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# The best pedagogical practices for teaching mathematics revisited: Using math manipulatives, children's literature, and GeoGebra to produce math confident young people for a STEM world

Joseph M. Furner 1\* 🗓

<sup>1</sup>Florida Atlantic University, Jupiter, FL, USA \*Corresponding Author: jfurner@fau.edu

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#### **ABSTRACT**

Today math teachers can best reach their students and show them how math surrounds us by using manipulatives, children's literature, and GeoGebra while teaching mathematics. These are some of the best pedagogical practices for teaching mathematics today. In our high-tech world, students need to be proficient in science, technology, engineering, and mathematics (STEM) fields. As endorsed by National Council of Teachers of Mathematics (2000) and stressed in common core state standards in mathematics, it is important that we teach with technology, address dispositions and math anxiety, and make the math that young people are learning pertinent and meaningful. Frequently, it may be best to start teaching young people geometry first as opposed to numbers, which are considered more abstract and difficult to learn. Geometry is one of the most tangible divisions of math and concentrating on this first can help students' whole view of mathematics and their insolences towards learning the subject. Nowadays teachers also need to be cognizant and checking for attitudes and dispositions toward learning mathematics, as math anxiety is an issue in today's classrooms. This paper will revisit the best pedagogical practices for teaching math the review of the use of math manipulatives, children's books, and GeoGebra to help teachers create mathematically confident young people.

MODESTUM

Keywords: math anxiety, GeoGebra, manipulatives, children's math literature, real-world connections, STEM

# INTRODUCTION

Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me ... and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come (Williams, 1988).

A good mathematics teacher is always an empathetic one. Another way of engaging the students and improving their learning in the subject is by showing empathy and understanding of what is going on in the students' lives (Maloney, 2022).

Math anxiety is a tangible phenomenon, and many young people are confronted with this when learning math at various grade levels Kindergarten through college today. Many people often go through such math anxiety through their whole life, and it can often affect decisions in life as well as the career selections one will make. It is imperative that all people feel confident in their ability to do math in an era that depend on so profoundly on science, technology, engineering, and mathematics (STEM) areas and problem solving. It really is a school's obligation to see that their students' value and feel confident in their ability to do math and use technology to learn because ultimately, all choices individuals make and choosing of careers may be resolute in part by their dispositions toward mathematics. Math anxiety is a real phenomenon that has been researched for many years now (Alday & Panaligan, 2013; Beilock & Willingham, 2014; Boaler, 2008; Quander, 2013; Richardson & Suinn, 1972; Scieszka & Smith, 1995; Williams, 1988).

Mathematics educators need to take it seriously and use research to address the problem in an age of STEM. Resources and websites for addressing math anxiety and improving attitudes as well as incorporating technology like GeoGebra, math manipulatives, photography, and children's literature books will all be shared and included in the paper. Today it is critical we help to create mathematically confident young people in STEM world we now live in.

## Math Anxiety Is a Real Phenomenon Andmanyindividuals Today Suffer from It

Today math anxiety continues to devastate our society and distresses our young peoples' success and achievement within this subject area of mathematics (Finlayson, 2014; Quander, 2013). Quander (2013) found elementary teachers need to help make students to be lifetime learners and advance a productive mathematical temperament so that they are prepared for advanced schooling and eventual careers, many which may be STEM related. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life. The awareness as educators to exam math anxiety levels, enthusiasm to learn mathematics, and using advanced technologies like GeoGebra to instruct and motivate learners is critical today in a global society of STEM and also can impact achievement areas of learners (Furner, 2019, 2022; Gonzalez-DeHass et al., 2017, 2023; Furner & Marinas, 2016, 2020).

Educational organizations like National Council of Teachers of Mathematics (1989) believe that mathematics teachers need to assess students' mathematical disposition regularly regarding: checking for confidence in using math to solve math problems, communicate thoughts, and reason; being adaptable in exploring mathematical concepts and employing a variety of approaches when solving problems; preparedness to persist in mathematical problems; interests, inquisitiveness, and ingenuity in doing math; student ability to reflect and monitor their own thinking and performance while doing math; and value and appreciate math for its real-life application, connections to other subjects and norms and as an instrument and idiom of its own.

Young people often say: "I like the math class because of the teacher" since the math teacher knows how to present developmentally the subject matter, creates a learning atmosphere advantageous to learning with empathy, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students to address math anxiety and better ways to teach and reach students (Chernoff & Stone, 2014; Dowker et al., 2016; Schoenfeld, 2022).

