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Pre-service mathematics teachers' geometric reasoning skills with technology-integrated guided inquiry approach

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ABSTRACT

This study was intended to examine the progress of the geometric reasoning skills of pre-service mathematics teachers (PSMTs) in the College of Teacher Education, Oromiya, Ethiopia. In the study, there were three different classes: EG1 (n=48), EG2 (n=38), and CG (n=30), where PSMTs in EG1 were treated with technology into guided inquiry-based learning (TGIBL), EG2 with guided inquiry-based learning (GIBL), and CG with a conventional learning approach. A geometric reasoning test was used to collect data. The data were analyzed using a one-way ANOVA and a paired sample t-test. The findings showed that the effect of using the GIBL model assisted by technology was higher when compared to the other two models. On the other hand, a paired t-test showed significant mean gains in reasoning skills in the TGIBL and GIBL approaches. Hence, technology must be integrated into teaching and learning geometry at CTEs to promote PSMTs' geometric reasoning skills.

Keywords: plane geometry, reasoning skills, guided inquiry, technology, pre-service mathematics teacher

INTRODUCTION

For the recent worldwide renovation, mathematics is a universal knowledge and skill that is fundamental, and it has significantly supported the improvement of science, engineering, and/or technology. In addition, according to Bayuningsih et al. (2018), mathematics education helps learners enhance their collaborative skills as well as logical thinking, analytical, systematic, critical, and creative thinking. Furthermore, learners should be able to use mathematical ideas in their daily lives as a result of their mathematical education (Hansen et al., 2020). However, learners' mathematics learning outcomes have declined all over the world, including Ethiopia (Ministry of Education [MoE], 2015, 2017; Pohjolainen et al., 2018; Preez, 2018; Simamora et al., 2017).

According to Pittalis and Christou (2010), geometry is a fundamental area of mathematics that supports learners' skills to think critically and solve mathematical problems. As geometry unifies all of the mathematical contents, it serves to visualize mathematical concepts. It supports pre-service mathematics teachers' (PSMTs) in developing their problem-solving abilities, conjecturing, deductive reasoning, logical argument, and proof skills (Dimla, 2018;). In addition, geometry learning can improve PSMTs' mathematical abilities, specifically their reasoning and spatial abilities (Pitttalis & Christou, 2010). However, geometry, in particular, is a mathematical content identified as a major area, where learners fail to develop an adequate understanding of the concept, reasoning abilities, and problem-solving skills (Adolphus, 2011; Dimla, 2018; Hua et al., 2019; Mamali, 2015; MoE, 2017; Suantoa et al., 2019). Furthermore, Fyfe et al. (2015) claimed that the misunderstandings of geometric concepts are the sources of the learning difficulties of other mathematical concepts. This indicates that an improvement in geometric reasoning skills is therefore essential for acquiring mathematical abilities among PSMTs.

In the same manner, the Ethiopian National Learning Assessment (ENLA) indicated that learners' performance in geometry was far below the minimum national standards (MoE, 2015, 2017, 2020). Additionally, ENLA (2017) stated that learners lack conceptual understanding, application knowledge, and reasoning abilities in geometry. Furthermore, Kasa (2015) also stated that prospective PSMTs have insufficient geometrical understanding. Thus, Van Hiele (1999) asserts that learning geometric concepts was attained when PSMTs transformed their thinking and reasoning skills.

Reasoning skills in mathematics are essential for achievement and serve as the foundation for learners' cognitive development (Barnes, 2019). It is a type of thinking that focused on drawing conclusions from a given premises. According to Llinares and Clemente (2014), reasoning is the act of deducing new knowledge from previously known knowledge, from the problem itself or from prior knowledge. In addition, reasoning ability is claimed to be the foundation for an understanding of geometric concepts and abstraction (Mueller & Maher, 2009; National Council of Teachers of Mathematics [NCTM], 2002). In other ways, PSMTs'

Table 1. Summary of research design

Group	Interventions		
Experimental group 1 (EG1)	Pre-test	TGIBL	Post-test
Experimental group 2 (EG2)	Pre-test	GIBL	Post-test
Comparison group	Pre-test	-	Post-test

learning achievement is mainly determined by their reasoning abilities (Nunes et al., 2007). This shows that, learning procedures and proofs without understanding of geometric concepts may unprepared PSMTs for geometric reasoning abilities.

Geometry reasoning encompasses three cognitive activities: visualization activities (visual representation of geometric statements or heuristic exploration), construction activities (using tools), and reasoning activities (a discursive process for the expansion of knowledge, as evidence) (Fujita & Jones, 2002; Jones, 2002). However, geometric concepts are abstract and difficult to understand, so this problem can produce poor reasoning skills in learning geometry among PSMTs (Jones, 2002). In addition, geometric language, visualization abilities, an ineffective instructional approach, and poor reasoning skills are some factors for unproductive learning of geometric concepts. The traditional approach to teaching geometry focuses more on how much the PSMTs can remember and less on how well they can think and reason (Idris, 2007). In this approach, PSMTs have difficulty using their geometrical reasoning skills to solve problems due to procedural learning, which has different results than developing their thinking and reasoning skills (Payadnya, 2019).

So, in colleges of teacher education (CTE), learning strategies that might enhance PSMTs' reasoning skills are required. Consequently, it is important to implement a student-centered approach in order to equip PSMTs with mathematical reasoning, problem-solving abilities, and meaningful learning in the teaching and learning of geometric concepts. Therefore, guided inquiry-based learning (GIBL) is a type of student-centered approach in which PSMTs solve geometric problems and draw conclusions, generate meaningful knowledge, actively engage, and process information at a higher level with the support of the classroom teacher (Darkis, 2020; Lazonder & Harmsen, 2016; Sebsibe & Feza, 2019).

