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The awareness of difficulties in solving rational inequality and a solution proposal

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ABSTRACT

This research aimed to determine the difficulties that the students experience in inequality solutions that contain inequality in general and rational (proportional) expressions, and to solve these difficulties with an activity. Totally 108 students, who were enrolled in the Department of Mathematics and Science Education, consisted respectively of 43 in the 2015-2016, 32 in the 2016-2017, and 33 in the 2017-2018 academic year, participated in the research. It was started with an activity to make students realize the difficulties they experienced and become aware of the stages which they had difficulty. After the activity, the difficulties, and the stages that students had difficulties were determined. At that point, they were informed about the solutions for the difficulties they faced and shown a road map for coping with the stages they had difficulties. At the end of the research, the rational inequality questions were asked to the students in the mid-term and/or final exams, and it was observed that the majority of students did not experience any of the problems which they had experienced before the activity. Another result of the study was that the students had less difficulty in the inequalities related to the concept of greatest integer, which they received only in university education.

Keywords: inequality, rational inequality, inequality solution, absolute value, greatest integer, student awareness

INTRODUCTION

It is stated that the subject of equation and inequality plays a significant role in algebra, linear programming, analysis, and geometry branches of mathematics (Tsamir & Bazzini, 2004), additionally, the subject of inequality is the focus of many topics of geometry (Kaplan & Acil, 2015). The subject of inequality is used in mathematical modelling that represents a real-life situation, abstraction (Karatas & Guven, 2010) and problem-solving strategies (Altun et al., 2007). The inequalities are used with functions such as absolute value, greatest integer, sign, exponential, logarithmic, trigonometric, and in topics such as trigonometry, set software, sorting. The inequalities are also used with the prefixes such as sets where the variable (or variables) is defined, the number of variables and variable order, linear, nonlinear, rational, and the absolute value depending on the function used, trigonometric and logarithmic. Students encounter with some of these (such as linear, rational, absolute value, trigonometric and logarithmic) at different class levels depending on their countries before the university education (Guzel et al., 2010), and with some (such as greatest integer and sign functions) only at the university education. It is suggested that students should learn to represent the situations including the subject of equations and inequalities and solve the forms of equivalent expressions, equations, and inequalities by constructing their meaning (NCTM, 2000). However, it is stated that students' concept images related to the algebraic expression and the equation are incomplete and inaccurate, students at all levels tend to have difficulties in the subject of inequality, have extreme challenges in interpreting inequality solutions, and students who make more mistakes in these subjects demonstrate low achievement levels (Stewart, 2016).

The concept of absolute value, which requires the topic of equation and inequality as a prior condition, is the premise concept of many issues such as series, sequences, convergence, divergence, limit, derivative (Cortes & Pfaff, 2000; Lee, 2002; Pomerantsev & Korosteleva, 2003; Sandir et al., 2007; Stafylidou & Vosniadou, 2004; Tsamir & Bazzini, 2004; Vlassis, 2004; Yenilmez & Avcu, 2009). The concept of absolute value is one of the concepts that require graph using and in which students have learning difficulties and strong prior condition relationships (Ciltas, 2011). In the studies conducted on the concept of absolute value, it has been determined that similar mistakes are made regardless of the class levels (Basturk, 2009; Ciltas et al., 2010; Parish, 1992; Sandir et al., 2007; Yenilmez & Avcu, 2009). It is stated that the missing information and misconceptions in the preliminary subjects turn into prejudice for the concept of absolute value, which is one of the most difficult subjects to understand in mathematics lessons in secondary education, negatively affect the achievement rate (Yenilmez & Avcu, 2009). According to Guntekin and Akgun (2011), the fact that the concept of absolute value is not well-understood causes mistakes in the trigonometric operations. According to Sandir et al. (2007), the most important reasons for conceptual misconceptions are that the definition and geometric interpretation of absolute value are not understood and that the range examination and solution sets are emphasized.

The research is a long-term study. The setup of the study was carried out in 2013. The processes of creating activity form items as a result of the observations by the researcher, checking the usefulness of the items with student feedback and rearrangement of the item were added to this date and the first data of the research was handled in the 2015-2016 academic year: the last data in the 2017-2018 academic year. The literature review was analyzed in three groups according to these research processes. Accordingly, the first review consisted of the process of research setup until 2013; the second review from 2013 to 2018 when the data collection process finished; and the last review from 2019 to 2021 when the research was submitted.

The studies reviewed until the date of research setup, as follows:

1. That the topic selected for the research was not regional was understood from the studies by Bazzini and Tsamir (2001) with the students from Israel and Italy; Lee (2002) with Chinese students; Pomerantsev and Korosteleva (2003) and Rowntree (2009) with American students; Blanco and Garrote (2007) with Spanish students; and Sandir et al. (2007) with Turkish students.
2. That the students made mistakes and inaccuracies in the topics of inequalities and equations were described by Cortes and Pfaff (2000), in their study, as

   “passing a term to the other side of the equation or passing an unknown term to the other side of the equation without changing its sign, making operation with only one side of the equation when simplifying the coefficient of the unknown, making operational error in multiplying both sides of the equation by (-1), furthermore, that some students reversed the division when dividing both sides by the same number, apart from these errors, that some students did not change the direction of the inequality sign when multiplying or dividing the inequality by a negative number.”

   In their study, Bazzini and Tsamir (2001) suggest that

   “the students who explore the inequalities in the traditional way have difficulties when they are offered the “non-traditional” tasks, have some problems when setting relationships with the quadratic inequalities, tend to multiply both sides of an inequality by a negative number without changing the direction of the inequality.”

   Lee (2002) collected the student mistakes in his study under the headings as

   “incorrect calculation of numbers, wrong number in a calculation, missing number, a number is written as a wrong fraction, wrong number in factorization, not change sign when move a term to the other side of the equality, a number is written as a wrong sum, a number is written as a wrong difference, wrong sign on one side of the equality”.