It is critical to know that there are two distinctions to math anxiety handlings: prevention and reduction and there are distinct strategies and methods to address each is different ways. Furner (1996) synthesized three steps to prevent math anxiety:

- 1. Employing "best practice" in teaching math like using math manipulatives, cooperative learning, dialogue of math, enquiring and conjecturing, justifying one's thinking, math journaling, using a problem-solving approach to teaching, interdisciplinary instruction of content, emerging technology, assessment as an integral part of instruction, etc.
- 2. Incorporating National Council of Teachers of Mathematics and state/common core math standards into the curriculum and/or instruction; and lastly, the importance of discussing feelings, attitudes, and appreciation of mathematics with students.

Research by Furner (1996) also found that there are three approaches to reduce/lower math anxiety:

- (1) Psychological procedures such as math anxiety managing, desensitization, therapy, group support, bibliotherapy, and deliberations/conversations about experiences,
- (2) As a math anxious learner feels less fear and dread toward math, he/she might begin to build their self-confidence by taking more math classes and exposure to higher level math concepts, and
- (3) Most research on math anxiety reduction has shown that until a person with math anxiety has confronted this math anxiety by some form of discussion/counseling no "best practices" for teaching mathematics will help to overcome this fear of the subject.

Today math teachers during the school year while teaching mathematics should use some advantageous instructional methods, which are advocated now for teaching mathematics using concrete-representational-abstract (CRA) model for teaching mathematics, as follows: First educators need to start with concrete using hands-on manipulatives like geoboards, then secondly, they must move to representational models in diagrams (or use virtual manipulatives like NLVM at: <a href="http://nlvm.usu.edu/">http://nlvm.usu.edu/</a>), and lastly, connect to abstract symbolism, where student understand and function at an abstract level completely (GeoGebra software works well at: <a href="http://www.geogebra.org/cms/en/">http://www.geogebra.org/cms/en/</a>). CRA model is really the bases for the best practices pedagogy for teaching mathematics starting with young people but should also be used at all levels of math instruction (Figure 1).

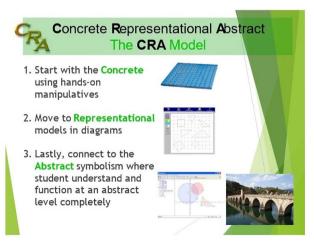


Figure 1. CRA model for teaching mathematics (Furner, 2023)

Jones (2012) found as discussed in her book, *Visualizing mathematics*, that it is essential that math teachers help students envision and create images of their mathematics comprehension so young people view math everywhere and as a large part of their being. Beilock and Willingham (2014) in their research have found that math teachers can help to address and reduce math anxiety. The author believes by using technology like GeoGebra along with the photography and children's books, teachers can make better connections and students are going to be more highly motivated to learn math (Furner & Marinas, 2014; Marinas et al., 2016).

## **Mathematics Anxiety Well-Defined**

Mathematics anxiety can be defined as a dread of math, which interferes with working with numbers and solving math problems or daily life experiences that involve numbers or mathematics. National Council of Teachers of Mathematics recognizes math anxiety as a problem and specifically included in its assessment practices. Standard #10 from National Council of Teachers of Mathematics prompts teachers to assess their students' mathematical dispositions; for example: self-confidence in employing math to solve word problems, communicate concepts, and reason mathematically. Math anxiety is often caused by a amalgamation of internal and external influences; nevertheless, educators cannot modify internal factors within the learner, so as teachers it makes sense to concentrate on what teachers can control (Chernoff & Stone, 2014). Math anxiety has been researched for almost fifty years and is a universal phenomenon, unfortunately not enough is being done today in schools to address it in how we approach teaching mathematics (Beilock & Willingham, 2014; Dowker et al., 2019; Geist, 2010). Poor dispositions toward math and carrying math anxiety around with you in life are grave impediments for students at all levels of schooling nowadays (Geist, 2010). Beilock and Willingham (2014) found in their study and summarized: "Because math anxiety is widespread and tied to poor math skills, we must understand what we can do to alleviate it" (p. 29). Poor teaching approaches are not the only cause of math anxiety. Because math anxiety can be seen in daily living activities as well as in class work or assignments, the need to have a multi-pronged approach is crucial to addressing it. Applying anxiety-reducing techniques in a multitude of activities and recurrently through instructional activities aids to address a variety of learner needs. This is like the idea of applying different management and organizations skills suited the situation. Research from Skagerlund et al. (2019) discovered that math anxiety can weaken math ability, they propose learners need to acquire approaches to bring about this so that it does not affect their working memory and number processing when they are doing math. Using different approaches before teaching a math activity allows the teacher to set a more focused and less anxious tone for a math learning using a variety of approaches to teaching. Employing procedures that lower anxiety and provide support just prior to beginning the math activity, as well as during the activity help signal to the anxious learner to a more positive approach to math schoolwork and the subject.