Moreover, because geometry involves abstract concepts in propositions and theorems, educational technologies need to be integrated into the teaching and learning processes (Wong et al., 2011). Alternatively, literatures also suggest that technology-supported learning approaches can improve the teaching and learning process if it is integrated with a meaningful method (Atnafu et al., 2015; Charles-Ogan & George, 2015; Eshetu et al., 2022; Gemechu et al., 2018; Sebsibe & Feza, 2019). Thus, integrating educational technology into GIBL approach gives PSMTs new opportunities to examine abstract concepts. Technology will support the investigation and exploration of mathematical concepts from different perspectives, resulting in the achievement of problem-solving and reasoning skills. Thus, by integrating technology into guided inquiry-based learning (TGIBL), PSMTs can engage in higher thinking levels (Eshetu et al., 2022; Melesse, 2014).

However, it has been noted that in many CTEs in Ethiopia, the mathematics teaching and learning process is teacher-centered, which is ineffective for internalizing and developing reasoning abilities about geometric concepts (Ahmad, 2013; Bekele, 2012; MoE, 2015, 2020; Sebsibe & Feza, 2019). Thus, studies have revealed that an instructional shift is yet to be practiced in CTEs in teaching mathematics and geometry, which remains an open question (MoE, 2015, 2020). Similarly, GIBL is more extensively applied in the science classroom than in the mathematics classroom (Caswell & Labrie, 2017; Gardner, 2012). Further, literature has stated that educational technology integration is not implemented during PSMT education in Ethiopia (MoE, 2015, 2020).

Therefore, this study was commenced to examine the impact of TGIBL and GIBL methods on PSMTs' reasoning skills in plane geometry. Thus, the study has important implications for PSMT education by lighting optimal learning settings and approaches. A 5E lesson plan with technology integration was employed for the study's objectives. Hence, the following research questions were proposed:

1. There is no significant mean difference between the mean scores of geometric reasoning skills among the three groups.
2. There is no significant mean difference between the pre-test and post-test mean scores of geometric reasoning skills in each group.

METHOD

Research Design

This study employed a non-equivalent pre-/post-test quasi-experimental research design. Since there were both treatment and comparison groups, a quasi-experimental design was employed (Creswell, 2012; Shadish et al., 2012). This form of research design includes the non-random distribution of subjects as well as the distribution of the treatments administered to the treatment group and the control group. Without using random assignment, this design is utilized to determine the causal effect of an intervention on its target population (White & Sabarwal, 2014). The outside factors were tightly controlled.

Therefore, the intervention uses experimental and comparison groups of PSMTs in their intact classes. The experimental group consisted of two classes, namely the first experimental class (EG1) and the second experimental class (EG2). EG1 group was treated with a TGIBL approach using a 5E learning model assisted by a dynamic geometric sketchpad (GSP), while EG2 group was treated with a GIBL approach using a 5E learning model. On the other hand, the conventional approach was employed in the comparison group. **Table 1** shows the summary of research design.

Table 2. Scoring rubric for assessment of geometric reasoning skills for PSMTs' responses

Score allotted	Criteria and descriptions for performance
4	❖ A correct and complete answer. • Notice helpful facts: make inferences and coherent chain of arguments from givens to conclusions. Give four points for a complete proof with justifications for each step. **Scoring note:** There are multiple valid ways of solving. Accept any valid method.
3	❖ Some of the answers are correct, but there is one significant error. • Notice helpful facts & make some inferences, but inability to precise, imprecise labeling, convoluted or clouded chains of argumentation. Give three points when the following response types: • Incomplete proof with one step missing. • Incomplete proof with 1 error in a statement or justification. **Scoring note**: There are multiple valid ways of solving. Accept any valid method.
2	❖ Some of the answers are correct, but there is more than one significant error. • Notice helpful facts & make an inference, but inability to organize information in coherent chain of arguments from givens to conclusions. Give two points for any of the following response types: • Incomplete proof with two steps missing. • Incomplete proof with two errors in a statement or justification. • Incomplete proof with one step missing and one error in a statement or justification. **Scoring note**: There are multiple valid ways of solving. Accept any valid method.
1	❖ The answer is incomplete but contains at least one correct argument • Recognition of some helpful facts but inability to make a logical deduction. Give one point for three errors: • Steps missing. • Error/s in a statement or justification. **Scoring note**: There are multiple valid ways of solving. Accept any valid method.
0	❖ The answer is not correct based on the process, or there is no response at all. • Lack of basic geometrical knowledge and terminology or lack of appropriate geometrical frame of reference. • Response demonstrates limited to no understanding of geometric proofs.

Note. Adapted from Georgia Milestones Assessment System (2015) & Ndlovu and Mji (2012)

Setting, Population, and Sampling

This research was conducted in Ethiopia, Oromiya Regional State, at CTEs. The population of the study was all PSMTs of the 2020 academic year who were registered for math-111 (plane geometry). A total of 116 PSMTs in which #EG1=48), (#EG2=38), and (#CG=30) were participated in the study.

