Mathematics teachers’ perceptions on general pedagogical knowledge for teaching pre-engineering students

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ABSTRACT
This study sought to investigate university mathematics teachers' perceptions on their general pedagogical knowledge (GPK) while teaching an applied mathematics course tailored for pre-engineering students at a public university in Ethiopia. Using a case study approach, data were collected through a Likert-scale questionnaire and semi-structured interviews. Five mathematics teachers were selected using purposive sampling to investigate into their perceptions about their capabilities and the teaching methodologies they used. Quantitative data were analyzed using SPSS statistics 22, while qualitative data were analyzed thematically. Teachers exhibited diverse levels of GPK. On average, the study revealed that the teachers held a positive self-assessment of their GPK. While some teachers were assured of their skills, others felt they needed more professional development in this domain. The findings of the study suggest that teachers’ perceptions on GPK are pivotal for understanding the professional development requirements of mathematics teachers, thereby guiding endeavors to enhance undergraduate mathematics instruction.

Keywords: general pedagogical knowledge, perception, learning activities, university mathematics teachers, classroom management

INTRODUCTION

Mathematics is a multifaceted and demanding subject. Effectively teaching it necessitates a profound understanding of the content and the pedagogy (National Council of Teachers of Mathematics, 2014). Central to successful teaching is the individual teacher’s knowledge and skill, which address the cognitive challenges of instruction (Chew & Cerbin, 2021; Walshaw, 2012). Ball et al. (2008) emphasize that effective mathematics instruction requires teachers to deeply understand the content they teach. Furthermore, teachers must employ strategies that promote inclusive mathematics learning for all students (National Council of Teachers of Mathematics, 2014). Essential to this is the ability to tailor instructional techniques to accommodate students’ varied needs and foster a supportive, inclusive learning atmosphere.

Teachers are pivotal in determining instructional quality, with specialized domain knowledge being a prized asset. Beyond mastering their subjects, teachers must adeptly convey and engage students in learning. In a seminal work, Shulman (1986) identified categories of teacher knowledge that form the professional foundation for teachers. Specifically, Shulman (1986) identified three primary dimensions crucial for exemplary teaching: content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK). CK encompasses subject matter knowledge, PCK involves teaching subjects relative to students’ pre-existing knowledge, and GPK centers on universal teaching strategies and learning processes.

König’s (2022) entry on “pedagogical knowledge” in the Encyclopedia of Teacher Education clarifies: Pedagogical knowledge, understood as universally applicable teacher knowledge across subjects, is termed “general pedagogical knowledge” to distinguish it from subject-specific “pedagogical content knowledge” (Shulman, 1987). König’s (2014) OECD review also introduced “pedagogical/psychological knowledge” (PPK) as an interchangeable term for GPK. This research utilizes “general pedagogical knowledge” synonymously with “pedagogical knowledge,” emphasizing teaching techniques applicable across multiple disciplines and contexts.

The international review of studies conducted for the OECD by König (2014) also discussed PPK as another term that refers to GPK and is sometimes used in the literature. In the present study, we use “general pedagogical knowledge” as a synonym for...
“pedagogical knowledge.” It is important to note that both terms encompass the understanding and application of teaching methods, strategies, and principles that are relevant across various subject areas and educational contexts.

Contrary to CK and PCK’s subject specificity, there is consensus on GPK’s universality across subjects in the professional teacher knowledge base (König, 2022). GPK stands as the foundational, universally applicable teaching knowledge (Ulferts, 2019). It encapsulates broad teaching strategies beyond specific subjects and addresses classroom management, student learning comprehension, assessment, and the broader educational environment (Shulman, 1987). Guerriero’s (2017) definition captures GPK’s breadth: GPK represents teachers’ expertise in creating universally effective teaching and learning environments.

Shulman’s (1986) teacher knowledge framework posits CK, PCK, and GPK as essential for exceptional teaching. Still, research predominantly focuses on CK and PCK, leaving GPK understudied (König, 2022; Trinidad-Velasco & Reyes-Cárdenas, 2020). Some scholars questioned the distinctions between PCK and GPK, perhaps contributing to the research gap. Yet, recent evidence confirms GPK’s unique dimension in teacher knowledge, accentuating the urgency for more GPK-centric studies (Hartati et al., 2019; König et al., 2018, 2022; Ulferts, 2019). One suggestion for GPK research shortfall might be the limited evidence linking it to superior teaching and student success (Ulferts, 2019). However, GPK concept has now been sharpened and solidified (Trinidad-Velasco & Reyes-Cárdenas, 2020).