#### **Thwarting Math Anxiety in Students**

Today there are several approaches schools can follow to help thwart math apprehension. In cooperation educators and parents play an important part in aiding to nurture positive attitudes toward mathematics. In many intervention programs, early intervention, and action aid to foster positive mathematics dispositions. Today the math education field has made more of a push to increase and reassure math literacy in schools, and laterally with that drive has established valuable resources to boost math capability. Research by Mammarella et al. (2018) show how important it is as instructors to detach the math from the anxiety levels and in their investigational results showed that children with severe math anxiety, nonetheless through no changing dyscalculia were explicitly impaired in the hands-on interference task, while learners with developing dyscalculia (with or deprived of mathematics anxiety) botched in the working remembrance duties. The above research findings contend how critical it is to distinguishing between the cognitive processes underlying the profiles of a child, which can have factors as teachers report preventative and lessening strategies as it correlates to math anxiety levels. A successful program established by Southeastern Consortium for Minorities in Engineering (SECME) is in schools today for high minority populations to stimulate and grow students interested in math, science, and engineering fields. SECME was initially an acronym for Southeastern Consortium for Minorities in Engineering. The organization is based out of Atlanta, Georgia at the Georgia Institute of Technology. SECME is a deliberate coalition to recommence and fortify the professional expertise of K-12 educators, to motivate and counsel students, and invest parents so that all their children can learn and achieve at higher echelons (SECME, n. d.) Many teachers find this program very useful to turn young people on to math and motivate them to like the subject more. The grades K-8 school years are critical to instilling confidence and powerful attitudes toward math in young people. Deterrence of math anxiety in students is all about instructor planning and employing the best possible teaching strategies in mathematics instruction (dos Santos Carmo et al., 2019). The way math anxiety is fixed in our schools, to put it simply, is better teaching to reach all students. Research by Finlayson (2014) contend using a constructivist style of instruction, which emphasizes the following practices:

- Using whole group instruction first then
- · Quest for student queries and interests
- Key resources should be manipulative materials
- Learning should be interactive constructing and building on what learners already know
- Instructor interacts/negotiates with students
- · Evaluating students through observations, interview, tests, etc. The process is as critical as end result
- Knowledge is dynamic/change with experiences
- Students work in groups



Figure 2. Math manipulative for teaching mathematics (Furner, 2023)

## Lowering/Overcoming Math Anxiety & Building Confidence is Key

Math anxiety reduction is much different from the prevention of such anxiety. While every educator would like to prevent a student from experiencing math anxiety, some come to school afraid and worried about learning math. Many math educators contends that a person who suffers from math anxiety needs to first lay the groundwork by coming to terms with their feelings and challenge their present views and comprehend they are not unaccompanied; secondly, a person has to change their views and negative thought and use intervention approaches to progress one's discerning that they may be efficacious with math; thirdly, a person needs to know themself, it is critical that a person knows his/her learning style/approach and that he/she apply such methods to doing math by successful people; and lastly, when a student has increased confidence and approaches for undertaking math then they must apply what they learned and how they actually go about doing the mathematics. Additionally, the problem for those who suffer from math anxiety is the condition of anxiety itself. Research by Rubinsten et al. (2015) found apprehensive learners often focus on negative impetuses more than positive stimuli, essentially making themselves more uneasy. The equivalent is true of individuals with math anxiety; the one difference is that for individuals with math anxiety, math is the negative stimuli (Rubinsten et al., 2015). From this it is suggested that math anxiety may be remedied through treatments designed to lower anxiety, such as cognitive social therapies and exposure therapies (by exposing a someone incrementally to that which they are fearful like doing math) (Rubinsten et al., 2015).