In this study, the three-stage sampling technique was employed. To begin with, two CTEs (Dambi Dollo and Shambu) were selected purposefully based on their equivalency in ICT facilities, academic staff, candidate enrolment, and demography. Purposive sampling is used when the researcher believes that useful data can be obtained under specific conditions (Fraenkel & Wallen, 2009). Following that, simple random sampling was employed to assign the two CTEs into experimental and comparison groups. Thus, Dambi Dollo CTE was allocated for treatment while Shambu CTE was sampled for comparison. Finally, the intact classes from Dambi Dollo CTE were assigned to EG1 (TGIBL) and EG2 (guided-inquiry) using a simple random sampling method. The experimental groups were in opposite shifts, so when one group left class for practicum, the other group stayed in CTE for intervention.

Instrument of Data Collection

The data related to PSMTs' geometrical reasoning skills were collected through geometry problem proof tests (GPPT) (pre- and post-test). GPPT was adapted, and it is a two-column format. It is a non-routine problem designed to measure PSMTs' ability in geometric reasoning. In addition, the scoring rubric was adapted, and the analysis procedure was determined based on the scoring rubric and looking at the score of each response to the question. **Table 2** shows methods of the scoring rubrics of geometry reasoning abilities (see **Appendix A** for sample rubric correcting of items).

Each item was scored using the criteria indicated in **Table 2**, and the total scores that the PSMTs received from the test were calculated using these scores. The highest and lowest scores for the test are 40 and zero, respectively. The pilot study was conducted at CTE, which was not selected for the main study, and used forty PSMTs that satisfied the minimum requirements (Bujang & Baharum, 2017).

Validity and Reliability

The validity of the instrument in the study relied on expert judgments. Thus, the content and face validity of the instrument were checked by professionals from mathematics education, measurement, and evaluation (Cohen et al., 2007; Kothari, 2004). Then, prior to the commencement of the pilot, some items were revised depending on the feedback and comments from these professionals on the clarity and errors in the answer keys.

Similarly, the reliability of the instrument in the study used to measure the degree of internal consistency to complement the validity (Fraenkel & Wallen, 2009). The inter-rater agreement or inter-rater reliability was used to measure the reliability of GPPT items. In this study, thus, one mathematics teacher educator and the researcher were to rate the items based on the criteria of the scoring rubrics in **Table 2**. The inter-reliability for the GPPT items was measured at a correlation coefficient of 0.834 (Liao et al., 2010), and the inter-rater agreement reliability (intra-class correlation/ICC average) of 0.827 shows a high value (Graham et al., 2012). If the ICC is greater than 0.75, the reliability is considered acceptable (Morgan et al., 2012). This shows that the items on the geometrical reasoning test are in good and can be used in the actual study.

Data Collection Procedure

Before the interventions were conducted, the researcher discussed with mathematics teacher educators and identified that they have computer knowledge but are unfamiliar with the usage of dynamic geometry software (DGS) as an instructional material. The mathematics teacher educators with master's degrees and teaching experience in CTEs were recruited for the intervention and comparison groups.

Following that, training was provided to mathematics teacher educators in the experimental groups on how to implement interventions, apply computer-assisted learning packages (GSP, GeoGebra) in GIBL settings, and guide the use of inquiry learning strategies. The training lasted for five days and included an overview of the TGIBL and GIBL approaches and how to implement 5E, which have five phases (engage-explore-explain-elaborate-evaluate) lesson plans (see **Appendix B**).

5E lesson plan was used to design and implement content and tasks in the GIBL and TGIBL approaches. Introductions of the lesson are usually followed by some practical examples or guided investigation, with each step clearly explained and interim questions formulated. Each task was followed by a discussion, and then examples and conclusions. The interventions were implemented for a whole semester (12 weeks) with four hours per week. In addition, PSMTs in the interventions were divided into groups of four to five, based on the classroom teachers' comments and their academic abilities. Following shows specific practices in each group.

Experimental group I/technology-integrated guided inquiry

PSMTs were divided into four-to-five members of heterogeneous groups. In this approach, activities that involve PSMTs to explore, investigate, discover, reflect on, and visualize the geometrical concepts, manipulating, animating, and dragging them, and analyzing and making conjectures using technology (GSP or GeoGebra), were given. In addition, YouTube videos were used. The instruction was based on 5E lesson plan (see **Appendix B**).

Experimental group II/guided inquiry-based learning

PSMTs were grouped into members of heterogeneous groups. In this approach, hands-on activities and manipulatives are used. In this case, concrete materials such as manipulatives were used to explore and investigate, conjecture geometric properties and concepts, and connect and model these properties and concepts through cutting, folding, pasting, connecting, and modeling activities. Hence, PSMTs are working collaboratively to think-pair, discuss, and investigate the activities given. In the meantime, teacher educators challenged PSMTs by asking questions like, How can you be sure it is correct? What is your explanation for what you have got? This elicits PSMTs to make further explanations and evaluations based on the collected evidence.

Comparison group/conventional teacher-centered method

In a comparison group, the conventional lecture method was used. The mathematics teacher educator followed the usual trend of the traditional lecture approach used in higher educational institutions in Ethiopia for their instructional tasks. In this trend, the mathematics teacher educator presented the content and worked on some sample examples while PSMTs took notes, passively listened to the lectures, and copied formulas. In this approach, no manipulatives, real-life examples, or technologies were used.

In the process of the interventions, the researchers followed its implementation process and provided feedback and comments at the end of classroom activities for further improvement of the interventions. Finally, after the completion of the interventions, a post-test was given to all groups.

Analysis of Data

The analysis was performed with one-way ANOVA because it has three factors namely TGIBL, GIBL, and conventional with one continues dependent variable and three different groups of participants involved could be taken as the other factor for using these inferential statistical tools at $\alpha=.05$ significant level (Cohen et al., 2007; Field, 2009). In addition, a paired sample t-test is employed to the changes between the pre- and post-test for each group.