The relevance of GPK, especially for mathematics teachers, has attracted recent attention. GPK has been identified as crucial for potent teaching and optimal student outcomes (König, 2014; Ulferts, 2019). Ulferts’s (2019) OECD meta-analysis concluded that strong GPK correlates with elevated teaching quality and improved student results. In the OECD’s ‘teaching as a knowledge profession’ (Ulferts, 2021), GPK is highlighted as an essential tool for navigating contemporary teaching challenges. Teachers equipped with robust GPK are better prepared to optimize learning opportunities, pinpoint student misconceptions, and tailor their instructional approaches.

GPK research has mainly concentrated on primary and secondary school teachers from diverse fields. However, university teachers have been largely overlooked, despite GPK’s acknowledged importance for this cohort (Ulferts, 2019). Assessing university mathematics teachers’ (UMTs) self-perceived GPK, especially for the ‘applied mathematics I’ course designed for pre-engineering students, is vital. It is imperative to delve deeper into UMTs’ self-perception about their GPK.

Existing research has scrutinized GPK of mathematics teachers and its interplay with other professional competencies, such as mathematical content knowledge, mathematical pedagogical content knowledge, and self-efficacy beliefs. Yet, further exploration is required, especially concerning university-level mathematics teachers, who play an integral role in molding the mathematical capabilities of future specialists. Expanding research in this domain promises invaluable insights into the specific challenges and prerequisites of tertiary-level mathematics instruction.

This study seeks to shed light on UMTs’ self-perceptions of their GPK, specifically in the context of “Applied Mathematics I.” This research aims to enhance the discourse on mathematics education, offering meaningful guidance for teacher training initiatives and curricular advancements. Central to our inquiry is the question: How do UMTs perceive their general pedagogical knowledge? By investigating five UMTs’ self-perceived GPK, this research endeavors to discern the professional development requirements for UMTs and promote undergraduate mathematics education quality.

**METHODOLOGY**

To examine UMTs’ GPK perceptions, we adopted the case study approach. Utilizing both quantitative (Likert-type scale questionnaire) and qualitative (semi-structured interviews) data collection techniques, the study sought to provide a comprehensive overview. The initial survey gauged UMTs’ self-assessed GPK. Participants responded to statements covering diverse GPK facets, ranging from lesson design to assessment methodologies. The subsequent semi-structured interviews allowed UMTs to expand on their survey responses, lending depth to their perceived GPK strengths and growth areas.

**Sample**

Five Assosa University, Ethiopia, mathematics teachers participated in this 2020/2021 academic year study, focusing on self-perceived GPK during their “applied mathematics I” course instruction for pre-engineering students. Out of seven potential candidates, one was excluded due to insufficient experience, and another was on maternity leave. Thus, five teachers (coded T1-T5) comprised our study sample. To ensure participant anonymity, code names were employed.

All participants are male, holding master’s degrees in mathematics, with specializations spanning modeling to numerical analysis. Two (T1 & T2) have undergraduate degrees in mathematics education, while the other three have undergraduate degrees in applied mathematics. Interviews, averaging five minutes, were transcribed verbatim, and analyzed for recurring themes, and patterns in teachers’ feedback.

**RESULTS**

To impart education effectively, teachers must possess robust GPK. This encompasses a comprehensive understanding of the curriculum, diverse instructional strategies, and appropriate assessment techniques. Teachers who excel often have an arsenal of skills tailored to boost student learning, allowing them to differentiate instruction based on individual student needs, and to offer feedback that carries genuine value. Such refined teaching expertise not only amplifies academic performance but also better equips students for future challenges.
Furthermore, the acumen of proficient teachers is not restricted to just pedagogical knowledge. They exhibit a profound grasp of their subject matter, deftly modify teaching methods according to the situation, foster resilient bonds with their students, and deliver feedback that is both constructive and actionable. However, the self-confidence of mathematics teachers in their GPK can be a spectrum. While some may exude confidence, others might express the desire for more extensive professional development. Understanding these self-perceptions is pivotal because they can significantly influence both teaching practices and the subsequent learning outcomes of the students. Table 1 provides a comprehensive overview of the descriptive statistics pertinent to GPK items examined in this study.

