

Preservice Teachers' Misconceptions in Solving Probabilistic Problems

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ABSTRACT

Concerns about the influence of misconceptions, culture and social setting on probabilistic reasoning of teacher trainees led to a study in this area to explore the reasoning of preservice teachers from a college of education in Ghana. This study investigates preservice teachers' misconceptions and difficulties in probabilistic problems solving before they learn Elementary Stochastic as part of their course of studies. The identification of preservice teachers' misconceptions will serve as a reference point for planning and enactment of lessons for effective training of teachers. The study employed explanatory sequential mixed methods research design. The participants in this study were 181 level 200 female preservice teachers offering a Diploma in Basic Education. Diagnostic test and semi-structured interview were used to collect data. Data collected were analysed descriptively. Also, content analysis was employed to analyse some parts of the data collected. The results indicate that preservice teachers had some probability difficulties such as conceptual difficulties, interpretation difficulties, and procedural difficulties. Similarly, the study found that preservice teachers entered college with the following probability misconceptions: equiprobability bias, representativeness bias, negative and positive recency effects, outcome orientation bias, and belief bias. This study shows the need for teaching and learning activities to focus on addressing probability misconceptions and difficulties in order to developed better probabilistic reasoning of preservice teachers.

Keywords: probabilistic reasoning assessment, critical thinking, equiprobability bias, representativeness bias, outcome orientation bias

INTRODUCTION

Probabilistic reasoning is important in decision-making (Kang & Park, 2019). Teachers, Lawyers, Preservice Teachers, and many others often have to make judgments about the likelihood or chance of their actions resulting in the meaningful end. Bryant and Nunes (2012) asserted that, "this reasoning allows us to work out the probability of particular outcomes, and thus, to understand the risks and possible benefits of acting in one way rather than another" (p. 3). However, probability misconceptions tend to limit peoples' ability in exercising good judgment under uncertainties (Anway & Bennett, 2004; Kaplar et al., 2021; Karatoprak et al., 2015).

The concept of probability is part of the mathematics curriculum for colleges in many countries such as Ghana, Spain, China, Germany, USA, and so on. Many countries have made probability an integral part of the school curriculum because the ability to make judgments under uncertainty is becoming part of basic literacy globally (Bilek et al., 2018; Morsanyi & Szucs, 2014). Despite the important role probability plays in the world, the specific training to teach probability is far from being a universal component of preservice courses for mathematics teachers (Estrada et al., 2018). The current way of training teachers does not provide essential skills needed to teach probability effectively (Khazanov & Prado, 2010; Morsanyi et al., 2013) and most training college students have probability misconceptions (Khazanov & Gourgey, 2009; Masel et al., 2015; Tenenbaum et al., 2011; Triliana & Asih, 2019). The challenging issue concerning misconceptions is that many students have difficulty in relinquishing misconceptions because the false concepts may have been deeply grained in the mental map of an individual (Kissane & Kemp, 2009; Tentori & Crupi, 2012). Gauvrit and Morsanyi (2014) explained misconception as students' conception that produces a systematic pattern of errors. Also, misconceptions can be seen as a belief about how something should be done which in itself is incorrect. The probability difficulties and misconceptions that are frequent in the literature are conceptual difficulties, interpretation difficulties, procedural difficulties, equiprobability bias misconception, representativeness bias misconception, negative and positive recency effects misconception, outcome orientation bias misconception and belief bias misconception (see e.g., Ang & Shahrill, 2014;

Khazanov & Prado, 2010; Morsanyi & Szucs, 2014). Paul and Hlanganipai (2014) conducted a study in South Africa on students' probabilistic misconceptions across three grades (10, 11, and 12) and found that equiprobability bias is the most common misconception among all the grades. On average, probability misconceptions were higher among the female participants compared to their male counterparts. This current study looked at the misconceptions in probability preservice teachers of colleges of education have from the Senior High School (SHS) in Ghana.

This study assumed that identifying preservice teachers' probability knowledge before teaching and learning of probability at teacher training college would aid in their better preparation to introduce probability formally to pupils at the basic level of education. To have a general knowledge about preservice teachers' probabilistic misconceptions, research from both developed and developing countries is needed. However, studies on probability misconceptions from Africa appear to be scarce. Therefore, this study provides perspective on how preservice teachers in Ghana reason about uncertainties. Since students who live in developing countries have different cultures or social experiences, the study provided a better picture of preservice teachers' probabilistic misconceptions across the globe. This is very important because we have students from both developed and developing countries being educated in one classroom in many countries.

Additionally, this study sought to provide an insight into how students were taught basic concepts of probability before entering into College of Education. Therefore, the findings of the study would provide the initial assessment of teaching and learning of probability units in basic school. This study was thus generally initiated to identify preservice teachers' probabilistic misconceptions and difficulties before taking College Elementary Stochastic course. Several studies (e.g., Khazanov & Prado, 2010; Masel et al., 2015; Tenenbaum et al., 2011; Triliana & Asih, 2019) suggested that targeting students identified probabilistic misconceptions in teaching would aid in their development of sound probabilistic reasoning. Therefore, it is important to identify the misconceptions students bring to statistics and probability classrooms.

To achieve the purpose of this study, the research question that guided the study was: What are preservice teachers' probabilistic reasoning difficulties and misconceptions? The aim here was to investigate preservice teachers' probabilistic misconceptions before introducing them to the Elementary Stochastic Course. The identification of preservice teachers' misconceptions and difficulties before teaching and learning of probability will serve as a reference point to organize teaching, training programs and to create opportunities for preservice teachers to confront their misconceptions and difficulties, and address them. Also, answering this research question would help advance knowledge based on Statistics Education by providing the initial assessment of religious belief and social experiences students in developing countries bring to college Stochastic Classroom.