   Tsamır and Bazzını (2004) stressed in their study that

   “participants implicitly and explicitly exhibited two intuitive beliefs: inequalities must result in inequalities and solving inequalities and equations are the same process.” and “students encounter difficulties in reaching single-value solutions to inequality tasks and that they tend to reject a single-value option when directly asked to consider it.”

   Blanco and Garrote (2007) listed the student inequalities in their study, as follows:

   “In passing from ordinary language to algebraic language in terms of an inequality,”

   “In the use and meaning that the students attribute to letters and to algebraic expressions,”

   “They do not take the real numbers as their reference set for their operations, but limit themselves to the natural numbers,”

   “To understand the meaning of interval,”

   “In the meaning of the variable,”

   “To understand the meaning of the greater than and less than signs,”

   “To use the greater than and less than signs, and, in general, inequalities to solve exercises,”

   “To interpret the result of an inequality,”

   “Operational errors,”

   “They give no semantic content to the inequality. They find no conceptual differences between equation and inequality,”

   “On handling expressions that involve the order relation of the real numbers,” and

   “Difficulty of connection between the visual-geometric and algebraic languages.”

Sandir et al. (2007) refer in their study that

> "the students have difficulties in solving these inequalities when two different inequalities are combined and given as the same inequality."

Almog and Ilany (2012) stress in their study that

> "students make errors related to the incorrect usage of logical connectors."

Ural (2012) claims in his study that the students, during solving a rational equation, use cross multiplication, applied to the methods of using necessary operations of eliminating the common factors in the numerator or denominator of both sides or by moving one side of the equation to another side, that the solving equations of the 2nd and 3rd degree are factors that make it difficult to solve the given rational equation.

3. The significance of students being aware of what they do and their knowledge: Vlassis (2004) used the expression of "conceptual change cannot fully occur without the students developing a meta-conceptual awareness of their symbolizing activities" in his study. Delice and Yilmaz (2009) claimed the significance of "how much of the knowledge the learner is aware of and how much s/he can apply to it." Gurbuz et al. (2011) determined in their study that the students and teachers did not regard "the inequalities among the difficult subjects."

The Studies Reviewed During the Data Collection Process for the Study

Sitrava (2017) suggests that

> "the pre-service teachers have basically three kinds of concept images related to the equation: equality, inequality and balance, the majority of the pre-service teachers define algebraic expression as expressions with unknowns and equations as equality and stated that they have deficient, and misconceptions related to these concepts."

Taqiyuddin et al. (2017) used the expression in their study that "the majority approached the question by doing algebraic operations. Interestingly, most of them did it incorrectly". Muttaqin et al. (2017) presented the approach of "transferring informal knowledge into formal mathematical knowledge with activities". Rosyidi and Kohar (2018) used the inequalities in their study to "discover the proving abilities of the students".

The Studies Reviewed After Finishing the Research

Agung et al. (2021) state in their study that

> "solving a rational inequality is difficult for mathematics students. Many of them make a mistake when solving such problem. Many students had mistaken of type errors in inequality rules. Most of them are solving the rational inequalities by treating it as a rational equation. This mistake happens because they misunderstand about the rule of inequality while they are performing multiplication. The other mistake is of type error in writing the solution. The students did not use number line so that leads to incorrect solution set."

Annizar et al. (2020) stress in their study that "in solving the rational inequality problem, students have understood the meaning of set of completion was value x which accomplished inequality to the problem."

It should be regarded that the formation of the premise concepts before the topic of inequality should be structured by students (Botty et al., 2015), feedback should be given, and common mistakes and misconceptions should be focused (Sarwadi & Shahrill, 2014). It is suggested that the content knowledge of the instructor is significant in understanding and analyzing student mistakes (Boz, 2004), that the mistakes decrease as a result of learning and teaching activities (Akyuz & Hangul, 2014; Gurbuz & Erdem, 2015), success (Uyangor & Dikkartin, 2012), and the awareness of the obtained knowledge increase (Delice & Yilmaz, 2009).

Associating mathematics with daily life is among the basic skills in the curricula of countries. Considering that many dimensions are rational in daily life and that relations are expressed with inequalities, it is important to consider these two concepts together. It is expected that this study, carried out with different groups for more than one year and with different groups, will be informative and guiding for students to realize their deficiencies in rational inequality solutions, to identify and eliminate their problems (i.e., awareness of their knowledge and practices). The research aims to make students aware of the difficulties they experience and to realize the stages in which they have a challenge in the solutions of inequalities that contain rational (proportional) expressions and to put forth an activity that enables to cope with these difficulties. The research was started with the researcher's observation throughout the years, at which the students experienced problems with the inequality solutions containing the rational (proportional) expressions. The activity has been planned to identify the difficulties experienced by the students and the stages in which they have difficulty to overcome the problem. The following sub-questions were asked to reach the answer to the main research question, as follows:

1. What is the contribution of this activity to solving the problem?
   a. How do the students become aware of the difficulties they experience and express (realize) the stages at which they have difficulties mathematically at the end of the study?
   b. How did the performed activity contribute to the students' solutions for rational inequality?

METHOD

Among the qualitative research patterns, the case study method, enabling to examine an event in-depth as to help students realize the difficulties, they experience in inequality solutions consisting of rational (proportional) expressions, the stages they had difficulties was employed in the research. The case study is a research method that investigates a recent phenomenon within the framework of its real environment and examines the situations in a multi-dimensional, systematically, and in-depth (Cohen et al., 1997). The case study enables an in-depth investigation of an uncontrollable event or phenomenon. The case under investigation is considered within its components. It is the method that is frequently applied in social sciences. In the study, among the case study patterns, the 'holistic multi-state pattern' was used. Several cases–related to the activity–to be investigated exist in this pattern. Each case is dealt with as a whole in its environment and compared to other cases. In this pattern, it is necessary to examine the same features for each case, to collect data about the same dimensions and to give information about the same problems (Yildirim & Simsek, 2008).