Every statistical test, whether it is parametric or non-parametric, starts with a number of assumptions about the data that it will be applied. However, parametric tests are regarded powerful than non-parametric test in sensing the differences existing between the groups and this is tenable only if the assumptions like normality of the distribution, homogeneity of variance, and independence of samples are met (Field, 2009). The sample independent observation is not violated because each PSMT in study did their own test by themselves independently.

Of the aforementioned assumptions, normality should be verified before conducting any inferential statistics. If any of the independent samples' normality is violated, parametric testing should not be used (Rietveld & van Hout, 2015). The skewness and kurtosis measurements were used to check whether the data for each of the groups were normally distributed (Field, 2009). According to George and Mallery (2003), normality is attained if the skewness and kurtosis values are between -2 and 2. **Table 3** shows the skewness and kurtosis of the pre- and post-test scores of PSMT's geometric reasoning skills.

Each skewness and kurtosis value shown in **Table 3** for the distribution of EG1, EG2, and CG on pre- and post-test geometric reasoning skills was in support of normality as those values are between -2 and +2 (George & Mallery, 2003).

Table 3. Normality test of PSMTs geometric reasoning skill test scores

Variables	Groups	n	Statistics			
			Pre-test		Post-test	
			Skewness	Kurtosis	Skewness	Kurtosis
Reasoning skill	EG1	48	-.169	-.969	-.655	.215
	EG2	38	-.182	-.865	-.184	-.644
	CG	30	-.233	-.863	-.401	-.349

Table 4. Levene's test on PSMTs' pre- & post-test scores of geometry reasoning skills

Variable	Group	n	Mean	SD	F	df1	df2	Sig.
Pre-test	EG1	48	10.46	4.42	2.387	2	113	.10
	EG2	38	10.53	3.56				
	CG	30	11.70	3.96				
Post-test	EG1	48	16.98	2.25	2.68	2	113	.07
	EG2	38	14.45	2.66				
	CG	30	12.87	3.22				

Note. SD: Standard deviation

Table 5. ANOVA test of pre-test score on geometric reasoning skills

	Sum of squares	df	Mean square	F	Sig.
Between groups	32.749	2	16.375	1.043	.36
Within groups	1,773.690	113	15.696		
Total	1,806.440	115			

Table 6. ANOVA test of post-test score on geometric reasoning skills between groups

	Sum of squares	df	Mean square	F	Sig.	η^2
Between groups	335.297	2	167.649	23.685	.000	.29
Within groups	799.841	113	7.078			
Total	1,135.138	115				

Similarly, homogeneity of variance must be checked to verify equality of variances between the groups. Thus, the homogeneity variance is tested using Levene's F-test. If the p-value is less than 0.05, the homogeneity variance is violated (Pallant, 2020). **Table 4** shows the results of Levene's test of PSMTs' pre- and post-test scores of geometric reasoning skills between the three groups.

Levene's analysis shown in **Table 4**, the equality of variance for the dependent variable in the pre-test as well as the post-test between the groups was attained (Pallant, 2020). As a result, all the assumptions for the parametric test ANOVA were not violated. In the analysis of the hypothesis, the data were computed using SPSS V20. The test results were obtained using the one-way ANOVA test to test the mean difference in geometric reasoning skills between the three groups. However, to test the differences between the groups, the pre-test result must be checked to see whether the groups are equivalent or not before the implementation of the interventions.

As it can be seen from **Table 5**, $F(2, 113)=1.043$, $p>.05$. This indicates that there is no significant mean difference between the groups on the pre-test score. This implies that there was no significant difference in the mean scores of the experimental and comparison groups. Thus, the groups were equivalent in their prior geometric reasoning abilities before the implementation of interventions. Therefore, it can be deduced that the differences observed on the post-test score is attributable to the effect of the interventions.

The hypothesis tested was [$\mathbf{H_0}$]: *There is no significant mean difference in geometric reasoning skills between the groups*.

To test the hypothesis, ANOVA was employed on post-test mean scores of the TGIBL, GIBL, and comparison groups to compare if there were any statistically significant differences between the groups. **Table 6** displays the analysis's findings.

In **Table 6**, ANOVA test result showed that $F(2, 113)=23.685$, $p<.001$, eta squared($\eta^2=.29$) indicated that there were statistically significant differences among the experimental and comparison groups. The eta squared value shows a large effect size. This indicates that 29% of the variation was the result of the interventions. However, to determine where the difference lies between the groups, Scheffe's test was employed for post-hoc analysis. **Table 7** displays result of post-hoc analysis.

From Scheffe's post-hoc analysis (**Table 7**), it can be deduced that there was a significant difference between the post-test mean scores of PSMTs taught using TGIBL ($x=16.98$) and those taught using GIBL ($M=14.45$) and the conventional approach ($M=12.87$) on geometric reasoning, which is in favor of PSMTs in the TGIBL group. However, no significant difference was established between PSMTs who were taught using GIBL ($M=14.45$) and the conventional approach ($M=12.87$).

Likewise, based on the mean gains between the pre- and post-test scores for each group, the geometric reasoning abilities of PSMTs in three groups were compared using a paired sample t-test. The results are displayed in **Table 8**. As shown in **Table 8**, the result of the paired sample t-test indicate that there was a significant difference in reasoning mean score of PSMTs in TGIBL and GIBL groups ($t[47]=14.14$, $p<.001$, $d=2.04$) and ($t[37]=8.13$, $p<.001$, $d=1.32$, respectively) the d-value shows a large effect size. This indicates PSMTs in TGIBL and GIBL approach influenced the improvement of geometric reasoning skills meanwhile no significant improvement by PSMTs in the comparison group ($t[29]=.22$, $p=.827$, $d=.04$) and d-value shows small effect size.

