understanding the concepts behind the formulas allows for a deeper understanding of the mathematical principles and their applications in real-world scenarios. This is why some educators advocate a more conceptual approach to teaching mathematics, where students are encouraged to explore and discover mathematical concepts through problem-solving and critical thinking.

Practice is an indispensable tool in learning mathematics; as the saying goes, "practice makes perfect." This is because practice helps students to understand and master the subject. Teachers unanimously agreed that practice is vital in mathematics education. Through repeated practice, students can easily remember and apply what they have learned. For instance, repeatedly solving or evaluating limits can help students perfect their skills and make them a habit (T_{02}). One teacher also noted that practice helped students engage with formulas and improve their ability to memorize them (T_{01}). Practice allows students to develop their problem-solving skills and critical thinking abilities. By encountering various types of problems, students can learn how to

approach each problem systematically and logically. This not only helps them in mathematics but also in other subjects and real-life situations. In addition, practicing math regularly can boost students' confidence and reduce their anxiety regarding the subject. It can also improve speed and accuracy, which are crucial in timed tests such as standardized exams. With consistent efforts and dedication, students can achieve mastery in mathematics and excel academically.

One participant emphasized the significance of learning-by-doing in mathematics, stating that practice is crucial for students to grasp the subject (T_{04}). According to him, students learn through practice and repetition. Another interviewee stressed the importance of practice in mathematics, stating that students need to practice a concept multiple times to understand it (T_{05}). Lastly T_{04} added that, mathematics requires more practice than any other subject due to the multitude of formulas and the need to remember previous lessons. The interviewee concluded that practice plays a vital role in students' mathematics learning. Besides emphasizing practice, both interviewees highlighted the importance of understanding mathematical concepts. T_{04} noted that simply memorizing formulas is not enough and that students must understand the underlying principles to apply them effectively. T_{05} echoed this sentiment, stating that students should focus on understanding the reasoning behind a concept rather than merely memorizing steps. Both interviewees mentioned the value of problem-solving in mathematics education.

Upon analyzing the general outlook of teachers regarding the nature of mathematics, including its teaching and learning, it becomes apparent that they are of the view that their beliefs about the nature of mathematics influence their mental constructs of how mathematics should be taught and learned. This is a significant finding as it highlights the importance of understanding the underlying beliefs that teachers hold about mathematics. It also emphasizes the need for teacher training programs that focus on helping teachers develop a deeper understanding of the nature of mathematics. In doing so, teachers will be better equipped to design effective instructional strategies that align with their beliefs about mathematics. Additionally, this understanding can help teachers identify potential misconceptions about mathematics and work towards correcting them. Ultimately, this will lead to more effective teaching and learning experiences for students, as well as a greater appreciation for the beauty and complexity of mathematics.

DISCUSSION

This study investigated teachers' beliefs regarding the nature of mathematics, the characteristics of teaching mathematics, and learning mathematics. Despite the differences in their backgrounds, the teachers shared similarities in their responses. They perceived mathematics as useful, rational, advantageous, and connected to their reasoning. Mathematics educators believe that one objective of studying mathematics in HEIs is to develop advanced mathematical and formal operational thinking, which requires teachers to guide students from tangible examples to theoretical concepts. However, the participating teachers did not mention advanced mathematical thinking as their goal.

The teachers who were studied have a perspective that regards mathematics as a dynamic and coherent system of knowledge refined through the resolution of real-world problems and is therefore useful in solving real-world issues. They recognize that mathematics is not an abstract subject and must be connected to real-life situations to address the most pressing concerns of humanity. Consequently, they assert that understanding mathematics is a crucial objective of education, and teachers must use various methods to assist their students in learning mathematics with understanding. Ernest (1989) proposed three philosophies for understanding the nature of mathematics: instrumental, platonic, and problem-solving. These philosophies can help teachers to identify their own perspectives on mathematics education. Those who subscribe to instrumentalism are more interested in the practical applications of mathematical knowledge in the external world, while Platonists emphasize the intricacy of the internal structure of knowledge itself. The problem-solving approach is based on the notion that mathematics is not a finished product but a result that is subject to change.

The teachers involved in the study had three views about the nature of mathematics: instrumental, problem-solving, and Platonist. Their instrumental view is reflected in their tendency to encourage students to solve problems practically (Cai, 2004). Teachers' problem-solving perspective is evident in their emphasis on the importance of revision, questioning, verification, and proof in mathematics. Platonist viewpoint of teachers is demonstrated by their focus on the coherence of mathematics and their encouragement of students to use generalized processes to solve problems. Teachers' beliefs about teaching and learning mathematics are consistent with their respective views, as presented in **Table 4**, because beliefs in the same row are thought to be theoretically consistent.