LITERATURE REVIEW ON PROBABILITY MISCONCEPTIONS AND DIFFICULTIES

Dollard (2011) investigated twenty-four (24) preservice elementary teachers' probabilistic situations thinking in the United States before they studied probability course as part of their Mathematics Course using a Task-Based interview. Dollard (2011) found that preservice teachers have a wide variety of misconceptions about the interpretation of probability. A study conducted by Karatoprak et al. (2015) in Turkey on prospective Elementary and Secondary School Mathematics Teachers' statistical reasoning showed that the prospective teachers have a good understanding of independence, large samples, and interpreting probability. However, the prospective teachers were found to have equiprobability bias and representativeness bias misconceptions. Karatoprak et al. (2015) results also showed that prospective teachers' understanding of the law of small numbers was problematic. There have been studies (e.g., Fielding-Wells, 2014; Fielding-Wells & Makar, 2015) that identified misconceptions about probability among prospective teacher training using different approaches. For instance, Fielding-Wells (2014) employed design-based research with the lens of exploratory research found that the participants see chance outcomes being inherently equiprobable which is a misconception. Equiprobability bias as a probability misconception is when the same probability of random experiment is assigned to different events irrespective of their actual chance or having the same degree of logical or mathematical probability (Merriam-Webster, n. d.). Similarly, Gauvrit and Morsanyi (2014) describe the equiprobability bias misconception as "a tendency to believe that every process in which randomness is involved corresponds to a fair distribution" (p. 1). Teacher trainees from SHS with this misconception think that when two fair dice are tossed once, the probability of obtaining a sum of 4 and a sum of 6 are all equally likely. However, the sum of 6 is more likely than a sum of 4. The chance of a sum of 4 occurring is 3 ways out of 36 ways as compared to a sum of 6 occurring 5 ways out of 36.

The representativeness approach which is another misconception in solving probability problems refers to events that are perceived as equally representative of their parent population and have an equal probability of happening (Hirsch & O'Donnell, 2001; Morsanyi & Szucs, 2014). Similarly, Anway and Bennett (2004) refer to representativeness misconceptions as "the tendency of students to erroneously think that samples which resemble the population distribution are more probable than samples which do not" (p. 1). In Anway and Bennett (2004) study, a senior student (equivalent to Level 300 student) worked with a faculty member on a project that involved three (3) elements (measuring the misperceptions of the students, teaching to correct these misperceptions, and measuring the improvement. The student in Anway and Bennett (2004) study developed the research instrument. However, in this current study, the instruments were developed by the researchers. Students with representativeness misconception think that when a coin is tossed eight times, they believe that the events, THHTHTH and HHTHTHT are most likely outcomes than HHHTHHT or TTHTTTT (Tsakiridou & Vavyla, 2015). This belief contradicts the basic property of true random experiments as independent (Nabbout-Cheiban, 2017). In this regard, they are considering the sample which has an even number of heads and tails as the likely outcome. Students with this misconception also believe the chance of occurrence of an event is dependent on its resemblance to the population (Hirsch & O'Donnell, 2001). However, each of the results is an equally likely outcome since the sample is too small to be representative of the population. Similarly, representativeness bias leads

students to hold the view that chance is a self-correcting measure (Batanero & Borovcnik, 2016). When several tosses of a fair coin result in a consecutive number of tails, many students believe that the next toss is more likely to be a head because of the previous occurrence of many tails. However, the occurrence of a tail and a head are still equally likely.

The positive recency effect misconception is the belief that the previous outcome will repeat itself in the next trials (Chiesi & Primi, 2009). An example is an experiment of rolling a die four times continuously and obtaining a 2, three times, and the belief that the next toss will result in a 2. The negative recency effect is the belief that the outcome obtained from repeated experiments will not happen in subsequent trials (Gravir, 2019). For example, in an experiment of rolling a die 4 times and obtaining a 6 in all cases, and the belief that when the die is rolled again it has no chance of being a 6 anymore because of the previous occurrence of a 6.

Humphrey and Masel (2014) explained outcome orientation which a misconception in probability as a way of treating the chance of occurrence or non-occurrence of an event as an affirmation of certainty rather than a measure of likelihood. Outcome orientation is a situation where the probability of an event has not been seen as an expectation or likelihood (Blanco & Chamberlin, 2019). Another common misconception in probability is belief known as the Gambler's Fallacy (Williams & Griffiths, 2013). Belief bias misconception is where students hold the view that the possibility of an event occurring is subject to some forces like God or wind or luck.

Conceptual difficulties in solving probability problems involve difficulty in understanding and using the concept of sample space, in making reasoning about probability events, and in interpreting some probability concepts such as discrete event and independent event (Gage, 2012; Sezgin-Memnun et al., 2010, 2019). Procedural difficulty arises from failure to carry out manipulations or algorithms despite having understood the concepts behind the probability problem (Arum et al., 2018; Jense & Kellogg, n. d.). An interpretation difficulty in probability is when a student is not able to be precise and unambiguous in interpreting probability problems (Konold, 2017; Pisarenko, 2018).

Constructivist Theory

The Mathematics Teaching Syllabus for colleges of education is based on the principle of constructivism. Constructivism states that learning is an active, contextualized process of constructing knowledge based on personal experiences of one's environment rather than acquiring it (University at Buffalo, 2021). Constructivists hold the view that students bring to class their own ideas. Bada (2015) established that if all students are to succeed then teachers have to use the constructivist approach in teaching them. Preservice teachers are to learn from their own experiences through activities. We must therefore identify the kind of knowledge they bring to class. Knowing the kind of knowledge preservice teachers bring to College Stochastic Classrooms would help structure teaching and learning at their level for conceptual understanding. Constructivist theory underpinned this study because students entering teacher training colleges from SHS have constructed some concepts in probability.

METHODOLOGY

The research design that was employed in this study was explanatory sequential mixed methods design. In this design, "the qualitative phase builds directly on the results from the quantitative phase" (Wisdom & Creswell, 2013, p. 2) to answer the same question. In this study, data were collected in two phases. In the first phase, quantitative data were collected and the results analyzed. Two weeks after, qualitative data was collected based on the results of quantitative data to better identify probabilistic misconceptions and difficulties of preservice teachers. **Figure 1** presents the data collection model. That is, quantitative data was collect with the diagnostic test instrument and analysed followed by qualitative data collection and analysis and finally, identification of probability difficulties and misconceptions.