Universe and Sampling

The research was carried out with 108 first class students studying at the Division of the Secondary School Mathematics Education in the Department of Mathematics and Science Education in the Faculty of Education in Erzincan, in the 2015-2016, 2016-2017, and 2017-2018 academic year, respectively with 43, 32, and 33 students who voluntarily participated in the study. It is obligatory to successfully complete the four-year of high school education, enter the examination by the Student Selection and Placement Center (OSYM) and get sufficient score in a definite type (such as numeric, verbal, and equal weight) to register at this program and any program in higher education. The field types, special conditions, if there is, and the quota tables are announced by the OSYM before the exam every year. The quotas may vary year by year. The students registered to a teaching department take the courses (compulsory, elective), whose contents were prepared by the Council of Higher Education (YOK). To register to the Department of Secondary School Mathematics Teaching, some special conditions such as gender, age are not considered. The students in the study group are those, who took general mathematics (which does not state in the curriculum changed in the 2018-2019 academic year) course. The general mathematics course consists of the contents as

> "the set of natural numbers, the set of integers, the set of rational numbers, the set of real numbers and their properties, quadratic equations and inequalities, analytical analysis of lines, analytical analysis of circles and related applications. Function concept, polynomials, rational functions, trigonometric functions, hyperbolic functions, exponential and logarithmic functions, and elementary functions consisting of their inverses, graphs of functions. Principle of induction, sum and product symbol properties, basic concepts of series and series, complex numbers, and their properties."

In the mathematics teaching program, the concept of inequality first take place in the 8^{th} class with the heading of "inequalities", in 9^{th} class with "first degree inequalities", in 11^{th} class with "inequalities and systems of inequalities with a quadratic unknown", in 12^{th} class with "exponential and logarithmic inequalities." The concept of absolute value take place first in the 9^{th} class with the heading of "first degree inequalities". The participants had not taken the subjects of "integer and signum" functions that require using inequality before the graduate education.

Data Collection Tools

In the research, the activity forms, which were formerly prepared, and the mid-term and final exams were used as the data collection tools. The activity forms were created as a result of the researcher's observations in years with identifying the common mistakes and mistakes made by the students about inequalities, developing solutions for them, checking whether the offers developed work in eliminating mistakes and mistakes and bracketing the useful suggestions. The statement:

> "The questions given in the two stages below are prepared to be used in research on rational, in particular, absolute-valued inequalities. Your personal information will never be shared. You need to answer the questions given sincerely because of the research pain. Volunteering will be the basis for participation. Accordingly, those who write "I do not want to participate" in their answers will be considered reluctantly and will not be taken into consideration in the research. Thank you in advance for your participation."

exists in the introduction part of the activity form. Among the stages, in, there is the statement the 1^{st} stage as "find the solution sets of the inequalities given below. Please briefly write the reason for your actions." and in the second stage, "If you examine the solutions of the inequalities given now and compare them with the solutions you have made, write the mistakes (or missing-excess) steps you made." There. The questions asked in the activities and exams are not given in this section in order to avoid repetition, since they are indicated in the tables for each academic year in the section of data analysis.

Data Collection and Analysis

The data were collected in the activity which was held in the first month of each academic year; 10 minutes were given to answer each question and five minutes to make an evaluation in the activity; in the exam, the examination period determined in the regulations was taken into consideration. The forms, which were filled during the activities, were coded as "student-year-number" and they were transferred one by one to the computer environment. As the conceptual structure was previously determined, the descriptive analysis was used in the data analysis (Yildirim & Simsek, 2008).

Table 1. Students' comments on their solutions by the years

Education in the academic year	**2015-2016**		**2016-2017**		**2017-2018**		**Total**	
The number of participants	**43**		**32**		**33**		**108**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
Those who made mathematical expression on solution, made a comment	36	84	28	88	16	48	80	74
Those who did not make mathematical expression on solution or any comment	7	16	4	12	17	52	28	26
Total	43	100	32	100	33	100	108	100
Themes of comments								
Not making table	20	55	8	29	5	31	33	41
Doing cross multiply	10	28	7	25	3	19	20	25
Ignoring the denominator	4	11	4	14	1	6	9	11
Operation error	-	-	6	21	2	13	8	10
Failure to bring into inequality form	-	-	-	-	5	31	5	6
Applying the wrong rule in absolute value	1	3	2	7	-	-	3	4
Multiplying both sides of an inequality by the same unknown	1	3	1	4	-	-	2	3
Total	36	100	28	100	16	100	80	100

The students' mathematical interpretations related to their solutions during the activity were gathered under the themes of "not making table", "doing cross multiply", "ignoring the denominator", "operation error", "failure to bring into inequality form", "Applying the wrong rule in absolute value", and "multiplying both sides of the inequality by the same unknown".

The students' solutions in both activities and exams were evaluated under the headings of "reaching the correct solution with the right steps" and "Inability to reach the right solution by making the right steps and making mistakes". The correct steps are determined as "bringing into inequality form" and "making a table" and their contents are given below.

Bringing it to the inequality form

Regulating inequality so that one side of the inequality sign remains zero.

Making a table

Determining the zeros and signs of the polynomials state in the numerator and the denominator according to the values of the variable and demonstrating them in separate lines by making a table. In the next line, determining the sign of the rational expression existing in the numerator and denominator with the help of denominator and denominator signs. Writing the solution set by stating that the value that makes the numerator zero also makes the rational expression zero and the value that makes the denominator zero cannot be a solution.

The mistakes made by students, who used the right steps and but did not reach the correct solution, were determined as "equation solving", "writing solution set", and "operation error".

FINDINGS AND COMMENTS

In this section, the answers of the participants to the research questions are presented in tables by the years. Besides, the answers of each participant were examined one by one and evaluated in detail.

The findings related to the sub-problem 'How do the students become aware of the difficulties they experience and express (realize) the stages at which they have difficulties mathematically at the end of the study?' are presented below.