Based on the data presented in Table 1, UMTs achieved the highest ratings for items GPK5 and GPK6 in GPK ratings. However, they scored noticeably lower for items GPK3 and GPK4. This disparity suggests that there are areas, specifically concerning items GPK3 and GPK4, where their GPK could be strengthened to boost their teaching efficacy. To address this, tailored professional development programs emphasizing items GPK3 and GPK4 could be advantageous for these teachers.

Table 2 illustrates the descriptive statistics for GPK items as reported by five teachers. This data provides insights into the teachers’ proficiency levels in GPK. It can serve as a benchmark for comparison with other groups or for monitoring progress over time. It is imperative, however, to understand that these statistics do not shed light on the actual effectiveness or quality of their teaching. The average score across these GPK items suggests that teachers are well-equipped with the skills needed to convey subject matter in a manner conducive to meaningful student comprehension. Among the teachers, T10 secured the highest score in GPK with a mean of 5.00, while T01 lagged with a mean of 4.08. It is crucial to delve into the reasons behind T10’s lower score and offer focused professional development to bolster his expertise in this area. Engaging in a knowledge-sharing session with T10 could also serve as a conduit for enhancing the teaching caliber of the entire group. However, one must be cautious in equating high scores in GPK with effective teaching. To truly understand the impact of teachers’ GPK on students, more in-depth research combined with classroom observation is indispensable.

Based on the teaching practices shared by the participating teachers, it is clear that their methods vary depending on both the topic at hand and the student’s understanding levels. T10 prefers the lecture method to introduce core concepts, supplementing it with discussions to engage the students. This is further enriched by question-and-answer sessions and individual assignments. T01 predominantly adopts a teacher-centric lecture method due to time constraints, whereas T01 blends both lectures and group discussions. T01 and T01’s strategies align but differ slightly; for instance, T01 employs a range of methods from problem-solving to project-based learning, contingent on the topic’s nature. T10 leans heavily towards group discussions and lectures, occasionally interspersing with a question-answer format to gauge and enhance student understanding.

In a study by Kasa et al. (2022), students’ perceptions of these mathematics teachers were examined. While students recognized their teachers’ comprehensive content grasp, they highlighted a gap in pedagogical acumen. Often, mathematics teachers exhibit a heightened self-perception of their GPK, equating to a firm understanding of student learning dynamics, lesson planning, and classroom management. However, such self-perceptions may not necessarily reflect their actual proficiency, underscoring the need for continuous professional development to amplify their GPK. Teachers need to improve their pedagogical skills through ongoing training to narrow the difference between how they view their teaching abilities and how their students perceive them.

The study also highlighted that UMTs typically justify their pedagogical choices based on the teacher-centric vs. student-centric dichotomy. Those leaning towards lectures, often labeled as teacher-centric, cited voluminous course content and the disruptive impact of the COVID-19 pandemic as driving reasons. On the other hand, proponents of student-centric methods, like

### Table 1. Descriptive statistics of general pedagogical knowledge items

<table>
<thead>
<tr>
<th>Item</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPK1</td>
<td>4.40</td>
<td>0.55</td>
</tr>
<tr>
<td>GPK2</td>
<td>4.60</td>
<td>0.55</td>
</tr>
<tr>
<td>GPK3</td>
<td>4.20</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK4</td>
<td>4.20</td>
<td>1.30</td>
</tr>
<tr>
<td>GPK5</td>
<td>4.80</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK6</td>
<td>4.80</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK7</td>
<td>4.60</td>
<td>0.55</td>
</tr>
<tr>
<td>GPK8</td>
<td>4.80</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK9</td>
<td>4.60</td>
<td>0.55</td>
</tr>
<tr>
<td>GPK10</td>
<td>4.80</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK11</td>
<td>4.20</td>
<td>0.45</td>
</tr>
<tr>
<td>GPK12</td>
<td>4.80</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Note: M: Mean, SD: Standard deviation; n=5; & Scale: 1 (strongly disagree) to 5 (strongly agree)

### Table 2. Descriptive statistics for items related to GPK as reported by teachers

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T01</td>
<td>4.58</td>
<td>0.49</td>
</tr>
<tr>
<td>T02</td>
<td>4.30</td>
<td>0.87</td>
</tr>
<tr>
<td>T03</td>
<td>4.08</td>
<td>0.28</td>
</tr>
<tr>
<td>T04</td>
<td>5.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T05</td>
<td>4.67</td>
<td>0.47</td>
</tr>
</tbody>
</table>
group discussions or project-based learning, emphasized benefits like fostering critical thinking, enhancing student engagement, and nurturing a collaborative learning ambiance.