Table 7. Scheffe post-hoc analysis of groups mean scores

Group (I)	Group (J)	Mean difference (I – J)	SE	Sig.	95% confidence interval	
					Lower bound	Upper bound
TGIBL	GIBL	2.53*	.577	.000	1.0988	3.9648
	CG	4.11*	.619	.000	2.5765	5.6485
GIBL	TGIBL	-2.53*	.577	.000	-3.9848	-1.0988
	CG	1.58	.649	.056	-.0311	3.1925
CG	TGIBL	-4.11*	.619	.000	-5.6485	-2.5765
	GIBL	-1.58	.649	.056	-3.1925	.0311

Table 8. A paired sample t-test comparison of pre-post score of geometric reasoning for three groups

Group	Observations	Paired difference		95% confidence interval		t	df	Sig.	d
		MD	SD	Lower bound	Upper bound				
TGIBL	Post-pre-test	6.5208	3.196	5.5929	7.4488	14.14	47	.000	2.04
GIBL	Post-pre-test	3.9211	2.972	2.9433	4.8978	8.13	37	.000	1.32
CG	Post-pre-test	.0667	1.659	-.55301	.68634	.22	29	.827	0.04

Note. MD: Mean difference & SD: Standard deviation

DISCUSSION AND CONCLUSIONS

The main objective of the study was to examine the effect of a technology-integrated guided inquiry approach on the geometric reasoning skills of PSMTs who studied a plane geometry course (math-111) in CTEs. This research compared three intact classes that received different treatments, such as TGIBL, GIBL, and conventional methods. A one-way ANOVA was conducted to analyze the difference between the group of PSMTs who were exposed to TGIBL, GIBL, and conventional methods of teaching.

The findings of the study revealed that the geometric reasoning of PSMTs who taught geometry using the TGIBL approach outperformed both PSMTs who taught using the GIBL and conventional methods of instruction. This study is consistent with previous research findings of Cesaria and Herman (2019), Donevska-Todorova (2015), Idris (2009), Murni and Jehadus (2019), Salifu (2020), Tezer and Cumhur (2017), and Yusuf and Afolabi (2010) in which learners taught using computer-based instruction attained higher reasoning abilities when compared with those taught using non-computer-based instruction.

With technology's (e.g., GSP) integration into the teaching and learning of geometry, the activities allowed PSMTs to explore, investigate, discover, reflect, and visualize the geometrical concepts so that they could generate reasoning abilities (Idris, 2009). Thus, the result indicated that the provision of technology can facilitate the reasoning and thinking abilities of PSMTs in learning plane geometry. However, the finding also showed no significant difference in geometric reasoning between PSMTs who were taught using GIBL and conventional methods. This contradicts Kandil's (2016) previous findings that guided inquiry has a positive impact on geometry learning.

In addition, a paired sample t-test was conducted to analyze the change between pre- and post-test and showed significant mean gains for PSMTs in the TGIBL and GIBL approaches. This finding is congruent with the prior studies by Dagnew and Mekonnen (2020), Donevska-Todorova (2015), and Kandil (2016) in which guided inquiry supported technology had a positive impact on PSMTs' reasoning abilities because technology offers multiple representations rather than single and static representations. Therefore, this study concluded that in improving PSMTs' geometrical reasoning abilities, the effect of using a guided inquiry approach aided by technology was higher when compared to a guided inquiry approach without technology and conventional approaches.

Recommendations

From the findings of this study, the following recommendations were forwarded:

1. Mathematics teachers should be trained on the effective use of computer-aided instructional approaches, i.e., the integration of technology, pedagogy, and content, through seminars, workshops, and conferences, in order to authorize their PSMTs in teaching and learning geometry.
2. Given that guided inquiry teaching techniques supported by technology are capable of enhancing PSMTs' reasoning skills in plane geometry, mathematics educators and instructors should use these approaches to deliver the geometric concepts.
3. In addition, further empirical studies are essential to develop more precise technological activities to be incorporated into guided inquiry learning strategies in other mathematical areas, such as calculus, algebra, etc.

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REFERENCES

Adolphus, T. (2011). Problems of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. *International Journal of Emerging Sciences, 1*(2), 143-152.

Ahmad, S. (2013). Teacher education in Ethiopia: Growth and development. *African Journal of Teacher Education, 3*(3), 1-20. https://doi.org/10.21083/ajote.v3i3.2850

Atnafu, M., Teshome, Z., & Kassa, M. (2015). Perception of civil engineering extension students at Addis Ababa University Institute of Technology in the teaching of applied mathematics. *Ethiopian Journal Education & Science, 10*(2), 51-78.

Barnes, A. (2019). Perseverance in mathematical reasoning: The role of children's conative focus in the productive interplay between cognition and affect. *Research in Mathematics Education, 21*(3), 271-294. https://doi.org/10.1080/14794802.2019.1590229

Bayuningsih, A. S., Usodo, B., & Subanti, S. (2018). Problem based learning with scaffolding technique on geometry. *Journal of Physics: Conference Series, 1013*(1), 012134. https://doi.org/10.1088/1742-6596/1013/1/012134

Bekele, A. (2012). *Methods of teaching and their implications for quality of student learning at Samara University* [Master's thesis, Addis Ababa University].