According to the participants, it is crucial to learn mathematics with understanding, and accomplished teaching should always promote this. Memorization, practice, and concrete experience are not considered good or bad but are necessary prerequisites for learning mathematics with understanding. Effective teaching practices utilize these means to make learning understandable. This study also raises questions about the intentions of university mathematics teachers. The main goal should be for students to acquire a system of knowledge, whether through remembering and other learning strategies or by focusing on facilitating mathematical inquiry (Burton, 2004). However, the rhetoric of mathematics education does not match the reality in Ethiopian public universities, where building mathematical concepts and progressing through mathematical thinking is emphasized, but not always implemented.

The participants asserted that while there is consensus on the criteria for evaluating good teachers and lessons, some may not meet these standards in their professional practice. The primary factors in being an effective teacher are having expertise in the subject matter and being proficient in its application. As for practical lessons, teachers believe that they should be centered on the students, understandable, easy, and concise, and progress from a tangible to a conceptual level of cognition. However, good teaching extends beyond subject matter expertise and practical lesson planning. Effective teachers also possess strong

communication skills, empathy, and the ability to adapt to various learning styles. They create a positive classroom environment that fosters student engagement and encourages critical thinking skills. Additionally, good teachers continuously reflect on their teaching practices and seek feedback from their students to improve. Teachers also should stay up-to-date with the current research and best practices in education. Ultimately, being a good teacher requires a combination of knowledge, skills, and personal qualities that enable them to inspire and guide their students towards success.

In this study, two significant theoretical perspectives on the belief system of mathematics teachers were identified, which were proposed by Ernest (1989). The study supports Ernest's (1989) argument that teachers' beliefs regarding the nature of mathematics serve as the foundation for their mental models of teaching and learning mathematics. Moreover, it was found that university mathematics teachers' beliefs about mathematics as a real-life knowledge source have a noticeable impact on their mental models of teaching and learning mathematics, where they highlight the relationship between mathematical concepts and real-life situations. The research also validates Ernest's (1989) assertion that teachers' ideas about teaching and learning mathematics are influenced by their social context and the limitations and opportunities presented by their knowledge.

The study highlights the importance of understanding the perspectives and experiences of mathematics teachers, as well as the impact of their beliefs on their teaching practices. It also emphasizes the need for ongoing professional development that supports teachers in developing a deeper understanding of mathematics as a source of real-life knowledge. Additionally, this study underscores the importance of creating a supportive learning environment that encourages students to make connections between mathematical concepts and real-life situations. Overall, this research provides valuable insights into the complex interplay between teachers' beliefs, social contexts, and knowledge in shaping their approach to teaching and learning mathematics.

CONCLUSIONS & IMPLICATIONS

Our study examined teachers' perspectives on effective mathematics teaching and learning, covering various aspects of mathematics such as practicality, logic, usefulness, and thought processes. The participants believed that memorization, practice, and concrete experiences could enhance their understanding of mathematics. Effective teaching practices enable students to learn mathematics through comprehension. Classroom activities are influenced by cultural, social, and individual factors. To promote learning and understanding, teachers must establish a suitable sociocultural and psychological environment. Additionally, teacher professionalism is critical for making informed decisions regarding all aspects of classroom learning and teaching.

To ensure effective mathematics teaching and learning, it is important for teachers to explicitly state their beliefs about the nature of mathematics and its teaching. Understanding teachers' beliefs is crucial to implementing mathematics education reforms that can enhance student learning. However, the teachers who participated in this study felt that the numerous reform efforts were not based on a bottom-up approach, and they had no input in the reforms. The study focused on the teachers' perspectives on the effectiveness of mathematics teaching, and the researchers found similarities and minor variations in the belief systems of the five participants. Further research is necessary to ensure that these similarities and differences are sustainable among university mathematics teachers. By involving teachers in the reform process and understanding their perspectives on effective teaching practices, we can create a more collaborative and successful educational system.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that the study, being part of a PhD dissertation, received approval from the ethics committee of Addis Ababa University on February 2, 2020. It followed ethical guidelines and obtained written informed consent from the five participating teachers. The authors further stated that the participants were also informed of their right to withdraw consent without facing any adverse repercussions.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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