After the questions of the activity were solved step-by-step and showing the ways by the researcher, the comments of the students related to their solutions are presented in the table below, in the themes by the years.

As it can be seen in **Table 1**, although 74% of the students participating in the activities made a comment expressing a mathematical meaning for their solutions, 26% did not make any comment expressing a mathematical meaning or made any comment. The highest rate of comments (88%) was made in the 2016-2017 academic year, on the other hand, the lowest (48%) was in the 2017-2018 academic year. In the distribution of the comments by themes, the highest (41%) is the theme of "not making a table" and the lowest (3%) is the theme of "multiplying both sides of the inequality with the same unknown".

Some of the mathematical comments by the students on their solutions are given below.

The opinion of the "St-2015-3" coded student is, as follows (**Figure 1**):

> "I valued. But I could not think it did not provide it after 0. I forgot that 0 cannot come to the denominator."

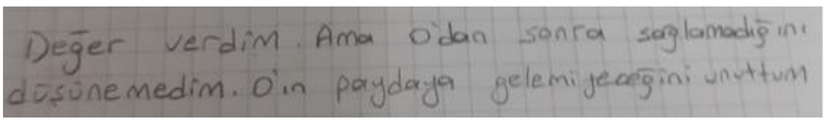

Figure 1. The opinion of the "St-2015-3" coded student in Turksih

The opinion of the "St-2016-1" coded student is, as follows (**Figure 2**):

> "I made a mistake in cross multiply in the system of inequality. I found the solution set wrong. I thought for each, but it's wrong."

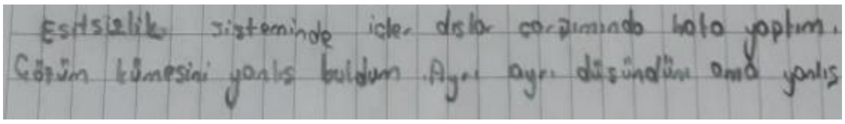

Figure 2. The opinion of the "St-2016-1" coded student in Turksih

The opinion of the "St-2016-15" coded student is, as follows (**Figure 3**):

> "I have lost the root that is '0', as I did not collect all the expressions on the *x* denominator".

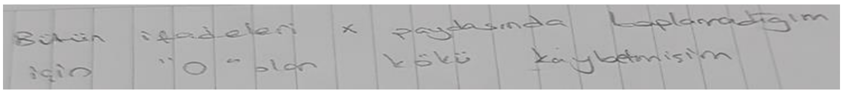

Figure 3. The opinion of the "St-2016-15" coded student in Turksih

The opinion of the "St-2017-15" coded student is, as follows (**Figure 4**):

> "I should have made a table, I had to make a sign analysis according to the table."

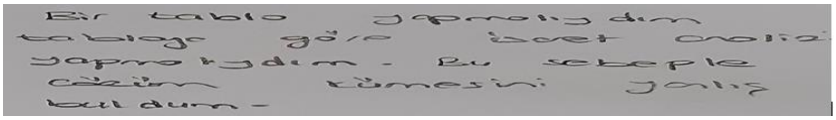

Figure 4. The opinion of the "St-2017-15" coded student in Turksih

There were even comments that were not mathematically meaningful among the students' comments. Some of them are, as follows:

> "I misunderstood the problem."

> "I could not go further as I was afraid to continue the post-operations."

> "There are problems in the steps, I need to correct it."

> "I finished my equation in a shorter time. I did not solve inequality."

> "I had trouble creating the equation."

> "It's all incorrect."

> "I could not find a solution as there was a mistake in the 2nd step."

> "(Pointing a step) I made a mistake after that."

> "I do not know how to solve it as I have not studied on the subject."

> "I've solved it incorrectly from the beginning. I forgot."

Findings related to the sub-question 'How did the performed activity contribute to the students' solutions of rational inequality?' are presented below.

The data obtained during and post-activity activity for the 2015-2016 academic year are given in **Table 2**.

As it is understood from **Table 2**, the number of students (43), who participated in the activity, is different from the number of students (48) attending the exam, that is, five students attended the exam without participating in the activity (these students were among the six students who had got zero point). None of the participants of the activity could reach the correct solution with the right steps. While 45 students, who took the exam, went to the solution by making a table, three students did not make a table during solving the problem. In addition, 13 students reached the correct result with the right steps and got full points, 32 students could not take full points as they made mistakes. The mistakes of the students collected under the themes of "solving equation (seven students)", writing solution sets (seven students), and "operation error (18 students)".

There are examples of students' mistakes with their themes below.

Table 2. Data obtained during and post-activity for the 2015-2016 academic year

2015-2016 activity		
Question: $\left\|\frac{1}{x+1}\right\| \geq 4$. Find the solution set of the inequality.		
Participating in the activity on inequality.		43
Reaching the right solution with the right steps.		0
2015-2016 mid-term exam		
Question 2 option B; find the solution set of $\frac{x^2+3x+1}{x^2-1} > 0$.		
Scores	0 point	6
	3 points	5
	4 points	1
	5 points	6
	8 points	12
	10 points	2
	12 points	1
	13 points	2
	15 points	13
Total		48
Creating a table	Yes	45
	No	3
	Total	48
Those who create table & solve question	Correct	13
	Incorrect	32
	Total	45
	Errors made	
	Solving equation	7
	Writing solution set	7
	Operation error	18
	Total	32

Writing a solution set: The St-2015-17 coded student took the numerator and denominator as the combined set of the solution sets of the equations instead of removing the elements that make the denominator zero from the intersection set of the solution sets of equations that have the numerator and denominator as shown in **Figure 5**.