Bujang, M. A., & Baharum, N. (2017). A simplified guide to determination of sample size requirements for estimating the value of intra class correlation coefficient: A review. *Archives of Orofacial Science, 12*(1), 1-11.

Caswell, C. J., & LaBrie, D. J. (2017). Inquiry based learning from the learner's point of view: A teacher candidate's success story. *Journal of Humanistic Mathematics, 7*(2), 161-186. https://doi.org/10.5642/jhummath.201702.08

Cesaria, A., & Herman, T. (2019). Mathematical reasoning in geometry learning using information and communication technology (ICT). *Journal of Physics: Conference Series, 1157*(4), 042101. https://doi.org/10.1088/1742-6596/1157/4/042101

Charles-Ogan, G., & George, N. R. (2015). Investigating difficult concepts in senior secondary school mathematics curriculum as perceived by students. *International Journal of Academic Research and Reflection, 3*(6), 67-74.

Chimuka, A. (2017). *The effect of integration of GeoGebra software in the teaching of circle geometry on grade 11 learners' achievement* [Unpublished master's thesis]. University of South Africa.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. Routledge. https://doi.org/10.4324/9780203029053

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson.

Dagnew, A., & Mekonnen D. (2020). Effect of using guided inquiry teaching method in improving grade eight students' concept of photosynthesis, primary school: Ethiopia. *International Journal of Innovative Research in Education, 7*(1), 01-16. https://doi.org/10.18844/ijire.v7i1.4655

Darkis, J. M. (2020). Views and challenges in teaching mathematics of elementary teachers in rural and urban school districts. *Journal of Critical Reviews, 7*(4), 107-112. https://doi.org/10.31838/jcr.07.04.19

Dimla, R. B. (2018). Probing students' levels of geometric thinking in geometry and their enacted example space function. *Journal of Education in Black Sea Region, 4*(1), 155-163. https://doi.org/10.31578/jebs.v4i1.162

Donevska-Todorova, A. (2015). Conceptual understanding of dot product of vectors in a dynamic geometry environment. *Electronic Journal of Mathematics & Technology, 9*(3).

Eshetu, D., Atnafu, M., & Woldemichael, M. (2022). The effectiveness of guided inquiry-based technology integration on pre-service mathematics teachers' understanding of plane geometry. *Journal of Pedagogical Research, 6*(4), 84-100. https://doi.org/10.33902/JPR202215241

Ethiopian National Learning Assessment (ENLA). (2017). *Ethiopian third national learning assessment of grade 10 and 12 students achievement*. NEAEA, Addis Ababa, Ethiopia.

Field, A. (2009). *Discovering statistics using IBM SPSS statistics*. SAGE.

Fraenkel, J. R., & Wallen, N. E. (2009). *How to design and evaluate research in education.* McGrow-Hill.

Fujita, T., & Jones, K. (2002). The bridge between practical and deductive geometry: Developing the 'geometrical eye'. In *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education* (pp. 383-391).

Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction, 35*, 104-120. https://doi.org/10.1016/j.learninstruc.2014.10.004

Gardner, K. (2012). An online community of inquiry for reflective practice in an operative dentistry course. *Journal of Dental Education, 76*(5), 641-650. https://doi.org/10.1002/j.0022-0337.2012.76.5.tb05298.x

Gemechu, E., Kassa, M., & Atnafu, M. (2018). Matlab supported learning and students'conceptual understanding of functions of two variables: Experiences from wolkite university. *Bulgarian Journal of Science and Education Policy, 12*(2), 314-344.

George, D., & Mallery, P. (2003). Descriptive statistics: Measures of central tendency, variability, deviation from normality, size, and stability. In *SPSS for Windows step by step: A simple guide and reference* (pp. 95-105). Allyn and Bacon.

Georgia Department of Education. (2015). *Georgia milestones assessment system. Study/resource Guide for Students and parents Geometry*. www.georgiastandars.org

Graham, M., Milanowski, A., & Miller, J. (2012). Measuring and promoting inter-rater agreement of teacher and principal performance ratings. *Center for Educator Compensation Reform*. https://files.eric.ed.gov/fulltext/ED532068.pdf

Hansen, A., Drews, D., Dudgeon, J., Lawton, F., & Surtees, L. (2020). *Children's errors in mathematics*. Sage.

Hua, X., Tang, M., Sun, S., & Han, Z. (2019). Inquiry into mathematics teaching in senior high school–Taking "planar analytic geometry" as an example. In *Proceedings of the 3rd International Seminar on Education Innovation and Economic Management* (pp. 312-315). Atlantis Press. https://doi.org/10.2991/seiem-18.2019.81

Idris, N. (2007). The effect of geometers' sketchpad on the performance in geometry of Malaysian students' achievement and van Hiele geometric thinking. *Malaysian Journal of Mathematical Sciences, 1*(2), 169-180.

Idris, N. (2009). Enhancing students' understanding in calculus trough writing. *International Electronic Journal of Mathematics Education, 4*(1), 36-55. https://doi.org/10.29333/iejme/229

Jones, K. (2002). Issues in the teaching and learning of geometry. In L. Haggarty (Ed.), *Aspects of teaching secondary mathematics: Perspectives on practice* (pp. 121-139). Routledge.

Kandil, S. (2016). *An investigation of the effect of inquiry-based instruction enriched with origami activities on the 7th grade students' reflection symmetry achievement, attitudes towards geometry and self-efficacy in geometry* [Master's thesis, Middle East Technical University].