Educators need to realize that there are important supportive techniques in a counseling setting when working with the math anxious. For example, some researchers (Ramirez et al., 2018) propose systematic desensitization as an effective approach for helping people reduce their math anxiety. Systematic desensitization in the framework of math anxiety, which can be a distinct and measured gradual exposure to math ideas that are producing students to develop anxiety and teaching learners how to manage such distress. When using systematic desensitization, a mutual practice in counseling today, learners come to understand that their math anxiety is a learned conditioned behavior, one they were not born with, and they can be trained to overcome it by constantly applying their self-monitoring strategies to become less math anxious. Some researchers advocate the use of relaxation in conjunction with repeated positive messages and visualizations to reduce math anxiety.

Operating from the academic perspective, Zemelman et al. (2012) summarize much evidence-based practices for teaching math, which include:

- (a) use of manipulatives (make learning math concrete) (Figure 2),
- (b) use cooperative group work,
- (c) use discussion when teaching math,
- (d) make questioning and making conjectures a part of math,
- (e) use justification of thinking,
- (f) use writing in math for: thinking, feelings, and prob. Solving,
- (g) use problem-solving approach to instruction; make content integration a part of instruction,
- (h) use of calculators, computers, and all technology,
- (i) being a facilitator of learning, and
- (j) assess learning as a part of instruction.

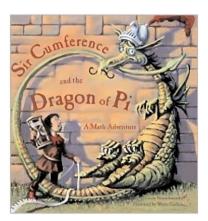
# HANDS-ON/MATH MANIPULATIVES & CONNECTING TO CHILDREN'S BOOKS & GEOGEBRA

"I think that children's literature offers a wonderful vehicle for helping teachers teach math well" (Marilyn Burns).

"If you want your children to be intelligent, read them fairy tales. If you want them to be more intelligent, read them more fairy tales" (Albert Einstein).



Figure 3. Children's literature books for teaching mathematics (Furner, 2023)



**Figure 4.** Book cover image for Sir Cumference and the Dragon of Pi (https://www.google.com/books/edition/Sir\_Cumference\_and\_the\_Dragon\_of\_Pi/3K6acVyi-MAC?hl=en)

As stated above it is important that math teachers read the children's book and fairy tales like *Sir Cumference and the Dragon of Pi, a math adventure* by Neuschwander (**Figure 3** and **Figure 4**), the son whose name is Radius (his title is referred to but not labelled in math terms) protects his father from dying, Sir Cumference, after unintentionally turning him into a fire breathing dragon. A math formula/equation is the solution to the problem here in this book. In the story Sir Cumference names Pi in this story. The story in the book tells how the math formula for  $\Pi$  was uncovered? The book offers a fictional story nevertheless the author's purpose is achieved when young people reading this book recall this math lesson and what Pi actually means.

#### Helping Young People Gain Math Confidence for a STEM World

We are living in an age of advancing technologies that are constantly changing. Children need to be literate as well as be very good at mathematics and problem solving in order to compete in a global society. A youngster's lack of confidence and ability to do mathematics may impact her entire life continually both in all choices they decide on a regular basis as well as forthcoming vocational choices. Educators in an age of STEM should be prepared to reach all learners and develop their confidence and ability to do mathematics so they can compete globally. Teachers today should check to see that all their students have positive attitudes and dispositions toward math (National Council of Teachers of Mathematics, 1989). It is very important to ensure our young learners are confident and well equipped in math in a STEM world if they are going to vie for such high-tech jobs now and in the future. Today, the USA and other countries are working to lead more young people into the fields of STEM so as countries all can better compete globally. Today it is critical teachers build math confidence in our students, educators need to directly address the matter of math anxiety as it manifests the issue as uncertainty or learned helplessness in observed math achievement. Many adults do not like mathematics. Sparks (2011) contends that as STEM fields become more significant for our young people to study, our schools and teachers need to do more to address math anxiety levels in learners, so our young people are confident in their ability to study fields associated to STEM areas.

Children's picture book like, *Math curse*, by Scieszka and Smith (1995) addresses the issue of math anxiety. The book is a great resource for educators to help anxious learners come to terms with the truth that not all individuals feel self-confident in their capability to do mathematics. The book commences with the teacher Mrs. Fibonacci, who says to her students that they can think of almost everything as a math problem, one student then starts thinking and worries and becomes overcome by the breath of math. His math anxiety then becomes a real curse, hence the title of the book, *Math curse*. Nevertheless, the character in the book ultimately comprehends that math is all around us and there is no way of avoiding it in daily life; therefore, the math anxious youngster in the story recognizes math as a means of making one's life easier. *Math curse* may be used as a form of bibliotherapy to prompt discussion on the topic of math anxiety and allow other students to discuss their feelings on the theme to comparisons to the character in the story. Isdell (2017) wrote another great book, *A gebra named Al*, about a young girl who struggles with her feelings toward math at the middle school level. This is also a wonderful book to incorporate in a bibliotherapy lesson to address math anxiety with students. Hebert and Furner (1997) feel that teachers need to take the time in their math instruction to address

such affective aspects of learning mathematics so that students can come to terms with their feelings toward mathematics. Sripatmi et al. (2023) conclude that using picture/children's books and such media help reach students better in teaching of math.