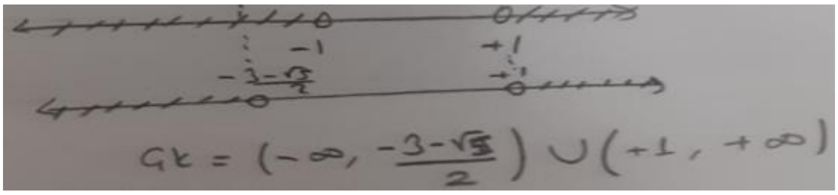

Figure 5. The opinion of the "St-2015-17" coded student's solution in Turksih

Equation solving: The St-2015-28 coded student stated that the quadratic equation in the denominator is always positive (that is, it does not have the roots), as shown in **Figure 6**.

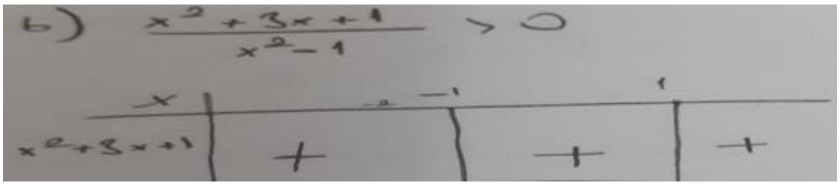

Figure 6. The opinion of the "St-2015-28" coded student's solution in Turksih

Operation error: Although the St-2015-8 coded student used the correct formulas for the equation stated in the denominator, s/he made a mistake in finding roots (took *b*, 1 instead of 3), as shown in **Figure 7**.

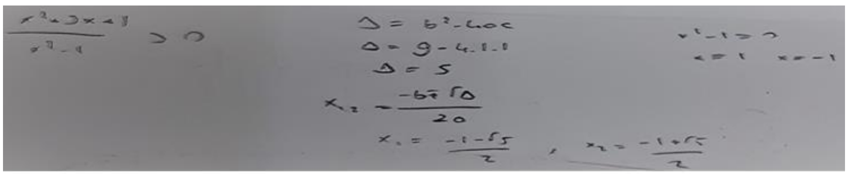

Figure 7. The opinion of the "St-2015-8" coded student's solution in Turksih

Table 3. Data obtained from the midterm exam for the 2016-2017 academic year during and post-activity

2016-2017 activity		
Question: Find the solution set of $\left\|\frac{2}{x}-3\right\| < 5$ inequality.		
Those participating in the activity on inequality.		32
Those reaching the correct solution with the right steps.		0
2016-2017 mid-term exam		
Question 2 option B; find the solution set of $\frac{1}{x-1}+1>x$.		
Inequality form	Bringing	36
	Not bringing	11
	Total	47
Scores	0 point	13
	2 points	3
	3 points	2
	4 points	6
	6 points	1
	8 points	6
	10 points	3
	12 points	13
Total		47
Creating a table	Yes	34
	No	13
	Total	47
Those creating table and solve question	The right ones	13
	Those who do wrong	21
	Total	34
	Errors made	
	Equation solving	6
	Writing solution set	8
	Operation error	7
	Total	21

The data obtained from the midterm exam during and post-activity for the 2016-2017 academic year are presented in **Table 3**.

As it can be seen from **Table 3**, the number of students participating the activity (32) is different from the number of students attending the exam (47), and 15 students attended the exam without participating the activity (13) of these students got zero point; two of them two points. None of the participants of the activity reached the correct solution with the right steps. While 36 students, who participated in the exam, brought the question to the inequality form, 11 students solved the question without bringing it to the inequality form. While 34 students, who took the exam, reached to the solution by making a table, 13 students did not make a table. 13 students, who reached to the solution by making a table, reached the correct solution with the right steps and got full points, while 21 students did not get full points as they made mistakes. The mistakes of students were grouped under the themes of "equation solving (six students)", writing a solution set (eight students), and "operation error (seven students)".

There are some examples of students' mistakes with their themes below.

Writing a solution set: The St-2016-16 coded student used an inappropriate symbol ("=" instead of "ϵ") in the set representation, as shown in **Figure 8**.

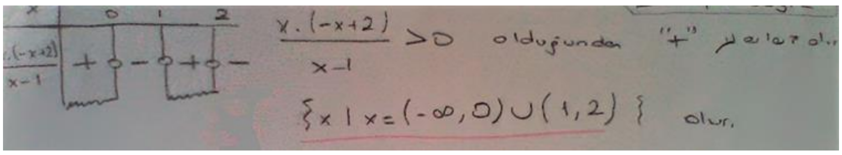

Figure 8. The opinion of the "St-2016-16" coded student's solution in Turksih

Equation solving: The St-2016-6 coded student solved the quadratic equation incorrectly by simplifying "*x*" from both sides of the equation without checking whether it is the opposite (**Figure 9**).

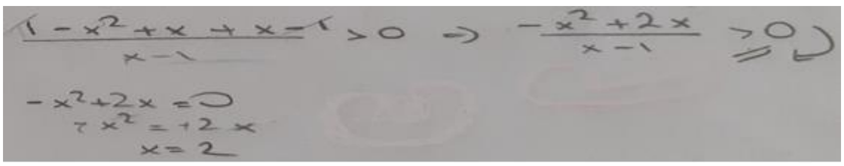

Figure 9. The opinion of the "St-2016-6" coded student's solution in Turksih

Operation error: The St-2016-28 coded student made a mistake by multiplying both sides of the inequality by "*x*-1" (by taking the product of [*x*-1] and 1/[x-1] without checking whether it is the opposite), as shown in **Figure 10**.