Kasa, M. W. (2015). *Mathematics teacher education and teachers' professional competence in Ethiopia* [Unpublished PhD dissertation]. Addis Ababa University.

Kothari, C. R. (2004). *Research methodology: Methods and techniques*. New Age International.

Lazonder, A. W., & Harmsen, R. (2016). Meta-analysis of inquiry-based learning: Effects of guidance. *Review of Educational Research, 86*(3), 681-718. https://doi.org/10.3102/0034654315627366

Liao, S. C., Hunt, E. A., & Chen, W. (2010). Comparison between inter-rater reliability and inter-rater agreement in performance assessment. *Annals Academy of Medicine Singapore, 39*(8), 613. https://doi.org/10.47102/annals-acadmedsg.V39N8p613

Llinares, S., & Clemente, F. (2014). Characteristics of pre-service primary school teachers' configure reasoning. *Mathematical Thinking and Learning, 16*(3), 234-250. https://doi.org/10.1080/10986065.2014.921133

Mamali, N. R. (2015). *Enhancement of learner's performance in geometry at secondary schools in Vhembe District of the Limpopo Province* [Doctoral dissertation, University of Venda].

Melesse, K. (2014). The influence of e-learning on the academic performance of mathematics students in fundamental concepts of algebra course: The case of Jimma University. *Ethiopian Journal of Education & Science, 9*(2), 41- 59.

MoE. (2015). *Education sector development program V*. https://planipolis.iiep.unesco.org/en/2015/education-sector-development-programme-v-esdp-v-2008-2012-ec-201516-201920-gc-programme-action

MoE. (2017). *Strategic policy for national science, technology and mathematics education*. http://www.moe.gov.et

MoE. (2020). *Why Ethiopia values STEM education?* https://www.moe.gov.et/ViewNews/21

Morgan, G. A., Leech, N. L., Gloeckner, G. W., & Barrett, K. C. (2012). *IBM SPSS for introductory statistics: Use and interpretation*. Routledge. https://doi.org/10.4324/9780203127315

Mueller, M., & Maher, C. (2009). Learning to reason in an informal math after-school program. *Mathematics Education Research Journal, 21*(3), 7-35. https://doi.org/10.1007/BF03217551

Murni, V., & Jehadus, E., (2019). Learning circle through GeoGebra media oriented to understanding concepts. *International Journal of Innovative Science and Research Technology, 4*(5), 1-4.

NCTM. (2002). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Ndlovu, M., & Mji, A. (2012). Pedagogical implications of students' misconceptions about deductive geometric proof. *Acta Academica, 44*(3), 175-205.

Nunes, T., Bryant, P., Evans, D., Bell, D., Gardner, S., Gardner, A., & Carraher, J. (2007). The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology, 25*(1), 147-166. https://doi.org/10.1348/026151006X153127

Pallant, J. (2020). *SPSS survival manual: A step by step guide to data analysis using IBM SPSS*. McGraw-Hill. https://doi.org/10.4324/9781003117452

Payadnya, I. P. A. A. (2019). Investigation of students' mathematical reasoning ability in solving open-ended problems. In *Journal of Physics: Conference Series,* 1200(1), 012016. https://doi.org/10.1088/1742-6596/1200/1/012016

Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics, 75*, 191-212. https://doi.org/10.1007/s10649-010-9251-8

Pohjolainen, S., Nykänen, O., Venho, J., & Kangas, J. (2018). Analyzing and improving students' mathematics skills using ICT-tools. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(4), 1221-1227. https://doi.org/10.29333/ejmste/81869

Preez, M. D. (2018). The factors influencing mathematics students to choose teaching as a career. *South African Journal of Education, 38*(2), 1-13. https://doi.org/10.15700/saje.v38n2a1465

Rietveld, T., & van Hout, R. (2015). The t test and beyond: Recommendations for testing the central tendencies of two independent samples in research on speech, language and hearing pathology. *Journal of Communication Disorders, 58*, 158-168. https://doi.org/10.1016/j.jcomdis.2015.08.002

Salifu, A. S. (2020). Effect of GeoGebra on pre-service teachers' achievement and perception of circle theorems at E. P. College of Education, Bimbilla-Ghana. *International Journal of Innovative Science and Research Technology, 5*(8), 1202-1215. https://doi.org/10.38124/IJISRT20AUG404

Sebsibe, A. S., & Feza, N. N. (2019). Assessment of students' conceptual knowledge in limit of functions. *International Electronic Journal of Mathematics Education, 15*(2), em0574. https://doi.org/10.29333/iejme/6294

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Houghton Mifflin.

Simamora, S. J., Simamora, R. E., Sinaga, B., & Calderon, O. (2017). Application of problem based learning to increase students' problem solving ability on geometry in class X SMA negeri 1 pagaran. *International Journal of Sciences Basic and Applied Research, 36*(2), 234-251.

Suantoa, E., Zakaria, E., & Maat, S. M. (2019). The development of the KARA module based on experiential learning approaches in the threedimensional geometry blocks topic for lower secondary school students. *International Journal of Innovation, Creativity and Change, 7*(11), 26-46.

Tezer, M., & Cumhur, M. (2017). Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects. *EURASIA Journal of Mathematics Science and Technology Education, 13*(8), 4789-4804. https://doi.org/10.12973/eurasia.2017.00965a

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children mathematics, 5*(6), 310-316.