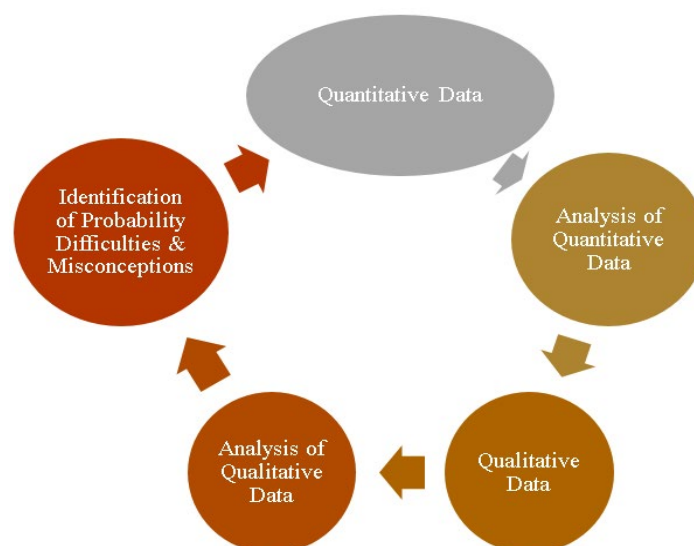


Figure 1. Data collection framework

Participants

The participants in this study were second year i.e., level 200 preservice teachers in a female college of education in the Volta region of Ghana. Out of 230 students offering the general programme in level 200 in the 2018/2019 academic year, 181 agreed to take part in the study. These 181 female Preservice Teachers in level 200 were purposively sampled for the study. Purposive sampling technique is non-probability sample which involves deliberate choice of a participant due to the qualities the participant possesses (Etikan et al., 2016). The participants were easily accessible to the researchers and were offering the general programme in Diploma in Education. The reason for considering Level 200 Preservice Teachers is that the Mathematics Curriculum requires Preservice Teachers to study Elementary Stochastic in the second semester of the second year of their training. The purpose of the course is to equip Preservice Teachers to teach the probability units effectively at Upper Primary and Junior High Schools and apply it in their daily lives.

Research Instruments

In this study, diagnostic test and semi-structured interview schedule were used to collect data. The diagnostic test was used to collect quantitative data while the semi-structured interview schedule was used to collect qualitative data. The instruments were designed by the researchers using related literature as a reference point. The reason for doing this was to ensure that the questions are culturally fit and also within the scope of SHS curriculum where they studied probability. To check validity of the instruments, the researchers solicited two experts' views on the instruments; on content coverage, content relevance, and duration participants could use in responding to the instruments. It was the view of the experts that the instruments would help collect appropriate information for the study. According to Cohen et al. (2018), an instrument is valid when in fact it measures what it intends to measure. The instruments were generally developed to identify probabilistic misconceptions and difficulties students bring to College Stochastic Classrooms. For reliability of the diagnostic test, the test items were piloted on thirty (30) level of 200 female teacher trainee from a mixed college of education in the volta region of Ghana. According to Koshy (2020), a minimum of 30 samples are required for plotting. Also, the rule of thumb is to test an instrument on at least 12 to 50 people prior to full-scale administration of an instrument (SAGE Publications, Inc., 2016). The statistical test used to ensure internal consistency of the diagnostic test items was Kuder-Richardson formula 20 (KR 20). Sabir (2013) stated that, KR20 formula is commonly used to measure the reliability of test with dichotomous answers. According to Fraenkel et al. (2019), one should attempt to generate a KR20 reliability coefficient of .70 and above to acquire reliable score. In this study, preservice teachers' responses to closed-ended test items were coded as right or wrong (i.e., all wrong options were coded as wrong) and entered into IBMSPSS using the codes 1 and 0 to represent right and wrong respectively. These codes were used to find the reliability of the test items. The reliability coefficient for the diagnostic test was 0.824. The researchers ensured that the diagnostic test was written under examination conditions. Piloting an interview schedule is rare and sometimes not necessary (Beebe, 2007; Campell, 2017; Marshall & Rossman, 2016). However, reliability and validity are conceptualized as trustworthiness, rigor and quality in qualitative paradigm (Golafshani, 2013). The step taken by the researchers to ensure validity and reliability of the qualitative data, was to provide a detailed account of the focus of the study to the participants, by so doing the researchers established trust and credibility

Diagnostic Test Items

Items 4i, 4ii, and 6 were adapted from Hirsch and O'Donnell (2001) and Anway and Bennett (2004). The rest of the items were constructed by the researchers. There were 12 items on the question paper. Some of the test items consisted of two-tier multiple-choice parts: The principal question and justification. The format was that some of the questions were closed-ended while others were open-ended and varied in type to identify probabilistic misconceptions and difficulties. These questions were based on probability and helped detect preservice teachers' probabilistic difficulties and misconceptions. Based on the literature that were reviewed, the options for each objective item were constructed (to identify probability misconceptions) and to some extent difficulties such that the choice by the participant could give a clear indication of the possible type of difficulty and misconception that existed. The approach has many advantages over the one-part items. Several studies (e.g., Chiesi & Primi, 2009; Hirsch & O'Donnell, 2001; Nabbout-Cheiban, 2017; Tsakiridou & Vavyla, 2015) reported that correct probabilistic reasoning is significantly overstated if no justification for the chosen answer is required. The literature (e.g., Kaplar et al., 2021; Karatoprak et al., 2015) provided support for teachers to assess students' probabilistic reasoning by using objective tests and written-in responses to gain insight into how they think probabilistically. The objective test helped check inconsistencies in teacher trainees' answers while written-in responses allowed researchers to assess teacher trainees' thinking. Initially, the participants were expected to response to the test items within forty-five minutes. However, upon the advice of two experts, the duration was changed from 45 minutes to one hour. The participants were given a maximum of one hour to respond to the items. The scripts were collected after one hour and marked within two weeks. The results helped to understand the extent to which learners could solve probability problems and how they reasoned about probabilistic situations before taking College Elementary Stochastic course. **Table 1** presents test items and measured probability misconceptions.

Table 1. Test items and probability misconceptions assessed

Item	Measured misconceptions
1	Decision-making ability
2, 3i, 3ii, 5	Representativeness bias
3i, 3ii, 4i, 4ii, 5	Positive and negative recency effects
8, 9	Equiprobability bias
6, 10	Outcome orientation bias
12	Without replacement situations
4i, 7, 10, 11	Basic ideas of probability

Semi-Structured Interview

The semi-structured interview schedule was used to collect data to supplement the quantitative data. The purpose of the interview was to illicit information from participants on conceptions and understandings of some probability concepts. The participants whose responses during the diagnostic test were not clear up-to what accounted for their solutions or answers were selected for interview. The interview also helped to identify preservice teachers' probabilistic misconceptions and difficulties. Twenty-five (25) of them were shortlisted for an interview because their responses to some selected items on the diagnostic test were not clear for the researchers to identify their difficulties or reasons behind their solutions. Ten (10) of the participants availed themselves for the interview sessions. The participants were interviewed individually for ten to twenty minutes in the lead researcher's office. The interviews were recorded and transcribed for analysis. The interview further helped the researchers to understand the ideas behind their solutions and also clarified some of the misconceptions that influenced their answers/solutions. The interview was done with the prior permission of the participants without their parents' consent since they were 18 years and above based on the college's student statistics.