Table 4. Data obtained from the final exam in the 2016-2017 academic year

2016-2017 final exam		1. Question: Find solution set of $\frac{2}{x-1} < \frac{1}{x+1}$	2. Question: Find solution set of $\left\lvert\frac{x}{1-x}\right\rvert > 1$
Inequality form	Bringing	38	35
	Not bringing	8	11
	Total	46	46
Scores	0 point	10	32
	8 points	12	8
	10 points	24	6
	Total	46	46
Creating a table	Yes	38	35
	No	8	11
	Total	46	46
Create table & solve question	Those with the correct solution	24	6
	Those with an incorrect solution	12	29
	Total	36	35
		Errors made	
	Equation Solving	-	
	Writing the Solution Set	5	9
	Operation Error	9	20
	Total	14	29

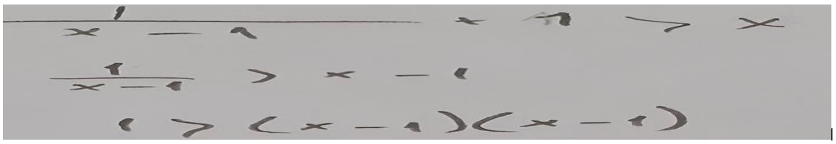

Figure 10. The opinion of the "St-2016-8" coded student's solution in Turksih

The data obtained from the final exam of the 2016-2017 academic year are given in **Table 4**.

As it can be seen from **Table 4**, 38 students, who took the exam, solved question 1 by creating a table and by bringing it to the form of inequality. Eight students solved question 1 without bringing it to the inequality form or making a table. While 24 students, who went to the solution by making a table, reached the correct solution with the right steps, 14 students could not get full points as they made mistakes. The mistakes of the students were grouped under the themes of "solution set writing (five students)" and "operation error (nine students)".

There are some examples of students' mistakes with their themes below.

Writing a solution set: The solution set written by the St-2016-30 coded student is far from a set written mathematically (**Figure 11**).

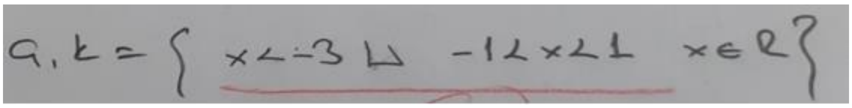

Figure 11. The opinion of the "St-2016-30" coded student's solution in Turksih

Operation error: The St-2016-22 coded student made a mistake in equalizing the denominator (multiplying in the wrong denominator) in the solution of the inequality (**Figure 12**).

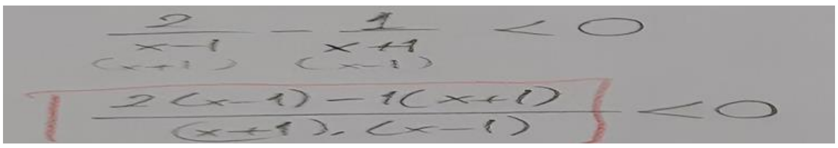

Figure 12. The opinion of the "St-2016-22" coded student's solution in Turksih

As can be seen from **Table 4**, 35 students, who took the exam, solved question 2 by bringing it to the form of inequality and making a table. On the other hand, 11 students solved question 2 without bringing it to the inequality form or making a table. While 38 students, who took the exam, reached to the solution by making a table, eight students did not make a table. While six students, who reached to the solution by making a table, reached the correct result with correct steps, 29 students could not get full points by making mistakes. The mistakes of the students were grouped under the themes of "writing solution set (nine students)" and "operation error (20 students)".

There are some examples of students' mistakes with their themes below.

Writing a solution set: The St-2016-6 coded students used “*x*=“instead of “*x*ϵ” in the solution set (**Figure 13**).

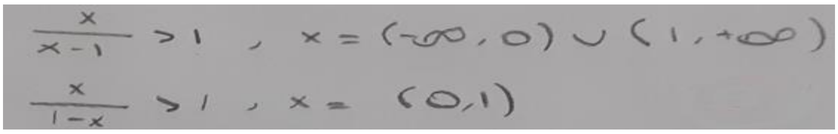

Figure 13. The opinion of the “St-2016-6” coded student’s solution in Turksih

Operation error: The St-2016-26 coded students made a mistake by multiplying every part of the inequality with “*x*-1” without checking whether it was the opposite (**Figure 14**).

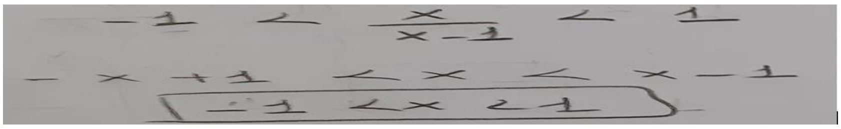

Figure 14. The opinion of the “St-2016-26” coded student’s solution in Turksih

Data obtained from the 2017-2018 academic year activity and mid-term exam are given in **Table 5**.

As it can be seen from **Table 5**, the number of students participating the activity (33) is different from the number of students attending the exam (46), and 13 students attended the exam without participating the activity (while these students got zero to eight points from the 3rd question, they got zero from the 5th question). None of those, who participated in the activity, reached the correct solution with the right steps. While 37 students, who took the exam, reached to the solution by bringing the 3rd question to the inequality form, nine students solved the 3rd question without bringing the inequality form. While 38 students, who took the exam, reached to the solution by making a table for the 3rd question, eight students did not make a table. Although 23 students, who reached to the solution by making a table for the 3rd question, found the correct solution with the right steps and got the full points, 15 students did not get the full score as they made mistakes. The mistakes of the students were grouped under the themes of “writing solution set (nine students)” and “operation error (six students)”.

There are some examples of students’ mistakes with their themes below.

Writing a solution set: The St-2017-24 coded student wrote the combination of the two sets instead of writing the common property (propositional form) provided by *x* after the vertical line in braces (**Figure 15**).

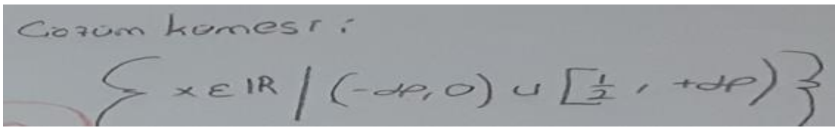

Figure 15. The opinion of the “St-2017-24” coded student’s solution in Turksih

Operation error: The St-2017-6 coded student made a mistake by multiplying all sides of the inequality by “x” without checking whether it was the opposite (**Figure 16**).