White, H., & Sabarwal, S. (2014). *Quasi-experimental design and methods: Methodological briefs-impact evaluation*. https://www.unicef-irc.org/publications/753-quasi-experimental-design-and-methods-methodological-briefs-impact-evaluation-no.html

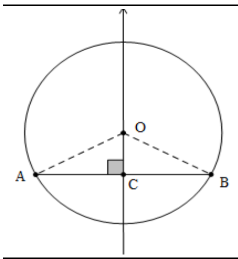
Wong, W. K., Yin, S. K., Yang, H. H., & Cheng, Y. H. (2011). Using computer-assisted multiple representations in learning geometry proofs. *Journal of Educational Technology & Society, 14*(3), 43-54.

Yusuf, M. O., & Afolabi, A. O. (2010). Effects of computer assisted instruction (CAI) on secondary school students' performance in biology. *Turkish Online Journal of Educational Technology, 9*(1), 62-69.

APPENDIX A: SAMPLE OF PSMTS RESPONSE AND RUBRIC SCORING (GPPT) OF ITEMS

Q4. Sararri handhuura geengoo keesa qaxxaamuree kordiitti parpeendikularii ta'u, koordicha wal-qixatti kuta [The line drawn through the center of a circle and perpendicular to a chord, bisects the chord]. Danaa [figure].

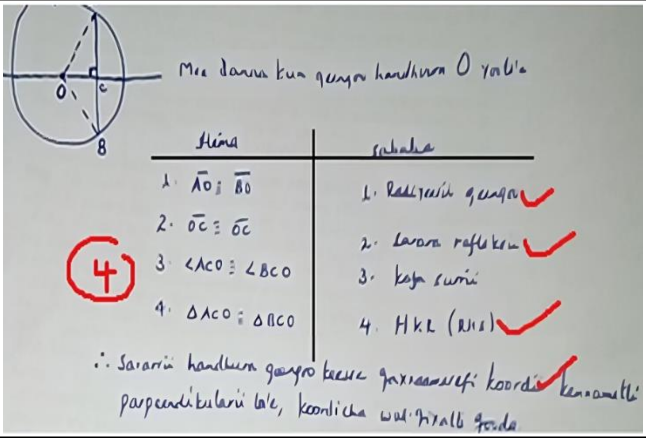
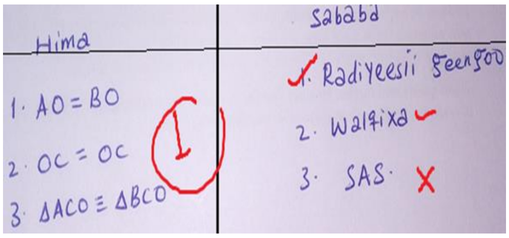
Table A1. Rubrics used for scoring the questions



Statements	Reasons
1. Join AO and BO	1. Construction
2. OC=OC	2. Reflexive property/common side
3. AO=BO	3. Radii of circle are equal
4. $\angle ACO = \angle BCO$	4. Given
5. $\Delta ACO \cong \Delta BCO$	5. RHS
6. AC=BC	6. From statement in step 5

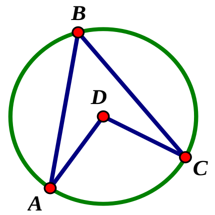
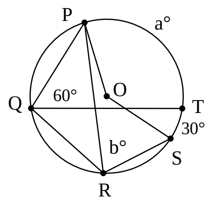
Hence, the line passes through center of circle and perpendicular to a given chord bisects that chord.

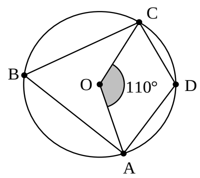
Table A2. Sample answer of the tests by PSMTs

PSMT-1	PSMT-2
	

APPENDIX B

Table B1. Sample of 5E lesson plan activities

<table>
<tr><th></th><th>Teacher activities</th><th>Student activities</th></tr>
<tr><td colspan="3">Objective: At the end of the lesson, PSMTs should evaluate the relationship between the central and its inscribed angle.
Prior knowledge: Central angle, inscribed angle, & chords
Materials: Paper cutter, hard paper, GSP, and/or GeoGebra</td></tr>
<tr><td>Engage</td><td>Motivate, inspire, & arouse curiosity & determine what students know & believe</td><td>Listening, participate answering, & think-pair-share</td></tr>
<tr><td>Explore</td><td>Facilitator & arranges exploration activities & guide without direct telling</td><td>What will happen when point A move? When C move? When point B move? What do you conjecture for your exploration? Write your hypothesis.
<table>
<tr><th>$m\angle ADC$</th><th>$m\angle ABC$</th></tr>
<tr><td>90.61°</td><td>45.31°</td></tr>
<tr><td>105.52°</td><td>52.76°</td></tr>
<tr><td>108.46°</td><td>54.23°</td></tr>
<tr><td>109.74°</td><td>54.87°</td></tr>
<tr><td>111.35°</td><td>55.67°</td></tr>
<tr><td>97.06°</td><td>48.53°</td></tr>
</table>

</td></tr>
<tr><td>Explain</td><td>Suggest comments</td><td>Explain their hypothesis found</td></tr>
<tr><td>Elaborate</td><td>Asking question to deepen their understanding & challenging with big hypothesis</td><td>Apply concepts in different activities, relationship between central & inscribed angle are formulated, & reasonable evidence & conclusion must be conceptualized</td></tr>
<tr><td>Evaluate</td><td>Formal & summative assessment</td><td>1. Compute following if $P\hat{Q}T = 60^0, \widehat{ST} = 30^0$, find value of arc PT $= a^0, P\hat{R}S = b^0, P\hat{O}S =?$


2. What is the value of angle $\angle CDA$? $\angle CBA$?

</td></tr>
</table>