#### Mathematics Standards Currently as They Relate to Using GeoGebra

Most schools and states in the USA today are adhering to the new common core math standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), which can be found at: <a href="http://www.corestandards.org/">http://www.corestandards.org/</a>. When math teachers relate real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make significant correlations in mathematics, students can recognize that geometry/shapes and mathematics surround us. GeoGebra is an ideal piece of math software to use to teach many of common core or state standards today. Furner and Marinas (2016) offer many premade GeoGebra activities that match up with common core math standards and with many more resources at <a href="https://www.matharoundus.com">www.matharoundus.com</a> and there are also many GeoGebra resources for teachers at: geogebra.org. GeoGebra is great software that can even involve programming and as a technology on the computer like gaming can be a powerful aid to motivate learners.

Technological tools being used as part of instruction is critical in today's world of STEM. Young people need to learn to succeed in math at much higher levels of generality, represent and solve multifaceted problems, and emphasize decision-making and reasoning more (National Council of Teachers of Mathematics, 1995). National Council of Teachers of Mathematics (1995) believes that mathematical power can arise from technology, which includes increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future, this also connects with gamification aspects by employing technologies to motivate learners. As part of the President's Council of Advisors on Science and Technology (PCAST) by Holdren et al. (2010) they issued a policymaking report with explicit commendations to government leaders given to safeguard that the USA is a frontrunner in STEM education in the upcoming decades. One major recommendation is to recruit and train upwards of 100,000 new STEM middle and high school mathematics teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong STEM content-specific pedagogical preparation for such fields. PCAST believes that teachers are the most important factor to address in ensuring excellence in STEM education of future young people for our nation. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics, 2006; National Educational Technology Standards for Teachers, 2008), educators' ineffective usage of technology skills has been cited in the research.

Software like GeoGebra, is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics, and calculus into a streamlined and integrated software package. This free dynamic mathematics software that can be downloaded free and accessed immediately at: **Error! Hyperlink reference not valid.**. GeoGebra allows students and teachers the freedom to use it both within the classroom and while at home or on the go. GeoGebra has a large international user and developer community with users from 190+ countries it has currently been translated into 55 different languages.

GeoGebra can be employed to demonstrate how mathematical formulas/equations can be used in commonplace ways modeling math. Research by Aydin and Monaghan (2011) uncovered that math teachers must explore the potential for learners to view mathematics in the real world through coding mathematical features of digital pictures using a dynamic geometry program like GeoGebra software. Mathematics teachers may find the following videos (Mathematics and Multimedia, n. d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their math classrooms.

#### Math Manipulatives, Children's Literature Books, & GeoGebra Explained Usage

Using a hands-on approach and math manipulatives are common in most math lessons today in schools in the USA and around the world. Concrete manipulatives are critical for students to developing understanding of math concepts (Furner & Worrell, 2017). Moore and Rimbey (2021) found that math manipulatives help better connect the math ideas for better understanding. Iqbal et al. (2021) found that using math manipulatives had a positive impact on student achievement in learning mathematics. Larson and



Figure 5. Tangrams in GeoGebra, children's book, & manipulative lesson (Furner, 2023)

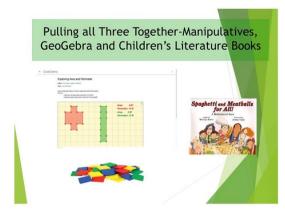


Figure 6. Color tiles with a book problem using GeoGebra (Furner, 2023)

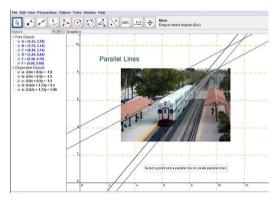


Figure 7. Examples of parallel lines in a photo with GeoGebra (Furner, 2023)

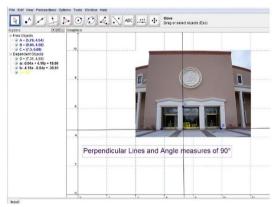


Figure 8. GeoGebra GGB demonstrates perpendicular lines on a photo (Furner, 2023)

Rumsey (2018) contend that children's literature in mathematics brings stories to life when teachers integrate literature and math manipulatives to make math lessons meaningful.