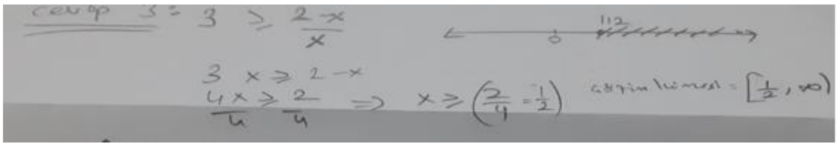

Figure 16. The opinion of the “St-2017-6” coded student’s solution in Turksih

As can be seen from **Table 5**, while 35 students, who took the exam, reached to the solution by bringing the 5th question to the inequality form, 11 students solved the 5th question without bringing the inequality form. Although 30 students, who took the exam, for the 5th question, reached to the solution by making a table, 16 students did not make a table. While two students, who reached to the solution by making a table for the 5th question, reached the correct solution with the right steps, 28 students could not get the full score as they made mistakes. The mistakes of students were grouped under the themes of “writing solution set (one student)” and “processing error (27 students)”.

There are some examples of students’ mistakes with their themes below.

Writing a solution set: The St-2017-29 coded student could not write the solution set (since s/he did not write the connectors, s/he could not decide whether it was the intersection or combination sets) (**Figure 17**).

Table 5. Data from the activity and midterm exam for the 2017-2018 academic year

2017-2018 activity			
Question: Find solution sets of inequalities.		$\frac{2}{x}-3<5$	$\left\|\frac{1}{x}+1\right\| \geq 1$
Participating in the activity on inequality.		33	33
Commenting on the solution.		16	16
Reaching the correct solution with the right steps.		0	0
2017-2018 mid-term exam			
2017-2018 mid-term exam		3. Question: $3 \geq \frac{2-x}{x}$. Find set of all real numbers that provide inequality.	5. Question: $\left\|\frac{2-x}{3}\right\| \geq x$. Find set of all real numbers that provide inequality.
Inequality form	Bringing	37	35
	Cannot bring	9	11
	Total	46	46
Scores	0 point	8	25
	1 point	-	1
	2 points	-	6
	3 points	1	1
	4 points	-	5
	5 points	1	2
	6 points	1	2
	7 points	-	1
	8 points	7	1
	9 points	5	-
	10 points	23	2
	Total	46	46
Creating a table	Yes	38	30
	No	8	16
	Total	46	46
Create table & solve question	Those with the correct solution	23	2
	Those with an incorrect solution	15	28
	Total	38	30
		Errors made	
	Equation solving	-	-
	Writing solution set	9	1
	Operation error	6	27
	Total	15	28

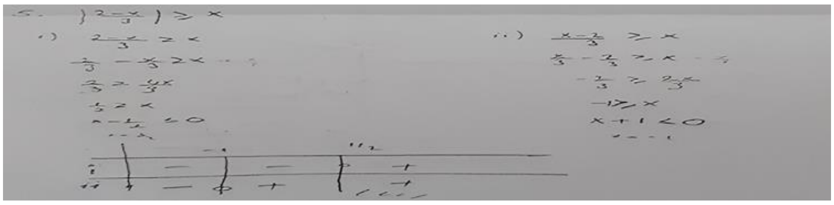

Figure 17. The opinion of the "St-2017-29" coded student's solution in Turksih

Operation error: The St-2017-32 coded student made a mistake as s/he used "∧" instead of "∨" connector (**Figure 18**).

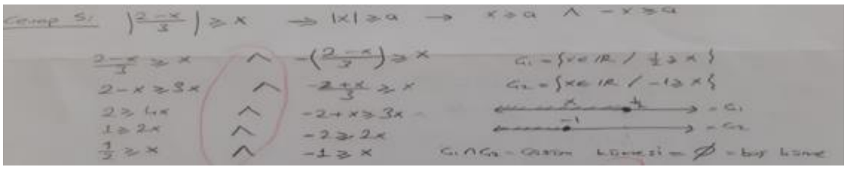

Figure 18. The opinion of the "St-2017-32" coded student's solution in Turksih

The data obtained from the final exam of the 2017-2018 academic year are given in **Table 6**.

As it can be seen from **Table 6**, 35 of 45 students, who took the exam, solved the question by bringing it to the form of inequality and making a table. On the other hand, 10 students solved the question without bringing it to the inequality form or making a table. While 12 students, who reached to the solution by making a table, reached the correct solution with the correct steps, 23 students could not get full points as they made mistakes. The mistakes of the students were grouped under the themes of "writing the wolution wet (11 students)" and "operation error (12 students)".

There are some examples of students' mistakes with their themes below.

Table 6. Data from the final exam for the 2017-2018 academic year

2017-2018 final exam		Question 5. $\left[\!\left[\frac{1}{x+1}\right]\!\right] = -3$. Find set of all real numbers that provide equality.
Inequality form	Bringing	35
	Cannot bring	10
	Total	45
Scores	0 point	1
	4 points	1
	5 points	4
	6 points	8
	7 points	2
	8 points	8
	9 points	9
	10 points	12
	Total	45
Creating a table	Yes	35
	No	10
	Total	45
Create table & solve question	Those with the correct solution	12
	Those with an incorrect solution	23
	Total	35
	Errors made	
	Equation solving	-
	Writing solution set	11
	Operation error	12
	Total	23

Writing a solution set: The solution set written by the St-2016- 30 coded student is mathematically far from a set writing.

Writing a solution set: The St-2017-28 coded student made a mistake by writing a "+" symbol (s/he must have intended to write "U") between the two sets during writing the solution set (**Figure 19**).

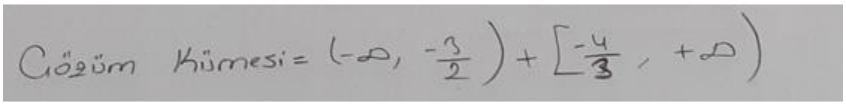

Figure 19. The opinion of the "St-2017-28" coded student's solution in Turksih

Operation error: The St-2017-28 coded student made mistake as s/he transferred the "-2" number to the left side of the inequality as "-2" (**Figure 20**).