Student-centric methodologies actively involve students, enhancing their comprehension levels. This approach also facilitates diverse learning by grouping varied achievers together, promoting self-reliance and critical thinking. Both $T_{oa}$ and $T_{oa}$ emphasized the significance of method adaptability, contingent on the topic and students’ understanding. This underscores the value of flexibility in teaching approaches, allowing teachers to cater to an array of student needs, thereby fostering engagement, inclusivity, and addressing challenges.

Regarding engagement, UMTs employ diverse learning activities. They universally resort to questioning to gauge student understanding and based on the topic nature, might opt for discussion-driven engagements. For instance:

- $T_{oa}$ leans towards problem-solving, reasoning, brainstorming, and practical tasks.
- $T_{oa}$ prioritizes problem-solving and practical exercises, especially when students face challenges.
- $T_{oa}$ focuses on query-response techniques, aligning them with exam expectations to ensure student attentiveness.
- $T_{oa}$ amalgamates group tasks, projects, and problem-solving activities.
- $T_{oa}$’s techniques encompass problem demonstrations, concept definitions, and question-answer sessions.

**DISCUSSION**

This study aimed to examine GPK of UMTs while imparting “Applied Mathematics I” to pre-engineering students via self-reports. The intent was to discern whether UMTs wield the requisite acumen to teach this pivotal course effectively, given its significant ramifications on students’ academic trajectories and professional futures. These insights could potentially refine teacher training programs, elevating the educational quality rendered to students. The findings spotlight areas demanding bolstered training and development for UMTs, thereby amplifying the pedagogical value they confer upon their students.

For efficacious teaching, especially in subjects like mathematics, a robust GPK-akin to CK-is indispensable. This necessitates discerning questions to pose, the presentation format, and ways to root abstract notions in tangible real-world scenarios. Pedagogical finesse is integral to fostering a classroom’s rhythm and ambiance, setting the stage for effective teaching (Das, 2015).

However, there is a perceived gap between UMTs’ self-assessments and their actual GPK. While they exhibit adeptness in gauging student understanding, they seemingly grapple with the nuances of diverse teaching strategies and practices. This divergence indicates the pressing need for professional enhancement, mentorships, and peer observations, to imbue UMTs with contemporary teaching techniques, accruing both to their benefit and to that of their students.

Drawing from Table 1, while UMTs adeptly assess student understanding, they seem to lag in their strategic instructional arsenal. Table 2 amplifies this, highlighting the disparities between $T_{oa}$ and $T_{oa}$’s scores. Their educational backgrounds might elucidate this variance. Historically, higher education teachers often lack pedagogical training, a potential detriment to their confidence and efficacy (Teshaile et al., 2018; Wondem, 2022). Against the backdrop of such findings, prioritizing professional development emerges as a pragmatic solution, arming teachers with cutting-edge methodologies, buttressing their confidence, and, in turn, uplifting the educational fabric for students.

Although lectures dominate higher education globally, their efficacy, especially in fostering higher-order thinking and analytical prowess, remains debated, especially in sectors like engineering (Hafeez, 2021). An amalgamation of teacher-centric and student-centric strategies, synchronized with course objectives and lesson contexts, offers a more holistic learning experience (Awacorach et al., 2021; Fatima, 2022; Muganga & Ssenkusu, 2019). This hybrid approach amalgamates direct knowledge transfer with active student participation, honing critical faculties essential for academic and professional accomplishments.

To cater to diverse learning styles, UMTs vary their pedagogical tactics. From probing queries to assess student understanding to real-world exemplifications, these strategies aim at enhancing participation and comprehension. Active engagements like hands-on tasks and group collaborations address diverse learning profiles, creating a classroom environment that is inclusive and fosters student growth.