Data Analysis

Content analysis (Columbia University, 2019; Luo, 2019) was carried out on the interview data obtained within the scope of this study. Descriptive statistics via the lens of item-by-item analysis was also employed in data analysis. One major reason for this was to ascertain how a participant response to similar items. To obtain a general picture of the participants correct probabilistic reasoning, both the correct and wrong answers were examined. Similarly, the percentage of the correct and wrong answers were calculated. The participants' responses were carefully analyzed manually to identify the main misconceptions and difficulties they had before teaching and learning of probability at the College of Education and to describe their probabilistic reasoning.

The researchers assigned a code to each participant concerning interview data. In the analysis of the interview data, the participants were referred to as student 01, 02, 08, and so on. The purpose of this was to protect the identity of the participants.

RESULTS

This section presents the results and discussions from the data analysis. The data collected through diagnostic tests and interviews were analyzed to answer the research question "*What are preservice teachers' probabilistic reasoning difficulties and misconceptions?*" The research question was aimed at revealing the probabilistic misconceptions and difficulties of preservice teachers before they take Stochastic Course at a College of Education in Ghana.

Item 1 tested fundamental ideas of probability for decision making. This item was: *Two containers, labeled M and P, are filled with green and red pens (container M with 20 green and 16 red pens and container P with 200 green and 160 red pens). Each container is thoroughly shaken. After picking one of the containers, you will reach in and without looking, draw out a pen. If the pen is red, you win GH¢100.00. Which container gives you the higher chance of drawing a red pen?*

- A. Container M (with 20 green and 16 red)
- B. Container P (with 200 green and 160 red)
- C. Equal chances from each container.

Twenty-eight out of 181 (representing 15.47%) of the students did not consider the ratio of green pens to that of red pens in each container before making choices. Seventeen of the participants representing 9.39% chose option "A" while 11 (representing 6.08%) of the participants chose option "B". This indicates that some of the students lack sound reasoning about random events in making good decisions. The participants used subjective judgment to compare the probabilities of red pens in both containers. However, one-hundred and fifty (representing 82.87%) of the participants chose the correct answer as "C" while 3 (representing 1.66%) of the students were undecided.

Item one did not meet the requirement set initially for interviewing participants but considering the fact that most participants got it right, the researchers were interested to know the reason behind their answers. The following were reasons given by some of the participants for their answers when interviewed on item 1.

Lead researcher: Why did you choose option C?

Student 96: "Equal chance from each container because both containers A and B contain some red and green pens so there is a possibility of picking a red."

Lead researcher: If you have the second chance would you change your answer, and why?

Student 96: No, because there is a possibility of drawing a red pen from both containers, hence equal chance.

Lead researcher: Why did you choose option C?

Student 102: "Each of the containers is having equal chances because both containers are having green pens more than the red pens and that probability of selecting red from each container is of equal chances."

Lead researcher: If you have the second chance would you change your answer?

Student 102: I can't tell now.

Lead researcher: Why did you choose option C?

Student 173: I chose the answer C because the chances of picking a red pen from each container are equal.

Lead researcher: If you have the second chance would you change your answer?

Student 173: "The probability that I can pick red pen from container A and red pen from container B are equal. If I have the second chance, I won't change the answer."

Lead researcher: Why did you choose option C?

Student 169: "Both containers have an equivalent number of green and red pens. So, no matter the container in which the red pen is being picked from, the probability of picking a red is the same."

Lead researcher: If you have the second chance would you change your answer?

Student 169: "I wouldn't change my answer if I were to be given a second chance."

Lead Researcher: Why not?

Student 169: "Because I am certain about my first answer."

Even though Students 96 and 102 chose the correct answer as "C", their reasons were misleading. Their comments suggested that they did not compare the probabilities of a red pen in each container. Their choice of an answer was only based on the mere presence of red pens in each container. This is an indication that decisions students make maybe right but are not necessarily grounded on correct reasoning about uncertainties. Nevertheless, Students 173 and 169 gave correct reasons for their chosen answers.

Similarly, one of the respondents who chose "A" concerning item one when interviewed provided these reasons to justify the answer.

Lead researcher: Why did you choose option A?

Student 128: I chose "A" because container A contains more green pens and fewer red pens.

Lead researcher: If you have the second chance would you change your answer?

Student 128: "If I have the second chance as to whether to change my answer or not, I will only do that when enough explanations are given to me as to why answer "A" is wrong other than that my answer will remain the same."

Lead researcher: But container B also contains more green than red, so why not container B?

Student 128: "In container A, the green pens are more than red pens only by 4 but in container B the green pens are more than red pens by 40."

The preceding comment reveals that Student 128 compared how many more green pens are in each container than red pens without relating it to the total number of pens in each container. This participant's response in the interview was consistent with the chosen answer. This finding suggests that care must be taken in teaching students how to compare probabilities.

Item 2 tests representativeness bias misconception. This question was: *A fair coin is to be tossed six times. Which of the following results is more likely?*

- A. HHTHHT
- B. THTHTH
- C. A and B are equally likely

Fifty-seven out of 181 (representing 31.49%) participants chose the correct answer "C". Similarly, 34 (representing 18.78%) participants chose "A" while 8 (representing 4.42%) participants were undecided. The 82 (representing 45.30%) preservice teachers who chose "B" as being "more likely" hold a misconception of representativeness because choosing "B" indicates a belief that the result of repeatedly tossing a coin must alternate or the coin naturally balances. The result suggests that in teaching probability, teachers must emphasize the laws of large numbers and small numbers as it relates to random events to address representativeness bias.

Item 3i tests positive recency, negative recency, and representativeness bias misconceptions. This question was: *A fair coin is tossed, and it lands tail. The coin is to be tossed a second time. What is the probability that the second toss will also be a tail?*

- A. $\frac{1}{4}$

- B. $\frac{1}{2}$
 C. $\frac{1}{3}$
 D. Slightly less than $\frac{1}{2}$
 E. Slightly more than $\frac{1}{2}$

One-hundred and seventeen (117) out of 181 the preservice teachers chose the correct answer as “B”. However, 7 out of 181 students and 5 out of 181 students chose “D” and “E” respectively showing negative recency and positive recency effects respectively. Also, students with representativeness bias may choose A, C, or D for reasons that the outcome is unlikely to be a tail in the next toss. Additionally, 39 out of 181 students picked option “A” as an answer indicating a lack of understanding of sample space. Furthermore, 12 of the participants chose “C” but one student could not choose any of the options with regards to item 3i.