Figure 5 shows concrete tangrams, tangrams on GeoGebra, and a children's book using tangrams.

**Figure 6** provides color tile manipulatives to model a math problem in the book, *Spaghetti and meatballs for all* written by Marilyn Burns, there is also a GeoGebra model of the problem shown. Furner (2018) found that using children's literature to teach mathematics was an effective vehicle in better reaching all students. Washington (2023) in her review of teaching mathematics creatively discusses the importance of using technology like GeoGebra, manipulatives and virtual forms, storytelling and children's literature in the teaching of mathematics as some of the best pedagogy for teaching the subject in a creative manner to better reach students.

Mathematics teachers may ask why it is important to make connections and motivate students about learning math while using GeoGebra? Teachers will find that when using GeoGebra, educators will be able to demonstrate a purpose for mathematics; develop relations between math ideas and shapes and; this software will show practical applications to math in life; it employs innovative teaching in the classroom; it stimulates through photography/modeling; it employs emerging technologies in math with many real world application; GeoGebra can aid in addressing math anxiety as a motivating form of technology so that students feel confident for all STEM fields when they complete high school. **Figure 7** shows examples of parallel lines in a photo with GeoGebra.

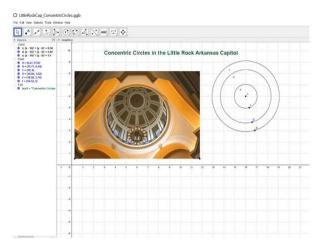


Figure 9. Circles & concentric circles in GeoGebra (Furner, 2023)

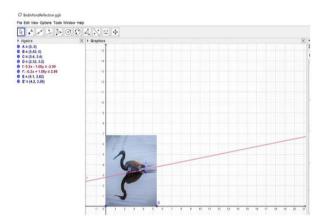


Figure 10. GeoGebra file showing photo & line of reflection (Furner, 2023)

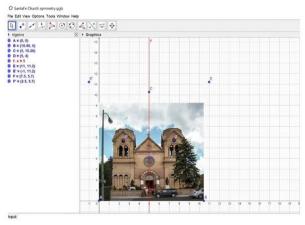


Figure 11. GeoGebra file of symmetry of photo of a building with symmetry (Furner, 2023)

**Perpendicular lines** create **right angles**, **90-degree angles**, and like in the GeoGebra file photo above (**Figure 8**) right angles and perpendicular lines are drawn on the photo with the GeoGebra software all allowing students to identify vocabulary and math ideas (Furner & Yahya, 2020).

The photo in **Figure 9** was imported into GeoGebra and then students were asked to draw **circles** and **concentric circles** like seen in the photo of the Little Rock Capitol building.

Reflections can often show up when taking photos of water, glass, or any other type of reflective surface. The photo above in **Figure 10** shows a photo of a **reflection** of a bird in water with a **line of reflection** draw in GeoGebra.

**Figure 11** shows a photo inserted into the GeoGebra software, a line was drawn through the center of the photo and then a **point** was selected and **reflected along the line** to show them as **symmetrical** to each other.

**Tessellating patterns** are patterns that repeat with the same **fundamental region** covering a space, with no **gaps** and no **overlaps** like seen in the chocolate bar rectangular pieces seen in **Figure 12**. Students can use the GeoGebra and move the pieces

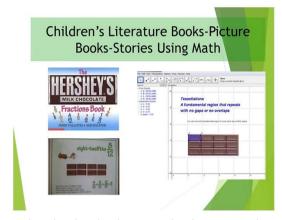


Figure 12. Photos of tessellations using chocolate bar, book, & GeoGebra (Furner, 2023)



Figure 13. GeoGebra designs by Daniel Mentrard Displaying Creativity at: https://www.youtube.com/watch?v=dioctXkvBrs

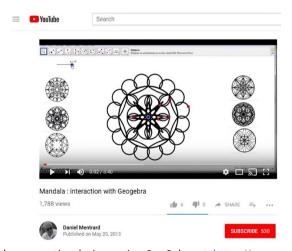


Figure 14. Daniel Mentrard's work shows creative designs using GeoGebra at: https://www.youtube.com/watch?v=g5-L\_J9lK3o

to test their hypothesis of the same shape repeating. By using GeoGebra and inserting photos into the software, students can then use the tools in GeoGebra to do the math, learn the vocabulary, and start better understanding mathematics concepts better.