**CONCLUSIONS**

To be successful in teaching mathematics, teachers need a solid grasp of both content-specific knowledge and general pedagogical skills. This pedagogical knowledge encompasses a suite of skills, including effective instructional strategies, adept assessment techniques, and classroom management tactics. A harmonious blend of deep mathematical understanding and pedagogical proficiency is vital for impactful teaching and learning in the realm of mathematics. Teachers with a profound grasp of mathematical principles, paired with the ability to convey these concepts lucidly, stand a greater chance of instilling a love for the subject in their students. Furthermore, those with a solid foundation in pedagogical techniques are adept at crafting lessons that are both engaging and tailored to address the myriad needs of their diverse student body.

UMTs displayed exemplary GPK ratings, particularly for items GPK5 and GPK6. However, they fell short in the areas of GPK3 and GPK4, underscoring potential areas for enhancement. There is a tangible benefit in crafting professional development modules zeroed in on GPK3 and GPK4 to bolster the prowess of these university teachers. An inspection of the data in Table 2 reveals that while $T_{oa}$ boasts the topmost score, $T_{oa}$ lags at the bottom. A deeper probe into the reasons behind $T_{oa}$’s subpar
performance, followed by targeted training interventions, is of the essence. Gleaning and disseminating the best practices from high-performers like T_{oa} could elevate the teaching standards across the board. However, it is crucial to note that a high GPK score is not a definitive marker of effective teaching. Classroom observations and additional research are pivotal in gauging the real-world implications on students’ learning trajectories.

Teachers in the study exhibited a diverse pedagogical approach. Their chosen methodologies were dictated by the specific topic at hand and the prevailing comprehension levels of their students. Some leaned towards a more traditional, teacher-centric style, while others embraced student-centric methods that prioritize active student participation and accommodate a spectrum of learning styles. It is imperative for teachers to maintain a fluid teaching style, continuously recalibrating their methods to best serve their students’ evolving needs. The selection of educational activities invariably hinges on the intrinsic nature of the subjects being taught.

These insights underscore an emergent need to furnish mathematics teachers with expansive avenues to hone their GPK. Such enhancement can be achieved via targeted professional development sessions, one-on-one coaching, and mentoring. Amplifying GPK acumen of mathematics teachers can pave the way for consistent, high-quality instruction across the board. These revelations also hint at the vast untapped potential in understanding GPK landscape of UMTs. Given the myriad facets of GPK yet to be explored fully, further research is imperative. Future studies should investigate the effectiveness of diverse teacher training modules and seek correlations between GPK and other crucial teaching competencies like self-efficacy beliefs and classroom management skills.

**Recommendations**

After meticulous analysis of the data and subsequent discussions from our research, we hereby present a detailed set of recommendations that should be taken into consideration. We propose the introduction of targeted professional development programs to address the noticeable gaps in GPK, particularly in areas GPK3 and GPK4. These programs should focus specifically on these areas, providing teachers with tools, strategies, and knowledge to enhance their general pedagogical skills. We also suggest leveraging high-performing teachers, such as T_{oa}, as assets. Their methodologies and teaching practices could be documented, studied, and shared with other teachers to uplift the overall teaching quality across the board. While scores in GPK are indicative, they are not conclusive proof of effective teaching. Therefore, a more comprehensive approach involving classroom observations is recommended. This will give a clearer picture of the practical application of GPK in real teaching scenarios and its impact on student learning.

Teachers should be encouraged to adopt a more adaptive teaching style, considering the varying needs of their students and the nature of the topic being taught. Workshops on the advantages and application of both teacher-centered and student-centered methods could be beneficial. Periodic assessments of teachers’ pedagogical skills are crucial. Regular feedback sessions can guide them on areas of improvement and acknowledge their strengths.

We also propose establishing mentorship programs, where novice or struggling teachers are paired with experienced ones can foster a culture of continuous learning. Regular coaching sessions can also help teachers refine their skills, keeping them updated with the latest in pedagogical practices. Lastly, we recommend creating platforms or forums, where teachers can share their experiences, challenges, and strategies. Such collaborative environments can lead to mutual growth and shared understanding.

In essence, a multi-pronged approach that combines professional development, research, collaborative learning, and regular assessments can help elevate the teaching standards in the realm of mathematics. This, in turn, will ensure students receive high-quality instruction, fostering a conducive environment for effective learning.

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**Declaration of interest:** No conflict of interest is declared by authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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