Item 3ii checks the consistencies of students’ responses to the item on positive recency, negative recency, and representativeness bias misconceptions. Item 3ii asked students to identify a specific reason (justification) for the correct answer to item 3i. This item was: Which of the following best describes the reason for the correct answer to the preceding item 3i?

- A. The second toss is less likely to be tails because the first toss was tail.
 B. There are four possible outcomes when you toss a coin twice. Getting two tails is only one of them.
 C. The chance of getting heads or tails on anyone toss is always $\frac{1}{2}$.
 D. There are three possible outcomes when you toss a coin twice. Getting two tails is only one of them.
 E. The second toss is more likely to be tails because the first toss was tail.

In general, there are inconsistencies in students’ answers. As students were not able to choose the corresponding option to their chosen answers in 3i. For example, while 117 students chose the correct answer in 3i, 101 students chose the correct answer in 3ii as “C”. Forty participants chose “A” while 31 chose “E” indicating negative recency effect as well as representativeness bias and positive recency effect respectively. Also, 6 participants chose “B” and 3 participants chose “D”. **Table 2** matched each option in 3i to its interpretation in 3ii with the number of respondents.

Table 2. Relationship between the ordering of answers to items 3i and 3ii

Frequency of students choosing options in item 3i (%)		Frequency of students choosing options in item 3ii (%)	
A	39 (21.55%)	B	6 (3.31%)
B	117 (64.64%) correct	C	101 (55.80%) correct
C	12 (6.63%)	D	3 (1.66%)
D	7 (3.87%)	A	40 (22.10%)
E	5 (2.76%)	E	31 (17.13%)
Undecided	1 (0.55%)		

Surprisingly, many of the participants who chose the correct answer in 3i could not choose the interpretation that matched their answers. This evidence suggests that students’ understanding of basic ideas were unstable.

Item 4i tests basic idea, positive and negative recency effects misconceptions. This item was: A box contains 6 balls: 2 are red, 2 are black, and 2 blue. Three balls are picked at random, one at a time. Each time a ball is picked, the color is recorded, and the ball is put back in the box. If the first 2 balls are black, what color is the third ball most likely to be?

- A. Red
 B. Black
 C. Blue
 D. Red and blue are more likely than black
 E. Red, black and blue are all equally likely

Eighty-nine (89) out of 181 gave the correct answer as “E”. However, 53 students answered “D” showing a negative recency effect. The possible reason for the 19 and 15 students who answered “A” and “C” respectively is belief bias that some colors are lucky. However, there may be other reasons behind their answers. Three students chose “B” while 2 were undecided.

Item 4ii tests the consistencies of the respondents on both positive and negative recency effects misconceptions. Item 4ii asked preservice teachers to identify a specific reason (justification) for the correct answer to item 4i. This item was: Which of the following best describes the reason for the correct answer in the preceding item (4i)?

- A. The third ball should not be black because too many black ones have already been picked.
 B. The picks are independent, so every color has an equally likely chance of being picked.
 C. Black seems to be lucky.
 D. The color red is more likely than any other color.

One-hundred and forty-six (146) students answered correctly by choosing option “B”. However, few students who gave correct answers in 4i answered differently and in a way that did not describe their chosen answers. This shows inconsistencies in students’

reasoning about random events. **Table 3** matched each option in 4i to its interpretation in 4ii and the descriptive statistics of respondents.

Table 3. Relationship between answers to 4i and 4ii

Frequency of students choosing options in item 4i (%)		Frequency of students choosing options in item 4ii (%)	
A	19 (10.50%)	D	12 (6.63%)
B	3 (1.66%)	C	8 (4.42%)
C	15 (8.29%)	A	15 (8.29%)
D	53 (29.28%)	B	146 (80.66%) correct
E	89 (49.17%) correct		
Undecided	2 (1.10)		

Fifty-seven students chose the correct answer in 4ii which does not describe their answer in 4i. This is an indication that preservice teachers' reasoning about uncertain situations is inconsistent. These inconsistencies may be due to the presence of positive and negative recency effects. The respondents who chose "C and D" concerning item 4i were expected to choose "A" in 4ii as their interpretation. However, this has not been the case as most students chose "B".

Item 5 tests representativeness bias, positive and negative recency effects misconceptions. This item was: *If you toss a fair coin and get tails 5 times in a row, what is the chance of getting a tail on the next toss?*

- A. 1
- B. Greater than $\frac{1}{2}$
- C. $\frac{1}{2}$
- D. Less than $\frac{1}{2}$

Fifty-three out of 181 (representing 29.28%) preservice teachers answered correctly by selecting option "C". However, 69 out of 181 (representing 38.12%) preservice teachers answered "A" showing a positive recency effect. Also, 40 (preservice teachers 22.10%) students answered "D" indicating the presence of a negative recency effect. It is also possible that students with representativeness bias would also choose "D". Similarly, 15 (representing 8.29%) preservice teachers answered "B" while 4 (representing 2.21%) preservice teachers were undecided. Many preservice teachers answered correctly item 3i on coin tossing but only a few students answered correctly on item 5 which is similar to item 3i. This finding reveals that preservice teachers' understanding of random event is unstable.

Item 6 tests outcome orientation and equiprobability biases misconceptions. The question was: *Suppose a particular outcome from a random event has a probability of 0.03. Which of the following statements represents correct interpretations of this probability?*

- A. The outcome will not happen.
- B. The outcome will certainly happen about three times out of every 100 trials.
- C. The outcome is expected to happen about three times out of every 100 trials.
- D. The outcome could happen, or it couldn't, either result is the same.

Sixty-seven out of 181 (representing 37.02%) preservice teachers chose the correct answer which is option "C" indicating good reasoning about probability. In all, 68 out of 181 (representing 37.57%) preservice teachers answered "A" and "B" showing outcome orientation bias misconception. Forty-six out of 181 (representing 25.41%) preservice teachers answered "D" showing equiprobability bias misconception.

Item 7 tests basic ideas on probability. The item was: *A fair die is tossed once. What is the probability of obtaining a 5?*

- A. 1
- B. $\frac{5}{6}$
- C. $\frac{1}{6}$
- D. 0

Seventy-three out of 181 (representing 40.33%) preservice teachers chose the correct answer which is option "C" showing the understanding of basic ideas. Thirty preservice teachers (representing 16.57%) chose "A" indicating a lack of fundamental ideas about probability. Sixty-four out of 181 (representing 35.36%) preservice teachers chose "B" and that seems they considered the number 5 instead of the number of times the 5 appeared on the die. Eleven (representing 6.08%) preservice teachers answered "D" indicating that the belief of some students that large numbers on the die are difficult to get. However, 3 (representing 1.66%) preservice teachers were undecided.