Daniel Mentrard has published some very creative GeoGebra creations on YouTube (**Figure 13** and **Figure 14**), these are also wonderful examples of how by using this free software, people can be very creative in creating interactive and artistic and appealing images and designs creating while applying imagination and creativity to their work and investigations.

While GeoGebra is a wonderful piece of software to use to teach mathematics, help to ease math anxiety, it can also be used to promote creativity in the classroom and beyond as students can use it to create, design, and even makes games and it can lead to assisting with gamification for young people as they learn math in our STEM world. This also relates to aspects of gamification, where students are learning and employing math software as well as also encouraging more creativity as they learn and create games and designs like shown here (Furner, 2021). See other creative endeavors for using GeoGebra in **Figure 15** and **Figure 16**.

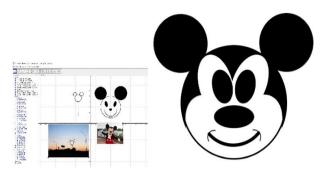
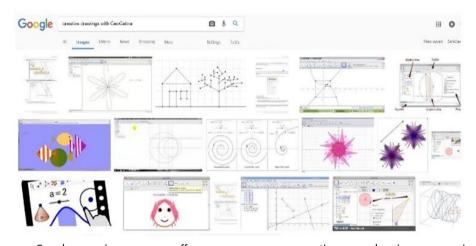


Figure 15. GeoGebra drawings from photos (Furner & Marinas, 2014)



**Figure 16.** Google images offers many creative drawings using GeoGebra (https://www.google.com/search?sca\_esv=a5e127585df5ae09&sca\_upv=1&sxsrf=ACQVn0-ltCyw\_Y7djYgPf-0VfMadczW7cg:1706660345806&q=Creative+drawings+in+GeoGebra&tbm=isch&source=lnms&prmd=ivhsnbmt&sa=X&ved=2ahU KEwjlxqedrYaEAxW-ZzABHZU6AlAQ0pQJegQIDhAB&biw=1536&bih=695&dpr=1.25)

## **SUMMARY OF FINDINGS**

Teachers of mathematics need to look deeper at their students' needs and address the math anxious students they have in their classrooms today to better prepare them for our high-tech world we now live in. As educators, we need to better prepare our young people for STEM, using technology, and having a strong and curious interest in mathematics. Math is best learned especially in the elementary levels when teachers use CRA model starting with concrete and manipulatives first. They need to make the connections to the representational through computer and GeoGebra with representational models and then getting to the abstract. Young learners will enjoy using technology like GeoGebra today while learning mathematics in a very meaningful and motivating way. Reading children's literature math books to students helps them see value and understanding of many math concepts in the real world with real world application. Math manipulatives, children's literature, and technology like GeoGebra are the keys to success for students when learning mathematics and it also better prepares them for a STEM world, where they are confident in their ability to do mathematics. Manipulatives can provide the concrete experiences for students, children's books offer great representational models of the math used in everyday life, and GeoGebra allows students an opportunity to use the math at abstract levels while employing the emerging technology. All of these today are considered best practices for teaching mathematics.

It is indeed an educators' responsibility to see that their students feel confident in their ability to do math and see a purpose for it in life, as ultimately a child's life: and all life decisions they will make, and vocations may be determined based on their temperament toward mathematics. As math teachers we must make the difference in our children's' feelings toward math while preparing them for a future with a greater STEM emphasis. In STEM world we live in now, it would be wonderful to see more young people when inquired on how they feel about mathematics say, "Math is my preferred course at school" or "I am great at geometry!" or "I can solve almost any word problem!" Math teachers today when employing all the "best pedagogical practices" and engaging math confidence building techniques in our schoolrooms today, educators and schools can produce more mathematically self-confident young people for the 21st century STEM world we now live in. Math manipulatives, children's math literature, and technology like GeoGebra are a large part of learning today and used to help cover math content as well to be a motivating factor in learning preparing our young people for a STEM world. These are some of the best pedagogical practices today for teaching mathematics to prepare young people for STEM.

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