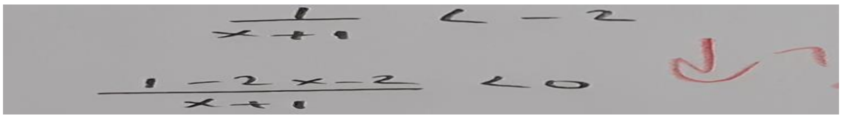

Figure 20. The opinion of the "St-2017-28" coded student's solution in Turksih

DISCUSSION AND CONCLUSION

In this section, the results reached as a result of the analyses of the data obtained from the activity and post-activity exams will be stated, discussed, and some recommendations will be presented.

At the end of the activity, the results related to the sub-question 'How do the students become aware of the difficulties they experience and express (realize) the stages at which they have difficulties mathematically at the end of the study?' are presented below.

Approximately three-quarters (74%) of students compared their solutions with the researcher's solution and made a mathematical comment about their solutions. The distribution of the comments to the themes from the highest percentage to the lowest percentage is, as follows:

Not making table (%41)

Cross multiply (%25)

Ignoring the denominator (%11)

Operation error (%10)

Failure to bring inequality form (%6)

Apply the wrong rule in absolute value (%4)

Multiply both sides of the inequality by the same unknown (%3)

Approximately a quarter of students (26%) expressed their comments without mathematical meaning or did not make any comment. Their mathematics literacy levels may be the reason for this.

The results reached related to the sub-problem as 'How did the performed activity contribute to the students' solutions of rational inequality?' are presented below.

None of the students (108 students), who participated in the activity, could reach the correct solutions with correct steps. This result demonstrates similarity with the result reached by Ural (2012) as "the success rate is 0% in case of mutual destruction of common expressions on both sides of equality and multiplication of their denominators in a rational equation", by Botty et al. (2015) as "the questions with high-difficulty level cannot be answered by any student", and by Taqiyuddin et al. (2017) as "many students fail in algebraic operations."

After the Activity

The number of students, who followed the right steps among those who made rational inequality questions in the exams, was more than the number of students, who participated in the activity. Participating in the activity contributed to the students' taking the right steps. The number of students, who got full points or insufficient points from the rational inequality questions in the exams, was more than the number of students, who participated in the activity and made a mathematical comment about the solution. Participating in the activity and making comments on their solutions contributed to students getting points from these questions. This result is similar to the result of Akyuz and Hangul (2014) as "the activities performed to reduce the students' mistakes in 'equations with a first-degree unknown'" and the result of Uyangor and Dikkartin (2012) as "success increases as a result of learning and teaching activities."

The number of students, who get full or insufficient points from the inequality questions consisting of absolute value, specifically the number of students, who get full points, is quite low. In addition, the number of students who get full or insufficient points in the greatest integer question is quite high. The reason for this situation may be that they did not receive the greatest integer at a course in their previous education and they first learned by correctly structuring at the university. Besides, they learned the absolute value without proper structuring in secondary, high school education and private teaching institutions, and they insist on the information they cannot configure (considering that they will make the operation easier without changing what they receive at the university). This result is similar to the result reached by Sandir et al. (2007) as "one of the most important reasons of conceptual misconceptions is that the definition of absolute value is not understood" and by Yenilmez and Avcu (2009) as "the concept of absolute value was a problem in the primary school years when the concept was first encountered" and the result by Taqiyuddin et al. (2017) as 'students are expected to have a good conceptual understanding in addition to the operational skill'.

The number of students, who get missing points due to their mistakes, is also quite high even though they used correct steps in inequality questions in exams. The reason for this is that the students do not perceive their mistakes as errors, as it can be understood from the percentages in their comments to the solutions they reached at the end of the activity. Errors of the students were determined to be as solving equation, writing solution set and operation error. The error of solving equation occurs specifically in solving the quadratic equation. This result is similar to the result of Gurbuz et al. (2011) as "'the second-and third-degree inequalities' subject is a problem with a high index of difficulty" both for students and teachers" and the result by Delice and Yilmaz (2009) as "students make efforts to reach results by factoring in quadratic equation solutions" and the result by Ural (2012) as "when the difficulties faced by the students are examined, 88% of the 3rd degree and 43% of the 2nd degree equation arises in the solution process." The most important of the operation error is cross multiplication. On the other hand, the errors of writing solution sets are the misusing or not using of logic connectors, using the wrong symbol and not being able to write the desired set-in accordance with set writing. This result demonstrates similarity with the result of Botty et al. (2015) as "students cannot scan the region determined by points that verify inequality", by Taqiyuddin et al. (2017) "students do not pay attention to exactly what an inequality solution means", and the result of Switzer (2014) as "the relationship between inequality and its graphic is not understood in-depth"

Sorting is made between items that can be compared (comparable) with a set and a correlation on it that provides certain conditions (reflection, inverse symmetry, and transition). The inequality, on the other hand, is the order that contains at least one unknown in which set is taken value and is the proposition at which values are assigned from the set in which this value is the unknown. Determining the values that make the proposition right will mean solving the inequality.

"Natural numbers", "integers", "rational (proportional) numbers", "real (real) numbers" sets, except for the complex number set and orthogonal (Cartesian) product sets, are the ordered sets. Absolute zero, relative (defined) zero (for negative-positive or neither) gain significance in the order relations in these sets.

The problems related to inequalities according to the sequence of priority-subordination are the definition of inequality, finding a solution set (algebraic solution or process step), writing the solution set, and graphical representation of the solution set in the set in which the inequality is defined (often requested as a scanning or geometric representation). Developing solutions with detailed analysis of each of the problems will be suitable for the successive structure of mathematics.

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