Item 8 tests equiprobability bias misconception. The question was: *Two fair dice are tossed once. Which of the following is more probable?*

- A. Obtaining a sum of 11
- B. Obtaining a sum of 10
- C. A sum of 11 and a sum of 10 are all equally likely

Twenty-eight out of 181 which represents 15.47% of preservice teachers chose the correct answer which is option “B”. However, 128 (representing 70.72%) preservice teachers chose option “C” indicating equiprobability bias misconception. Similarly, while 20 (representing 11.05%) preservice teachers chose “A”, 5 (representing 2.76%) preservice teachers were undecided.

The number of preservice teachers who provided appropriate responses were less than one-fifth of the preservice teachers who took part in the study. This reveals that equiprobability bias was very high and stable among preservice teachers sampled. What this implies is that teaching and learning activities for training of Basic School Teachers should be structured to address equiprobability bias misconception, else, they would carry it to the classroom. **Figure 2** presents a sample of preservice teachers’ difficulty in solving the next three items.

Figure 2 reveals the preservice teacher’s inability to apply appropriate strategies to solve items 9, 10, and 11 probability problems.

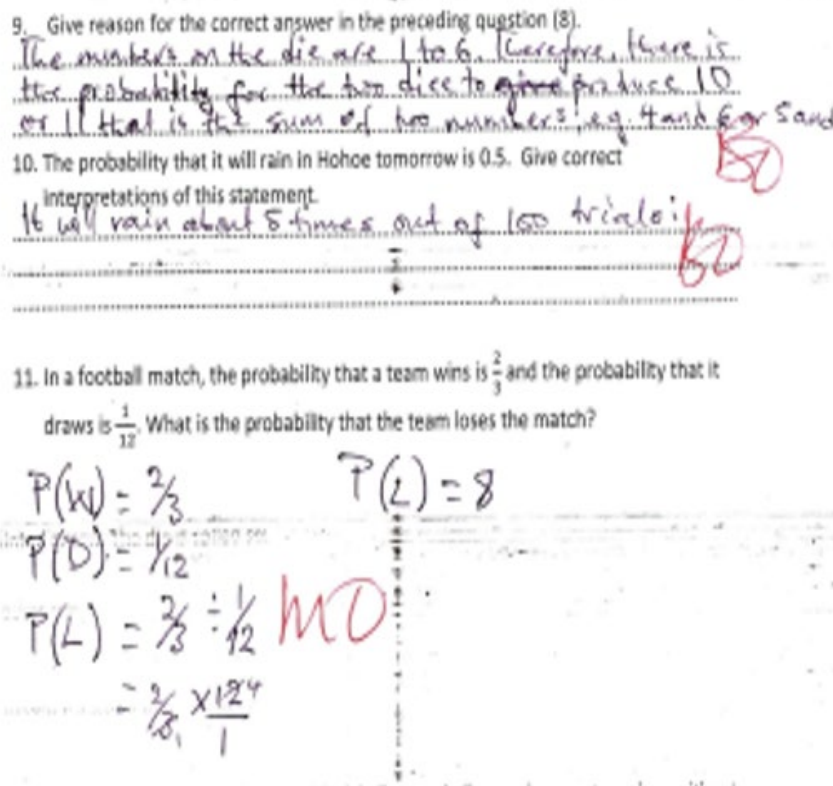


Figure 2. A sample of how a preservice teacher tackled items 9, 10, and 11

Item 9 asked students to give a reason for their answer to the preceding item 8. Students who chose the correct answer which is option “B” did not provide good reasons that could match their chosen answer to item 8. The possible reason for their answer is a belief or a guess. For example, “the sum of 10 will be more probable”. The following reasons in **Table 4** were provided by some of the preservice teachers for choosing “C” on item 8.

Table 4. Some preservice teachers’ reasons for choosing their respective options

Student	Students’ justification for the chosen answer in 8
Student 23	“The probability of getting the sum of 11 and 10 are all equal because the probability of getting 11 is $\frac{1}{2}$ and that of 10 is also $\frac{1}{2}$.”
Student 90	“Since in option A, it is possible of obtaining a sum of 11, and B too it’s also possible of obtaining a sum of 10, I choose option C because two fair dice are tossed once and there might be a possibility of obtaining the sum of 11 and a sum of 10. So, they are all equally likely.”
Student 13	“As for two fair dice, there is the probability that you can get 5 and 6 which sum up to 11 or 6 and 4 which sum up to 10, so they are all equally likely.”
Student 122	“This is because the probability of obtaining either sum depends on how the dice are held.”
Student 169	“A sum of 11 and a sum of 10 are equally likely because two fair dice are tossed once and it’s likely 11 or 10 can likely be obtained. Any side decided by God will occur.”

The reasons provided by the preservice teachers affirm the presence of equiprobability bias in almost all cases including those who chose the correct answer. Preservice teachers’ responses were consistent with their chosen answer “C” indicating equiprobability bias is stable among the preservice teachers. Additionally, *Student 169* was influenced by religious belief. It must be noted that the preservice teachers in this study were Christians and Muslims but their responses did not reveal much about the roles played by the religious background on how they reasoned about uncertainties. The finding of this item suggests that tutors of colleges of education need to provide preservice teachers with the opportunity to confront their misconceptions to develop a conceptual understanding of probability among their preservice teachers.

Item 10 tests outcome orientation bias misconception and understanding of probability statement. The problem was: *The probability that it will rain in Hohoe tomorrow is 0.5. Give a correct interpretation of this statement.*

Surprisingly, quite a number of the preservice teachers could not provide any response. Also, many of the preservice teachers who responded to this item failed to give a correct interpretation of the statement as 'the chance of rain or no rain in Hohoe are the same'. Twenty-one preservice teachers did not respond while one hundred and sixty responded. **Table 5** presents some of the interpretations given by the preservice teachers on item 10.

Table 5. Some preservice teachers' Interpretation of item 10

Student	Interpretation of the probability statement
Student 131	"This means that it may rain in Hohoe tomorrow."
Student 42	"Meaning that it will not rain at all."
Student 16	"The probability that it will rain in Hohoe is 1 out of 20."
Student 106	"The outcome is expected 5 times out of every 100 trials."
Student 144	"This means that it is not probable it will rain in Hohoe."

Student 131 seem to have some idea about item 10 but failed to consider the extent of the possibility in her interpretation. Students 42 and 144 responses as shown in Table 5 revealed outcome orientation misconception since their interpretation indicates affirmation of certainty rather than a measure of likelihood. Every probability between zero and one has a possibility of occurrence. Therefore, it is important teachers' stress the degree or the extent of the possibility of an event as it relates to probability. This is crucial for preservice teachers' applicability of probability in their daily lives. The preceding interpretations given by the preservice teachers indicate conceptual difficulties and interpretation difficulties. Additionally, the preservice teachers' interpretation suggests they were unable to apply the fundamental concept of probability in their life. The possible reason for preservice teachers' inability to interpret simple probability statements may be due to the failure of some teachers at the SHS to relate probability to real-life situations.

Item 11 tests students' ability to identify possible ways something can happen and apply appropriate strategies. None of the preservice teachers were successful in solving this problem. The item was: *In a football match, the probability that a team wins is $\frac{2}{3}$ and the probability that it draws is $\frac{1}{12}$. What is the probability that the team loses the match?*

The following reasons were given by some of the preservice teachers on item 11 when they were interviewed on the strategies, they applied to obtain the solution to the problem.

Lead researcher: What first step did you consider in solving this problem?

Student 20: "The first step to take is to state the probability that a team wins is $\frac{2}{3}$ and the probability that it draws is $\frac{1}{12}$ to guide you solve for the probability that the team loses the match."

Lead researcher: How did you apply it in your solution?

Student 20: "I subtracted the probability of the team that won from the probability that the team draws."

Lead researcher: how did you figure this out?

Student 20: "I figured it out by doing what I think I know."

Lead researcher: explain what you think you know about this problem?

Student 20: "I can't explain because I sincerely don't have a clue about the question." The question was well understood but I don't know how to solve it.

Lead researcher: What step did you follow to solve this problem?

Student 18: "The good step to solve this question is to find out the probability of the team losing since the probability of the team winning has been given. So, I just subtract the probability of the team winning from one to get the probability that the team will lose. That is $1 - \frac{2}{3} = \frac{1}{3}$."

Lead researcher: how did you arrive at this?

Student 18: I arrived at this from practices of previous examples.

Student 18 seems to think that there are only two possibilities that is a win or lose. This respondent displayed interpretation difficulties. Since the participants were unable to identify three possibilities involved as a win, lose, and draw and add the three probabilities and equate it to one as shown below:

$$P(L) + 2/3 + 1/12 = 1$$

$$12 P(L) + 9 = 12$$

$$12 P(L) = 3$$

$$P(L) = 1/4$$

The probability that a team loses the match is $\frac{1}{4}$.

Item 12 tests the participants' understanding of without replacement situations. A box contains 9 red balls and 6 black balls. Two balls are chosen at random without replacement one after the other. Find the probability that both are different in color.

Figure 3 and Figure 4 present the samples of how some preservice teachers responded to item 12.

12. A box contains 9 red balls and 6 black balls. Two balls are chosen at random without replacement one after the other. Find the probability that both are of different colour.

$$P(R) = \frac{3}{9}$$

$$P(B) = \frac{3}{6}$$

$$P(\text{of different colour}) = \frac{3}{9} + \frac{3}{6}$$

$$= \frac{3}{9} \times \frac{6}{2}$$

$$= \frac{2}{3}$$

$$P(\text{different colour}) = \frac{2}{3}$$

Figure 3. A sample of how a preservice teacher tackled item 12

12. A box contains 9 red balls and 6 black balls. Two balls are chosen at random without replacement one after the other. Find the probability that both are of different colour.

$$9 \text{ red balls} + 6 \text{ black balls} = 15$$

$$= \frac{9}{15} + \frac{6}{15}$$

$$= 1$$

$$9 \text{ red balls} + 6 \text{ black balls} = 25$$

$$= \frac{9}{15} * \frac{6}{15} + \frac{9}{15} * \frac{6}{15}$$

$$= \frac{6}{25} * \frac{6}{25}$$

$$= \frac{12}{25}$$

9 red balls and 6 black balls = 2(RB)
~~9 * 6~~
 = 2(54)

Figure 4. A sample of how a preservice teacher tackled item 12

Figure 3 and Figure 4 reveal a lack of understanding of the problem and the application of appropriate strategies. To obtain the reason behind some of these responses provided by the participants on item 12, some of them were interviewed concerning their understanding of the problem and the strategies they applied to solve the problem. In all, none of the respondents succeeded in solving item 12. These were some of the reasons preservice teachers gave to justify their answers when interviewed.

Lead researcher: What first step did you consider in solving this problem?

Student 24: "Firstly, I added the number of balls in the container that resulted in 15."

Lead researcher: How did you apply it in this case?

Student 24: “By expressing the number of red balls which are 9 over the total number of balls. Also, expressing the number of black balls which are 6 over the total number of balls.”

Lead researcher: Did you consider the phrase ‘without replacement’?

Student 24: “I solved it this way because I didn’t know that the phrase ‘without replacement’ actually must be considered.”

Lead researcher: Explain how you solved this problem?

Student 9: “So with my understanding, I decided to find the probability of choosing either of the colors and add them.”

Lead researcher: Explain how you arrived at this?

Student 9: “I used this method based on my understanding logically.”

Lead researcher: What first step did you consider in solving this problem?

Student 15: I find the total number of balls by adding 9 and 6 to get 15.

Lead researcher: How do you understand this problem?

Student 15: “Because the balls are picked without replacement, and the two balls picked are of different colors, so the total number of balls will decrease while each color will be the same.”

Lead researcher: How did you apply your understanding in solving this problem?

Student 15: “Therefore the probability of picking one color multiplied by the probability of picking the other color which causes the change in the total number of the balls.”

Lead researcher: Why did you multiply?

Student 15: “I can’t explain.”

The preceding comments of the respondents revealed the uses of subjective reasoning in interpreting the “without replacement” situations. The respondents’ inability to solve problems about “without replacement” situation was due to lack of understanding of probability statements and the ability to apply appropriate strategies. The respondents faced conceptual difficulties, interpretation difficulties and procedural difficulties. The preceding comments by the respondents indicate that students come to the College of Education with little or no idea on “without replacement” situations in probability.

However, the expected solution for item 12 is as follows: The probability that the two balls are of different colors

$$\begin{aligned}
 &= [P(R_1) \times P(B_2)] \text{ OR } [(P(B_1) \times P(R_2))] \\
 &= \left(\frac{9}{15} \times \frac{6}{14}\right) + \left(\frac{6}{15} \times \frac{9}{14}\right) \\
 &= \frac{54}{210} + \frac{54}{210} \\
 &= \frac{9}{35} + \frac{9}{35} = \frac{18}{35}
 \end{aligned}$$

DISCUSSION OF THE FINDINGS

The findings in this study suggested that several Preservice Teachers entered probabilistic classrooms at college with little or no idea of fundamental concepts of probability. Additionally, preservice teachers’ inability to solve probability problems were due to being unable to identify all possibilities associated with a particular event and applied appropriate strategies which were as a result of conceptual difficulties, interpretation difficulties, and procedural difficulties. Conceptual difficulties showed a failure to grasp the concepts in a problem and inability to see the relationships between concepts in a problem. Interpretation difficulties occurred when preservice teachers were not able to interpret a concept appropriately as a result of misconceptions. Procedural difficulties occurred when preservice teachers were unable to carry out manipulations or algorithms, after concepts were understood. This finding suggests that the current way of teaching probability do not give students the necessary exposure to real-world scenarios (Sharp et al. 2021), hence their failure to make correct interpretation of probabilistic situations. The influence of religion and culture on how preservice teachers’ reason about uncertainties has not been strong. This finding contradicts the results of Amir and Williams (1999) cited by Sharma (2016) and Sharma (2006) that students’ reasoning appeared to have been influenced by religious belief and superstitions. The contradiction might be due to the locations and the participants involved in the studies.

Furthermore, these misconceptions: equiprobability bias, negative and positive recency effects, representativeness bias, outcome orientation, and belief bias impeded students’ ability to solve probability problems. For instance, student 15 said that some colors are lucky when responding to a probability question involving colours. This finding is in agreement with the finding

by Khazanov and Prado (2010) who studied students' misconceptions in Introductory College Statistics. Khazanov and Prado's (2010) results showed that probabilistic misconceptions are widespread. The misconceptions were found to have limited students' ability to solve probabilistic problems correctly.

Also, findings from this study concur with the results of Anway and Bennett (2004) and Karatoprak et al. (2015) who investigated college students' misconceptions in introductory college statistics and found representativeness and equiprobability bias as common misconceptions among students. Similarly, the findings of the study align with Ang and Shahrill (2014) who identified the equiprobability bias, beliefs bias, and representativeness misconceptions among secondary school students. Furthermore, the findings of this study confirm the results of Khazanov and Gourgey (2009) that students' probabilistic misconceptions are common.

Similarly, the findings of this study agree in part with the findings of Blanco and Chamberlin (2019) who conducted a study on preservice elementary teachers' misconceptions in probability and statistics. While this current study agrees with Blanco and Chamberlin (2019) that equiprobability bias is higher among preservice teachers, they disagreed on the representativeness bias. Also, Blanco and Chamberlin (2019) found representativeness bias, and outcome orientation misconceptions as the least misconceptions among the participants in their study but this current study identified representativeness bias as common misconceptions among the participants. Further study in the area of representativeness misconception is needed to ascertain whether or not the differences in the results may be due to the social settings or cultural differences since these studies were conducted in developed and developing countries. However, both studies are conclusive on outcome orientation misconceptions as the least.

Furthermore, the findings of this study affirm the results of Arum et al. (2018) who investigated students' difficulties when solving probabilistic problems. Their study revealed students having several probabilistic difficulties. Arum et al. (2018) study categorizes students' difficulties in probabilistic problem-solving as understanding the probabilistic problem, choosing and using appropriate strategies, and computational process in solving the problem. Besides, the findings of this study are similar to Astuti et al. (2020) who investigated students' difficulties and misconceptions in solving probability problems and found that students exhibit four probabilistic misconceptions when solving probability problems. These are errors in interpreting questions, errors in the procedure of proving the theorems of probabilities, misconceptions in the application of Bayes' rule, and errors in calculating the possibility of an event.

CONCLUSIONS, IMPLICATIONS, RECOMMENDATIONS, AND LIMITATIONS

Based on the results and discussions, it is concluded that preservice teachers entered the college of education stochastic classroom with many probabilistic misconceptions and difficulties in solving probabilistic problems. The misconceptions identified from this study are equiprobability bias, representativeness, positive recency effect, outcome orientation bias, belief bias, and negative recency effect.

Preservice teachers displayed conceptual difficulties, interpretation difficulties, and procedural difficulties in solving probabilistic problems. Teaching of probabilistic concepts should provide students with the opportunity to interpret probability statement while addressing probabilistic misconceptions and how to manipulate algebra in probability problems.

The equiprobability bias, representativeness bias, positive and negative recency effects were higher among all the participants sampled. This suggests that teaching and learning activities should focus more on addressing equiprobability bias, representativeness bias, positive and negative recency effects, and understanding of probabilistic statements. This would prepare preservice teachers to teach and apply probability effectively and efficiently after graduation.

The findings suggest that preservice teachers lack the necessary skills for college of education Stochastic course. So, tutors should design their lessons to take into account the fundamental ideas on probability before building on with what college of education stochastic seeks to add to their knowledge. Additionally, teachers who introduce probability formally to students should make the lesson more practical by designing activities that reflect real-life situations for learners to confront any prior misconceptions they acquired from social experiences and culture.

Furthermore, teacher educators should focus on training teachers on how to identify probabilistic misconceptions and how to address them. This will help preservice teachers to acquire the relevant skills for teaching probability. The preservice teachers in this study were all females; another study could examine the role of gender on probabilistic misconceptions which could shed light on clarifications on male and female reasoning and decision-making ability concerning uncertainties and this will inform teachers on how to structure teaching and learning in single schools or mixed schools. Moreover, a study could be conducted on other areas of probabilistic misconceptions such as conjunction fallacy and human control. This will help in how to structure teaching to address students' needs in these areas of misconceptions. A study could also be conducted on Senior High School Teachers' pedagogical content knowledge use for teaching probability units, since the participants in this study were taught probability at High Schools and they performed below the expected standard. The limitation of this study was the selection of one College of Education in Ghana. The decision to use one college was based on accessibility. Also, the sample size was small for a country with more than forty-six Colleges of Education. Nevertheless, this study sought to provide an insight into the state of probability teaching at basic and senior high schools since the participants came from all the sixteen regions of Ghana where they were taught probability up to selection "with and without replacement" situations involving two events (Ministry of Education, 2010).

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